

MODELING OF TWO-PHASE FILTRATION IN DEFORMABLE POROUS MEDIA

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Abstract: The paper considers the process of two-phase filtration in a porous medium, taking into account the compressibility of the medium. The process mathematically modeled and solved by the large particle method. The effect of changing the distance between oil wells on the filtration process has been investigated. According to the results obtained, the correct choice of the distance between oil wells has a positive effect on the oil production process.

AMS Subject Classification: 76T99

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1. Introduction

The movement of a fluid through a porous material can be influenced by the pressure gradient, which has the potential to alter the shape of the porous medium. Conversely, if the medium is compressed, it can induce the movement of the internal fluid. This compression modifies the characteristics of the material, including its ability to conduct fluids. The compression also leads to a reduction in porosity and an increase in resistance to fluid flow. Consequently, there exists a intricate relationship between the deformation of a solid body [21].

The study of how porous media deform and fluid flows within them is the fundamental concept behind poroelasticity theory. This theory holds great sig-

nificance in various geophysical, engineering, and industrial applications [8]. For instance, it helps understand phenomena like soil liquefaction during earthquakes, gravity-driven events such as rapid landslides and mudflows [4, 12], erosion of land due to heavy rainfall and floods, damage to man-made structures caused by flooding [4], oil and gas extraction [11, 20], oil refining [4, 13], and carbon dioxide storage in geological formations [4, 12] within the energy field.

Different models have been developed to address the deformation of the porous medium, each employing distinct approaches. In the explicit coupled approach [3, 15], the simulation involves conducting multiphase fluid flow calculations for each time step and subsequently performing geomechanical deformation calculations at selected intervals. The frequency of geomechanical updates depends on the magnitude of pore volume changes. On the other hand, the iteratively coupled approach [15] involves coupling multiphase flow and deformation at each time step through a nonlinear iterative process. By adhering to the constraints of iteration, this approach can yield fully convergent outcomes [19]. In the fully integrated approach [16], fluid flow and deformation are simultaneously solved using a set of unified equations [18].

When a medium undergoes deformation, its properties undergo changes, especially through compaction, which results in a decrease in porosity. Consequently, the reduction in fluid pathways causes a decline in permeability [9, 21]. Therefore, it becomes essential to incorporate the correlation between conductivity and tissue tension into the primary equations [17]. The relationship between the pressure field and conductivity is represented by linear and exponential dependencies in [1], while the dependencies that describe the relationship with porosity are presented in [7].

The paper [10] proposed the Multiphase Darcy-Brinkman-Biot (DBB) approach, which is a comprehensive and interconnected model for simulating two-phase flow in deformable porous media. This model is based on the micro-continuum approach, which is derived using the method of volume averaging and fundamental physical principles. The study demonstrates that a unified set of partial differential equations can be employed to represent multiphase flow in porous media, regardless of whether the pore-scale, continuum-scale, or hybrid-scale representation is used.

In [7], the mathematical model describing the movement and interaction of particles in a multicomponent plasma is numerically solved, focusing on the electric arc synthesis of carbon nanostructures. The large particle method is utilized to carry out the computations. By employing this method, the computational workload and hardware requirements are reduced without compromising the accuracy of the numerical calculations. In [10], the stability of numeri-

cal schemes based on the modified "large particles" method is investigated for non-stationary, non-isothermal multi-phase filtration processes. The stability criteria are considered as a whole for the modified "large particles" method scheme.

This article investigates the effect of changing the distance between two wells in an oil reservoir on a two-phase filtration process in a deformable porous medium. A mathematical model suitable for the process was created and numerically solved.

2. Statement of the problem

Let us consider a reservoir where two oil wells have been opened. We assume that oil and water phases are involved in the filtration process in a deformable porous medium.

To describe the process, we will use a one-dimensional mathematical model of two-phase filtration in a deformable porous medium. Here we consider that oil phase as first phase, and water phase as second phase.

First phase mass conservation equation

$$\frac{\partial}{\partial t}(m\rho_1 s_1) + \frac{\partial}{\partial x}(\rho_1 u_1) = 0, \quad (1)$$

where s_1 is the saturation of the first phase, ρ_1 is the density of the first phase, m is the porosity, u_o is the velocity of the first phase.

Second phase mass conservation equation

$$\frac{\partial}{\partial t}(m\rho_2 s_2) + \frac{\partial}{\partial x}(\rho_2 u_2) = 0, \quad (2)$$

where s_2 is the saturation of the second phase, ρ_2 is the density of the second phase, u_2 is the velocity of the second phase.

Assuming $\rho_1 = \text{const}$, $\rho_2 = \text{const}$, we rewrite equation (1) - (2) in the following form.

First phase mass conservation equation

$$\frac{\partial}{\partial t}(ms_1) + \frac{\partial}{\partial x}(u_1) = 0. \quad (3)$$

Second phase mass conservation equation

$$\frac{\partial}{\partial t}(ms_2) + \frac{\partial}{\partial x}(u_2) = 0. \quad (4)$$

The phase filtration rates will be determined according to the Darcy's law

$$u_1 = -\frac{K k_1}{\mu_1} \frac{\partial p}{\partial x}, \quad u_2 = -\frac{K k_2}{\mu_2} \frac{\partial p}{\partial x}, \quad (5)$$

where K, k_1, k_2 are the absolute and relative phase permeability, μ_1 is the oil viscosity, μ_2 is the water viscosity, p is the pressure.

We will take into account the change in porosity in the same way as in [14, 19]

$$m = m_0 + \beta_m (p - p_0), \quad (6)$$

where p_0 is a fixed pressure, m_0 is the porosity coefficient at $p = p_0$, β_m is the reservoir elasticity coefficient.

It has been experimentally shown that not only porosity, but also permeability change significantly with changes in reservoir pressure [1]

$$K = K_0 (1 - a_K (p_0 - p)). \quad (7)$$

Let us add to the system of equations the obvious equality

$$s_1 + s_2 = 1, \quad (8)$$

dependencies for viscosities

$$\mu_1 = \text{const}, \mu_2 = \text{const}, \quad (9)$$

and relative phase permeabilities [1]

$$k_1 = \begin{cases} \left(\frac{0.85-s_2}{0.8}\right)^{2,8} \cdot (1 + 2.4 \cdot s_2) & 0 \leq s_2 \leq 0.85, \\ 0 & 0.85 \leq s_2 \leq 1, \end{cases} \quad (10)$$

$$k_2 = \begin{cases} 0 & 0 \leq s_2 \leq 0.85, \\ \left(\frac{s_2-0.2}{0.8}\right)^{3.5}, & 0.85 \leq s_2 \leq 1. \end{cases} \quad (11)$$

To derive an equation for pressure, we sum equations (3)-(4). As a result, we get

$$\frac{\partial}{\partial t} [m (s_1 + s_2)] + \frac{\partial}{\partial x} (u_1 + u_2) = 0.$$

Using expressions (6) and (8), we get the following

$$\frac{\partial m}{\partial t} = \frac{\partial}{\partial t} (m_0 + \beta_m (p - p_0)) = \beta_m \frac{\partial p}{\partial t}.$$

Finally, the pressure equation will take the form

$$\beta_m \frac{\partial p}{\partial t} = \frac{\partial}{\partial x} \left[\left(\frac{Kk_1}{\mu_1} + \frac{Kk_2}{\mu_2} \right) \frac{\partial p}{\partial x} \right]. \quad (12)$$

By setting the initial

$$p(x, 0) = p^0, \quad s_1(x, 0) = s_1^0, \quad s_2(x, 0) = s_2^0, \quad (13)$$

and boundary conditions

$$\left. \frac{\partial p}{\partial x} \right|_{x=0} = 0, \quad p(x_1, t) = p_1^*, \quad p(x_2, t) = p_2^*, \quad \left. \frac{\partial p}{\partial x} \right|_{x=L} = 0, \quad (14)$$

where p^0 is initial pressure, s_1^0, s_2^0 are initial saturations of the first and second phases, x_1 is the coordinate of the first well, x_2 is the second well coordinate, L is the length of filtration area, p_1^* is the bottomhole pressure of the first well, p_2^* is the bottomhole pressure of the second well. System (3)-(14) is a closed system of equations that describes two-phase filtration in a deformable porous medium.

3. Solution method

To solve the problem (3)-(14), we use the large particle method [2, 5].

We introduce the Euler grid for solving the system of nonstationary equations (3)-(14),

$$\Omega_{th} = \{t_{j+1} = t_j + \tau, \quad j = \overline{1, T}; \quad x_{i+1} = x_i + h, \quad i = \overline{1, N}\}.$$

The medium is modeled by a system of fluid particles corresponding to a given moment of time with an Euler grid cell. Let us divide the calculation of each time step into two stages [2, 6].

Step 1. We ignore the effects associated with the shift of the elementary cell. We approximate equations (3) - (14) in a discrete space:

$$\begin{aligned} \beta_m \frac{p_i^k - p_i^{k-1}}{\tau} = & \frac{1}{h^2} \left\{ \left[K \left(\frac{k_1}{\mu_1} + \frac{k_2}{\mu_2} \right) \right]_{i+\frac{1}{2}}^k \cdot p_{i+1}^k \right. \\ & - \left\{ \left[K \left(\frac{k_1}{\mu_1} + \frac{k_2}{\mu_2} \right) \right]_{i+\frac{1}{2}}^k + \left[K \left(\frac{k_1}{\mu_1} + \frac{k_2}{\mu_2} \right) \right]_{i-\frac{1}{2}}^k \right\} p_i^k \\ & \left. + \left[K \left(\frac{k_1}{\mu_1} + \frac{k_2}{\mu_2} \right) \right]_{i-\frac{1}{2}}^k \cdot p_{i-1}^k \right\}, \end{aligned} \quad (15)$$

$$A_i \cdot p_{i-1}^{\sim} - B_i \cdot p_i^{\sim} + C_i \cdot p_{i+1}^{\sim} = -F_i, \quad (16)$$

where

$$\begin{aligned} A_i &= \frac{\tau}{\beta_m h^2} \left[K \left(\frac{k_1}{\mu_1} + \frac{k_2}{\mu_2} \right) \right]_{i-\frac{1}{2}}^k, \\ B_i &= 1 + \frac{\tau}{\beta_m h^2} \left\{ \left[K \left(\frac{k_1}{\mu_1} + \frac{k_2}{\mu_2} \right) \right]_{i-\frac{1}{2}}^k + \left[K \left(\frac{k_1}{\mu_1} + \frac{k_2}{\mu_2} \right) \right]_{i+\frac{1}{2}}^k \right\}, \\ C_i &= \frac{\tau}{\beta_m h^2} \left[K \left(\frac{k_1}{\mu_1} + \frac{k_2}{\mu_2} \right) \right]_{i+\frac{1}{2}}^k, \\ F_i &= p_i^k, \end{aligned}$$

we solve the system of linear equations (19) by the Thomas' algorithm [6].

The Thomas algorithm coefficients are determined by the following formulas

$$\begin{cases} \alpha_{i+1} = \frac{C_i}{B_i - A_i \alpha_i}, & i = \overline{1, N}, \\ \beta_{i+1} = \frac{F_i + A_i \beta_i}{B_i - A_i \alpha_i}, & i = \overline{1, N}. \end{cases} \quad (17)$$

We find the pressure by the found coefficients as follows

$$p_i^{\sim} = \alpha_{i+1} \cdot p_{i+1}^{\sim} + \beta_{i+1}. \quad (18)$$

From the boundary conditions, we calculate the initial values of the conductivity coefficients and the pressure values in the wells and at the end of the field at each time step in the following form.

$$\begin{aligned} \alpha_1 &= 1, \quad \beta_1 = 0, \\ p_N^k &= \frac{\beta_N}{1 - \alpha_N}, \\ p_{N_1}^k &= p_1^*, \quad p_{N_2}^k = p_2^*. \end{aligned}$$

We determine the filtration rate in points according to the values of the found pressure at the nodes

$$(u_1)_{i+\frac{1}{2}}^{\sim} = - \left(\frac{K k_1}{\mu_1} \right)_i \frac{p_i^{\sim} - p_{i+1}^{\sim}}{h}, \quad (u_2)_{i+\frac{1}{2}}^{\sim} = - \left(\frac{K k_2}{\mu_2} \right)_i \frac{p_i^{\sim} - p_{i+1}^{\sim}}{h}.$$

Step 2. Find the values of porosity, saturation and permeability at each time step:

- porosity

$$m_i^{k+1} = m_0 + \beta_m (p_i^\sim - p_0),$$

- saturation of the first and second phases

$$(s_1)_i^{k+1} = \frac{1}{m_i^{k+1}} \left[m_i^{k+1} (s_1)_i^k + \frac{(u_1)_{i-1/2}^\sim - (u_1)_{i+1/2}^\sim}{h} \tau \right],$$

$$(s_2)_i^{k+1} = 1 - (s_1)_i^{k+1},$$

- relative phase permeabilities

$$(k_1)_i^{k+1} = \begin{cases} \left(\frac{0.85 - (s_2)_i^{k+1}}{0.8} \right)^{2.8} \cdot \left(1 + 2.4 \cdot (s_2)_i^{k+1} \right), & 0 \leq (s_2)_i^{k+1} \leq 0.85, \\ 0 & 0.85 \leq (s_2)_i^{k+1} \leq 1, \end{cases}$$

$$(k_2)_i^{k+1} = \begin{cases} 0 & 0 \leq (s_2)_i^{k+1} \leq 0.85, \\ \left(\frac{(s_2)_i^{k+1} - 0.2}{0.8} \right)^{3.5}, & 0.85 \leq (s_2)_i^{k+1} \leq 1, \end{cases}$$

- absolute permeability

$$K_i^{k+1} = K_0 (1 - a_K (p_i^\sim + p_0)).$$

4. Results and discussion

In computational experiments to study the stress-strain state of a saturated porous medium, the following parameter values were used: $m_0 = 0.3$, $K_0 = 4.8 \cdot 10^{-15} \text{ m}^2$, $(s_1)_0 = 0.6$, $(s_2)_0 = 0.4$, $p_0 = 10^6 \text{ Pa}$, $p^0 = 10^6 \text{ Pa}$, $p^* = 5 \cdot 10^5 \text{ Pa}$, $\mu_1 = 1.3 \cdot 10^{-3} \text{ Pa} \cdot \text{s}$, $\mu_2 = 1.768 \cdot 10^{-3} \text{ Pa} \cdot \text{s}$, $\beta_m = 5 \cdot 10^{-9} \text{ Pa}^{-1}$.

The calculation results are presented graphically in Figures 1-4.

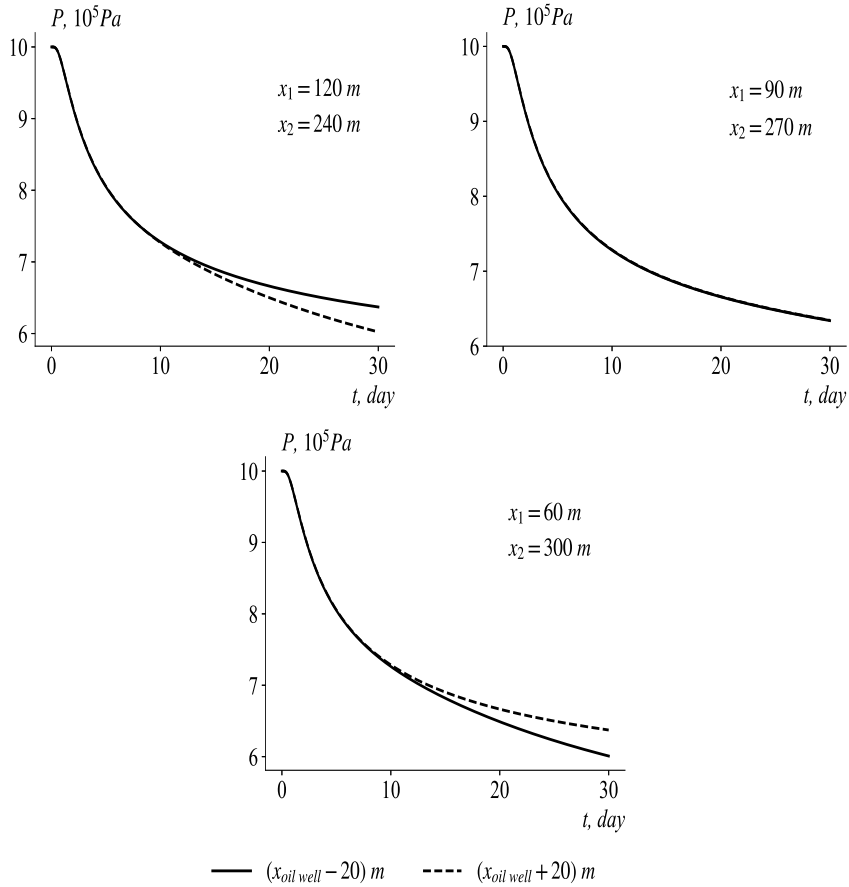


Fig 1. Dynamics of the influence of the distance between wells on the pressure field.

Figure 1 shows the dynamics of reservoir pressure changes at selected points 20 meters from the left and right sides of the first oil well at different distances between two oil wells. It can be seen from the graph that at close distances between wells ($x_1 = 120m, x_2 = 240m$) reservoir pressure decreases more in the inter-well zone, than symmetrical distances between wells ($x_1 = 90m, x_2 = 270m$) on both sides of the oil well, it can be seen that the reservoir pressure decreased uniformly, and with increasing distance between wells ($x_1 = 60m, x_2 = 300m$), on the contrary, reservoir pressure decreased less in the inter-well zone.

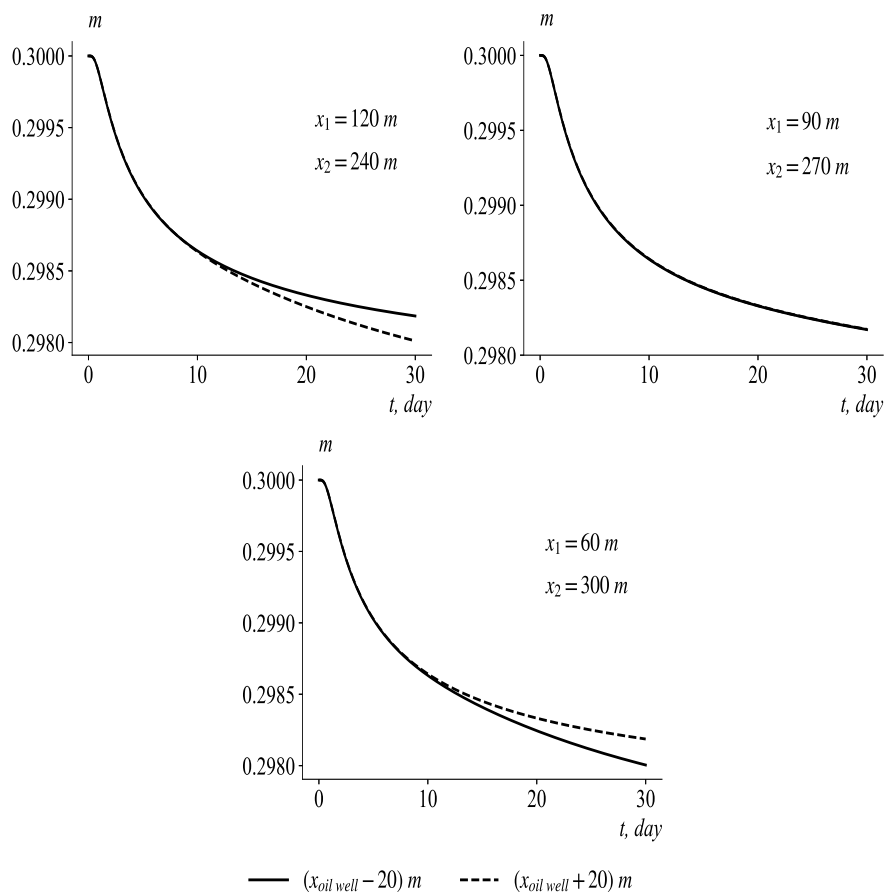


Fig 2. Dynamics of the influence of the distance between wells on porosity.

Figure 2 shows the dynamics of changes in the porosity of the medium at points selected 20 meters from the left and right sides of the first oil well at different distances between two oil wells. According to the graphs, at a distance between wells of 120 m, it can be seen that the porosity of the medium decreases faster than with symmetrical distances between wells, and at a distance of 240 m, the porosity of the medium decreases more slowly in the interwell zone. $K, 10^{-3}\text{Da}$.

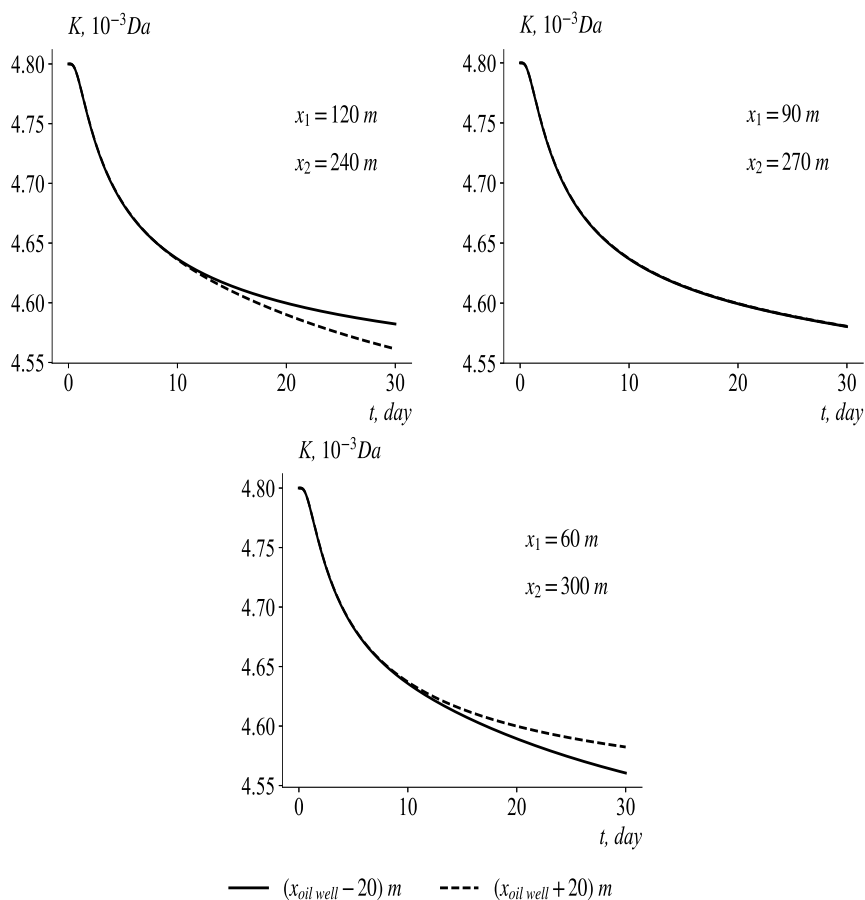


Fig 3. Dynamics of the influence of the distance between wells on permeability.

Figure 3 shows the dynamics of changes in the absolute permeability of the medium at points selected at a distance of 20 m from the left and right sides of the oil well at different distances between two oil wells (120 m, 180 m, 240 m). The graphs show that the absolute permeability of the medium in the interwell zone decreases faster at a close distance between the wells, and decreases more slowly at a far one.

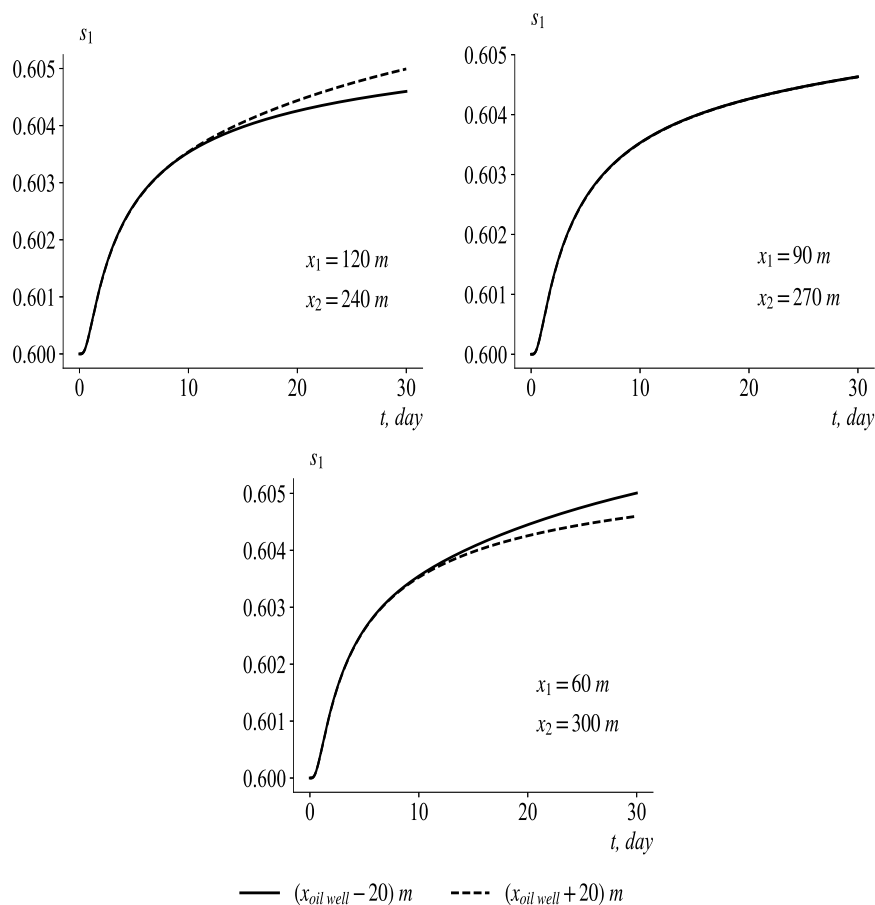


Fig 4. Dynamics of the influence of distance between wells on saturation.

Figure 4 shows the dynamics of changes in oil saturation at different distances between two oil wells. We see that oil saturation increased faster as the interwell zone narrowed and increased more slowly as the interwell zone expanded.

Conclusions. This study considers a two-phase filtration process in a deformable porous medium at various distances between two oil wells. A suitable mathematical model for the process was created and solved by the large particle method. According to the obtained results, decreasing in the distance between the oilwells, brings to the faster decrease in the reservoir pressure, porosity and permeability than the symmetrical distance between the oil-wells, when the distance between the oil-wells increases, the decreasing in the values of these hydrodynamic indicators slows down. Conversely, it has been observed that oil

saturation increases faster in the interwell zone as the distance between wells decreases and increases slowly as the distance between the oil-wells increases.

References

- [1] K.S. Basniev, I.N. Kochina, V.M. Maksimov, *Underground Hydromechanics*, Nedra, Moscow (1993).
- [2] O.M. Belotserkovsky, Yu.M. Davydov, *Method of Large Particles in Gas Dynamics*, Nauka, Moscow (1982).
- [3] D. Bevillon, R. Masson, Stability and convergence analysis of partially coupled schemes for geomechanics reservoir simulations, In: *7th Eur. Conf.*, Italy, Math. Oil Recovery, Baveno (2000).
- [4] H.H. Bui, G.D. Nguyen, A coupled fluid-solid SPH approach to modelling flow through deformable porous media, *International Journal of Solids and Structures*, **125** (2017), 244-264.
- [5] G.T. Bulgakova, A.V. Baizigitova, A.R. Sharifullin, Model of matrix acid treatment of carbonates: influence of sediment on the dissolution process, *Ufa : UG ATU. T.*, **13** (2009), 256-264.
- [6] V.F. Burnashev, U.A. Nazarov, Attenuation of detonation waves in layers of homogeneous and inhomogeneous monodisperse gas suspension in sharply expanding pipes *International Journal of Applied Mathematics* **36**, No 2 (2023), 281-290; DOI: 10.12732/ijam.v36i2.7.
- [7] V.F. Burnashev, K.K. Viswanathan, Z.D. Kaytarov, Mathematical modeling of multiphase filtration in a deformable porous medium, *Computation*, **112**, No 11 (2023).
- [8] E. Detournay, A. Cheng, Fundamentals of poroelasticity. Analysis and design methods, *Chapter of Analysis and Design Method*, **2** (1993), 113-171.
- [9] B. Fayziyev, A phenomenological model of suspension filtration in porous medium, *International Journal of Applied Mathematics*, **33**, No 3 (2021), 511-521; DOI: 10.12732/ijam.v33i3.10.
- [10] J.C. Francisco, I.C. Bourg, Modeling multiphase flow through and around multiscale deformable porous materials, *Advancing Earth and Space Science*, **57** (2021), e2020WR028734; DOI: 10.1029/2020WR028734.

- [11] H. Ghasemzadeh, Multiscale multiphysic mixed geomechanical model for deformable porous media considering the effects of surrounding area, *Journal of Petroleum Geomechanics*, **3** (2019), 79-99.
- [12] D. Hewitt, J. Nijjer, M. Worster, J. Neufeld, Flow-induced compaction of a deformable porous medium, *Physical Review E*, **93** (2016); DOI: 10.1103/PhysRevE.93.023116.
- [13] E.M. Khramchenkov, M.G. Khramchenkov, Mathematical model of multiphase nonisothermal filtration in deformable porous media with a simultaneous chemical reaction, *Journal of Engineering Physics and Thermophysics*, **93** (2020), 191-200.
- [14] J.M. Makhmudov, A.I. Usmonov, J.B. Kuljanov. Problem of anomalous filtration in nonhomogeneous porous medium, *International Journal of Applied Mathematics*, **36**, No 2 (2023), 189-203; DOI: 10.12732/ijam.v36i2.4.
- [15] S.E. Minkoff, C.M. Stone, S. Bryant, M. Peszynska, M.F. Wheeler, Coupled fluid flow and geomechanical deformation modeling, *J. Pet. Sci. Eng.*, **38** (2003), 37-56.
- [16] K. Morgan, R.W. Lewis, I.R. White, The mechanisms of ground surface subsidence above compacting reservoirs and their analysis by finite element method, *Appl. Math. Modell.*, **4** (1980), 217-224.
- [17] V.C. Mow, J.M. Mansour, The nonlinear interaction between cartilage deformation and interstitial fluid flow, *J. Biomechanics*, **10** (1977), 31-39.
- [18] S.A. Sadrnejad, H. Ghasemzadeh, E. Taheri, Multiscale multiphysic mixed geomechanical model in deformable porous media, *J. for Multiscale Computational Engineering*, **12**, No 6 (2014), 529-547.
- [19] A. Settari, D.A. Walters, Advances in coupled geomechanical and reservoir modeling with applications to reservoir compaction, *Paper presented at SPE Reservoir Simulation Symposium Houston TX* (1999).
- [20] A.I. Sukhinov, L.A. Grigoryan, A.A. Sukhinov, Mathematical modeling of filtration of a two-phase compressible fluid based on the modified adaptive method of minimum corrections, *Informatics, Computer Science and Control*, **86** (2016), 96-109.
- [21] Y. Xiong, H. Xu, Y. Wang, W. Zhou, L. Wang, K. Feng, The variation mechanism of petrophysical properties and the effect of compaction on the

relative permeability of an unconsolidated sandstone reservoir, *Marine and Petroleum Geology*, **92** (2018), 754-763.