**International Journal of Applied Mathematics**

**Volume 36 No. 3 2023, 335-355**
ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)
**doi:** http://dx.doi.org/10.12732/ijam.v36i3.3

# $T$ AND $ST$-COLORING OF CORONA PRODUCT AND EDGE CORONA PRODUCT OF GRAPHS

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**Abstract:** When coloring a graph's vertices, a particular technique known as $T$-coloring is used to ensure that the absolute difference between the colors allocated to the end vertices of any edge will not be an element of a predetermined set $T$ of non-negative integers that includes zero. A type of $T$-coloring of a graph known as $ST$-coloring is one in which there is a noticeable absolute difference between the assigned colors of each edge's end vertices. Here, we discuss these colorings on the Corona product and the edge corona network of graphs. We obtain a few findings on the chromatic numbers associated with the $T$ and $ST$-colorings, as well as the span and edge span of these graph products.

**AMS Subject Classification:** 05C15, 05C76
**Key Words:** $T$-coloring, corona product, edge corona product

Received: December 30, 2022 

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## 1. Introduction

Graph colouring is one of the most well-known, often utilized, and in-depth studied research topics in the field of graph theory. Graph colouring is used in diverge research areas such as computer science, electronics, construction projects, and other engineering undertakings. The development of wireless communication has led to the emergence of the channel assignment problems [3]. Transmitters using particular frequencies interfere in a Channel assignment problem, because of distance restrictions. Two interfering transmitters must be assigned frequencies that do not belong to a prohibited set of non-negative integers in order to prevent this type of interference. In order to conserve spectrum, effective channel assignments are necessary. This Channel assignment problem was first developed in a graph theoretic way by Hale [6], which was named as $T$-coloring problem. He denoted the transmitters as the vertices of the graph, with an edge developing between two vertices if the matching transmitters clashed. $T$-coloring of a graph is defined as a map that distributes non-negative numbers to the vertices in such a way that the absolute values of the differences between the non-negative integers allocated to the vertices of an edge must not belong to a finite set $T$. The $T$-set can vary for every interfering transmitter. In such case, Roselin et al. [17] defined a version of $T$-coloring called as $ST$-coloring of graphs. This variant of $T$-coloring is defined with a strong requirement added to $T$-coloring that for every distinct edge of the graph, the absolute values of the differences between the non negative integers assigned to the vertices of an edge must be distinct.

Graph products provide tractable investigation of many graph characteristics. In recent years, the Cartesian product, [13] hierarchical product, Kronecker product, [7], and others have been employed to emulate complex networks. The Corona Product [5] and Edge Corona Product (or Edge Corona network) [8] are also helpful for creating a big graph from smaller ones. They are new in mathematics. Since they are still evolving, they require further work. Even if many aspects of the concept are currently unclear, its potential to answer future riddles cannot be disregarded. They are employed in arithmetic, biology, social science, chemistry, and even astronomy. Researchers are fascinated by graph operations and their products. Researchers are intrigued to the Corona product and Edge Corona product of one graph on itself because they make clones of the same graph, generating a vast structure comparable to a giant star. Scholars are fascinated by graph operations like product. Corona product and Edge Corona product or network architectures excite researchers more than other graph operations. It is interesting to study this graph's structure and applications. Formal definitions of Corona product and Edge Corona product of graphs are discussed in the next section.

Over two decades have been spent researching the $T$-coloring problem. Hale [6] proposed it initially by expressing a number of frequency assignment issues in terms of graph theory. The work of Cozzens and Roberts [4] was expanded upon by Cozzens and Wang [3], and Tesman [21] by taking into account different frequency interference limitations. The $T$-span and $T$-edge span of the swing graph, theta mesh, and shadow graph of a cycle, as well as the crown graph, series-parallel graph, generalised theta graph, and wrapped butterfly network were estimated by Sivagami [19, 20] in 2016 and 2018. Recently, Roselin and Raj [16] provided a variety of insights relating to the span, edge span, and chromatic numbers relating to $T$-coloring of wheel graphs and by using these findings, they obtained various results relating to these parameters of some non-perfect graphs such as Petersen graphs, Double Wheel graphs, Helm graphs, Flower graphs, and Sun Flower graphs in [15]. T-coloring on a number of graph operations, including Union, Join, Cartesian product, and Tensor product, was studied by Roselin et al. [14] in 2022. The $ST$-coloring of numerous graph operations, including Union, Join, Cartesian product, Tensor product, and Corona product of graphs, was studied by Moran et al. [12, 13].

The corona product of two graphs was first developed by Frucht and Harary [5] in 1970. In [9], et al. investigated the star edge coloring of the Corona product of Path and some other graphs and discovered the associated chromatic numbers. Then in 2017 [11], total coloring of corona product of two graphs $G$ and $H$, was studied by Mohan, in which they assumed $H$ as a cycle, a complete graph or a bipartite graph. In 2020, Lu et al. [10] investigated the acyclic chromatic numbers that correspond to the corona product of paths or cycles and any simple graph. In the same year, Singh et al. [18] reviewed an article on Corona product of graphs. After that the local edge antimagic colouring of the path and cycle corona product, also known as the path corona cycle, cycle corona path, path corona and cycle corona cycle, was presented by Aisyah et al. [1] in 2021.

Edge corona product was first proposed by Haynes and Lawson [8]. In 2020, Wang et al. [22] studied Edge corona product as an approach to modeling complex simplical networks. In the study, they suggested a simple model for complex networks termed simplical networks that may describe group interactions in real networks, denoted by a parameter, by repeatedly applying the edge corona product. Then, they gave a thorough examination of the model's topological features that were pertinent, the majority of which were parameter-dependent. They demonstrated that the generated networks exhibit a number of striking features of actual networks, including non-trivial higher-order interaction, a power-law distribution of vertex degree, a small diameter, and a high clustering coefficient. In 2020, Lu et al. [10], et al. investigated the acyclic chromatic numbers that correspond to the Edge Corona product of paths or cycles and any simple graph.

Organization of the paper: In Section 2, some basic definitions which will be presented which will be used in the sections to follow. Section 3 contains two theorem on Chromatic numbers of the Corona product and Edge Corona product of two graphs and Section 4 contains a study on $T$-coloring of Corona Product and Edge Corona product of graphs. Similarly, in Section 5 a study on $ST$-coloring of Corona Product and Edge Corona product of graphs is presented, and finally in Section 6 a conclusion is provided on the whole work.

## 2. Preliminaries

The following definitions will be used in the sections to follow.

**Definition 1.** **$c_T$-span** ([20]): $c_T$-span, $sp^c_T(G)$ is the maximum value $\mid c(u) - c(w) \mid$ over all the vertices and the minimum of $sp^c_T(G)$ is known as $sp_T(G)$, where the minimum is taken over all $T-$coloring $c$ of $G$.

**Definition 2.** **$c_T$-edge span** ([20]): The $c_T-$edge span, $esp^c_T(G)$ is the maximum value $\mid c(u) - c(w) \mid$ over all the edges $(u, w)$ and the minimum of $esp^c_T(G)$ is known as $esp_T(G)$, where the minimum is taken over all $T$-coloring $c$ of $G$.

**Definition 3.** **Corona Product** ([5]): Consider two graphs with $m$ vertices and $p$ edges and, $n$ vertices and $q$ edges, respectively, as $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$. The graph produced by taking one copy of $G_1$ and $m$ copies of $G_2$ and attaching the $i^{th}$ vertex of $G_1$ to each vertex in the $i^{th}$ copy of $G_2$ is known as the corona product of graphs $G_1$ and $G_2$. Then $\mid V(G_1 \circ G_2) \mid = m(n + 1)$ and $\mid E(G_1 \circ G_2) \mid = p + nq + mn$.

**Definition 4.** **Edge Corona product** ([8]): Let $G_1$ and $G_2$ be initial graph with order $m$ and size $p$, $G_2$ be the graph with order $n$ and size $q$ respectively. The edge corona $G_1 \diamond G_2$ of $G_1$ and $G_2$ is generated by making one copy of $G_1$ and $p$ copies of $G_2$, identifying each vertex of the $i^{th}$ copy of $G_2$ to the two end vertices of the $i^{th}$ edge of $G_1$.

## 3. Chromatic numbers of the Corona product and Edge Corona product of two graphs

Let $V(G_1) = \{v_1, v_2, v_3, ...., v_m\}$ and $V(G_2) = \{v_{m+1}, v_{m+2}, v_{m+3}, ...., v_{m+n}\}$. Now, let us rename the vertices of $G_1$ after Corona product with $G_2$ as $\{v_{0j} : j = 1, 2, 3, ...., m\}$ and the vertices of $i^{th}$ copy of $G_2$ as $\{v_{ij} \mid j = m+1$ to $m+n\}$, where $i$= 1 to $m$. Similarly, let us rename the vertices of $G_1$ after edge corona product with $G_2$ as $\{v_{00k} : k = 1, 2, 3, ...., m\}$ and the vertices of the copy of $G_2$ corresponding to the edge $(v_{00i}, v_{00j})$ as $\{v_{ijk} \mid k = m+1$ to $m+n\}$, in which $v_{ijk} = v_{jik}$ and $i, j$= 1 to $m$.

Now we shall discuss the chromatic numbers of the Corona product and Edge Corona product of two graphs respectively.

**Theorem 5.** *For any two graphs $G_1$ and $G_2$,*

$$\chi(G_1 \circ G_2) = \begin{cases} \max\{\chi(G_1), \chi(G_2)\} & \textit{if } \chi(G_1) \neq \chi(G_2) \\ \chi(G_1) + 1 & \textit{if } \chi(G_1) = \chi(G_2) \end{cases}.$$

*Proof.* Let $G_1$ and $G_2$ be two graphs with the chromatic numbers $\chi(G_1)$ and $\chi(G_2)$ respectively, and $c$ be the coloring function defined on the vertex set of $G_1 \circ G_2$, which assigns colors to all the vertices of $G_1 \circ G_2$.

**Case I: $\chi(G_1) > \chi(G_2)$.** Then all the vertices $v_{0j}s$ can be colored with $\chi(G_1)$ colors. Now since the vertex $v_{0j}$ is adjacent to all the vertices of $j^{th}$ copy of $G_2$. Then all the vertices of $j^{th}$ copy of $H$ can be colored by the same set of colors, which were assigned to $v_{0j}$s excluding $c(v_{0j})$ the color assigned to $v_{0j}$. Therefore with $\chi(G_1) - 1$ colors, all the vertices of any copy of $G_2$ can be colored. Thus $\chi(G_1)$ colors are sufficient enough to color all the vertices of $G_1 \circ G_2$. Hence, $\chi(G_1 \circ G_2) = \chi(G_1)$.

**Case II: $\chi(G_1) < \chi(G_2)$.** Again, since $\chi(G_2)$ colors are required to color any copy of $G_2$, let us assign $\chi(G_2)$ colors to all the vertices of every copy of $G_2$. Then rest of the vertices $v_{0j}$s can be colored with same set of colors, provided that $c(v_{0j}) \neq c(v_{jl})$ as $v_{0j}$ is adjacent to all the vertices of $j^{th}$ copy of $G_2$. Thus $\chi(G_2)$ colors are sufficient enough to color all the vertices of $G \circ G_2$. Hence, $\chi(G_1 \circ G_2) = \chi(G_2)$.

**Case III: $\chi(G_1) = \chi(G_2)$.** Again, all the vertices $v_{0j}s$ can be colored with $\chi(G_1)$ colors. Now since the vertex $v_{0j}$ is adjacent to all the vertices of $j^{th}$ copy of $G_2$, then no any vertex of $j^{th}$ copy of $G_2$ can be colored by the color of $v_{0j}$, and apart from this, the same set of $\chi(G_1) - 1$ colors, which were assigned to the vertices of $G_1$ can be assigned. But $\chi(G_2) = \chi(G_1)$. Therefore one new color is to be assigned to color all the vertices of $j^{th}$ copy of $G_2$. Thus all the vertices of $G_1 \circ G_2$ can be colored with $\chi(G_1) + 1$ colors.

Therefore, for any two graphs $G_1$ and $G_2$,

$$\chi(G_1 \circ G_2) = \begin{cases} \max\{\chi(G_1), \chi(G_2)\} & \text{if } \chi(G_1) \neq \chi(G_2) \\ \chi(G_1) + 1 & \text{if } \chi(G_1) = \chi(G_2) \end{cases}.$$

□

**Theorem 6.** *For any two graphs $G_1$ and $G_2$,*

$$\chi(G_1 \diamond G_2) = \begin{cases} \chi(G_1) + 2, & \textit{if } \chi(G_1) = \chi(G_2) \\ \max\{\chi(G_1), \chi(G_2)\} + 1 & \textit{if } \mid \chi(G_1) - \chi(G_2) \mid = 1 \\ \max\{\chi(G_1), \chi(G_2)\} & \textit{if } \mid \chi(G_1) - \chi(G_2) \mid \geq 2 \end{cases}.$$

*Proof.* Let $G_1$ and $G_2$ be two graphs with the chromatic numbers $\chi(G_1)$ and $\chi(G_2)$ respectively, and $c$ be the coloring function defined on the vertex set of $G_1 \diamond G_2$, which assigns colors to all the vertices of $G_1 \diamond G_2$.

**Case I: $\chi(G_1) = \chi(G_2)$.** Then with $\chi(G_1)$ colors every vertex of $G_1$ can be colored. Since, all the vertices of the copy of $G_2$ corresponding to the edge $(v_{00i}, v_{00j})$ are adjacent to $v_{00i}$ and $v_{00j}$. Hence, any vertex of this copy of $G_2$ cannot be assigned the colors which are assigned to the end vertices $v_{00i}$ and $v_{00j}$. Since, $\chi(G_1) = \chi(G_2)$, Therefore two more colors is to be introduced apart from the colors assigned to the vertices of $G_1$ to color all the vertices of this copy of $G_2$. In the same way all the vertices of any copy of $G_2$ corresponding to any edge can be colored. Thus with $\chi(G_1) + 2$ colors, all the vertices of $G_1 \diamond G_2$ can be colored. Hence, $\chi(G_1 \diamond G_2) = \chi(G_1)$. Thus, for $\chi(G_1) = \chi(G_2)$,

$$\chi(G_1 \diamond G_2) = \chi(G_1) + 2.$$

**Case II: $\mid \chi(G_1) - \chi(G_2) \mid = 1$.** Let, $\chi(G_1) - \chi(G_2) = 1$. Then with $\chi(G_1)$ colors every vertex of $G_1$ can be colored. Since, all the vertices of the copy of $G_2$ corresponding to the edge $(v_{00i}, v_{00j})$ are adjacent to $v_{00i}$ and $v_{00j}$. Therefore any vertex of this copy of $H$ can't be assigned the colors of the end vertices $v_{00i}$ and $v_{00j}$. Since, $\chi(G) - 1 = \chi(H)$. Hence, with the set of remaining $\chi(G_1) - 2$ colors assigned to vertices of $G_1$, all the vertices of the corresponding copy of $G_2$ can not be colored, which will require a new color as $\chi(G_1) - 1 = \chi(G_2)$. In the same way all the vertices of any copy of $G_2$ corresponding to any edge can be colored. Thus with $\chi(G_1) + 1$ colors, all the vertices of $G_1 \diamond G_2$ can be colored. Hence, $\chi(G_1 \diamond G_2) = \chi(G_1) + 1$.

The proof is similar for the case $\chi(G_2) - \chi(G_1) \geq 2$. Thus, for $\mid \chi(G_1) - \chi(G_2) \mid \geq 1$,

$$\chi(G_1 \diamond G_2) = \max\{\chi(G_1), \chi(G_2)\} + 1.$$

**Case III:** $\mid \chi(G_1) - \chi(G_2) \mid \geq 2$**.** Let $\chi(G_1) - \chi(G_2) \geq 2$. Then with $\chi(G_1)$ colors every vertex of $G_1$ can be colored. Since all the vertices of the copy of $G_2$ corresponding to the edge $(v_{00i}, v_{00j})$ are adjacent to $v_{00i}$ and $v_{00j}$, therefore any vertex of this copy of $G_2$ can't be assigned the colors of the end vertices $v_{00i}$ and $v_{00j}$. So, with the set of remaining $\chi(G_1) - 2$ colors assigned to vertices of $G_1$, all the vertices of the corresponding copy of $G_2$ can be colored. In the same way all the vertices of any copy of $G_2$ corresponding to any edge can be colored. Thus with $\chi(G_1)$ colors, all the vertices of $G_1 \diamond G_2$ can be colored. Hence, $\chi(G_1 \diamond G_2) = \chi(G_1)$.

The proof is similar for the case, $\chi(G_2) - \chi(G_1) \geq 2$. Thus, for $\mid \chi(G_1) - \chi(G_2) \mid \geq 2$

$$\chi(G_1 \diamond G_2) = \max\{\chi(G_1), \chi(G_2)\}.$$

Hence for any two graphs $G_1$ and $G_2$,

$$\chi(G_1 \diamond G_2) = \begin{cases} \chi(G_1) + 2, & \text{if } \chi(G_1) = \chi(G_2) \\ \max\{\chi(G_1), \chi(G_2)\} + 1 & \text{if } \mid \chi(G_1) - \chi(G_2) \mid = 1 \\ \max\{\chi(G_1), \chi(G_2)\} & \text{if } \mid \chi(G_1) - \chi(G_2) \mid \geq 2 \end{cases}.$$

□

## 4. $T$-coloring of Corona Product and Edge Corona Network

### 4.1. $T$-coloring of Corona Product of graphs

In this section we discuss some of the results of $T$-coloring of Corona product of two arbitrary graphs $G$ and $H$.

**Theorem 7.** *Any $T-$sets of positive integers that includes zero, as well as any two graphs $G_1$ and $G_2$,*

*i.* $esp_T(G_1 \circ G_2) = \min\{esp_T(G_1), esp_T(G_2)\}$,

*ii.* $sp_T(G_1 \circ G_2) = \min\{sp_T(G_1), sp_T(G_2)\}$.

*Proof.* $(i)$ Considering $G_1 \circ G_2$ contains subgraphs that are isomorphic to both $G_1$ and $G_2$. Hence, by sub graph property of $T-$coloring, $esp_T(G_1) \leq esp_T(G_1 \circ G_2)$ and $esp_T(G_2) \leq esp_T(G_1 \circ G_2)$. Hence,

$$esp_T(G_1 \circ G_2) \geq \min\{esp_T(G_1), esp_T(G_2)\}. \tag{1}$$

Let $f$ and $g$ be two $T$-colorings of $G_1$ and $G_2$, respectively such that $esp_T^f(G_1) = sp_T(G_1)$ and $esp_T^g(G_2) = esp_T(G_2)$. Let $c$ be a coloring on $(G_1 \circ G_2)$ defined as:

$$c(v_{ij}) = \begin{cases} f(v_j), & \text{if } i = 0 \text{ and } j \in \{1, 2, 3, ..., m\} \\ f(v_i) - g(v_j) - g(v_{k+m}), & \text{if } i \in \{1, \ldots, m\} \text{ and } j \in \\ & \{m+1, \ldots, m+n\} \end{cases},$$

where $k \in \{1, \ldots, n\}$. Now, in $G_1 \circ G_2$, the edges can be distributed in three disjoint and distinct sets as $\{(v_{0j}, v_{0l}) : j, l \in \{1, \ldots, m\}\}$, $\{(v_{0j}, v_{jl}) : j \in \{1, \ldots, m\}$ and $l \in \{m+1, \ldots, m+n\}\}$ and $\{(v_{ij}, v_{il}) : i \in \{1, \ldots, m\}$ and $j, l \in \{m+1, \ldots, m+n\}\}$. Now for the edges, $\{(v_{0j}, v_{0l}) : j, l \in \{1, \ldots, m\}\}$, $(v_{ij}, v_{lm}), (v_{a,b}, v_{c,d}) \epsilon E(G_1 \circ G_2)$. Then,

$$\mid c(v_{0j}) - c(v_{0l}) \mid = \mid f(v_j) - f(v_l) \mid \notin T.$$

Thus $c$ is a $T-$coloring of $G_1 \circ G_2$. Hence,

$$\mid c(v_{0j}) - c(v_{0l}) \mid = \mid f(v_j) - f(v_l) \mid \leq esp_T(G_1). \tag{2}$$

Again for the edges $\{(v_{0j}, v_{jl}) : j \in \{1, \ldots, m\}$ and $l \in \{m+1, \ldots, m+n\}\}$

$$\begin{aligned} \mid c(v_{0j}) - c(v_{jl}) \mid &= \mid f(v_j) - f(v_j) - g(v_l) + g(v_{k+m}) \mid \\ &= \mid g(v_{k+m}) - g(v_l) \mid \notin T. \end{aligned}$$

Thus $c$ is a $T-$coloring of $G_1 \circ G_2$. Hence,

$$\mid c(v_{0j}) - c(v_{jl}) \mid = \mid g(v_{k+m}) - g(v_l) \mid \leq esp_T(G_2). \tag{3}$$

Similarly for the edges $\{(v_{ij}, v_{il}) : i \in \{1, \ldots, m\}$ and $j, l \in \{m+1, \ldots, m+n\}\}$:

$$\begin{aligned} \mid c(v_{ij}) - c(v_{il}) \mid &= \mid f(v_i) - g(v_j) - g(v_{k+m}) - f(v_i) - g(v_l) \\ &+ g(v_{k+m}) \mid = \mid g(v_j) - g(v_l) \mid \notin T. \end{aligned}$$

Thus $c$ is a $T-$coloring of $G_1 \circ G_2$. Hence,

$$\mid c(v_{ij}) - c(v_{il}) \mid = \mid g(v_j) - g(v_l) \mid \leq esp_T(G_2). \tag{4}$$

Now from equations (2), (3) and (4) for any edge $(v_{ij}, v_{ab}) \in E(G_1 \circ G_2)$

$$| \ c(v_{ij}) - c(v_{ab}) \ | \leq \min\{esp_T(G_1), esp_T(G_2)\} \tag{5}$$

$$\Rightarrow esp_T(G_1 \circ G_2) \leq esp_T^c(G_1 \circ G_2) \leq \min\{esp_T(G_1), esp_T(G_2)\}. \tag{6}$$

Therefore from equations (1) and (6),

$$\Rightarrow esp_T(G_1 \circ G_2) = \min\{esp_T(G_1), esp_T(G_2)\}. \tag{7}$$

$(ii)$ Considering $G_1 \circ G_2$ contains sub graphs that are isomorphic to both $G_1$ and $G_2$. Hence, by sub graph property of $T-$coloring, $sp_T(G_1) \leq sp_T(G_1 \circ G_2)$ and $sp_T(G_2) \leq sp_T(G_1 \circ G_2)$. Hence,

$$sp_T(G_1 \circ G_2) \geq \min\{sp_T(G_1), sp_T(G_2)\}. \tag{8}$$

Again, from equation (5), for any edge $(v_{ij}, v_{ab}) \in E(G_1 \circ G_2)$

$$| \ c(v_{ij}) - c(v_{ab}) \ | \leq \min\{esp_T(G_1), esp_T(G_2)\}.$$

Since, for all graphs $G$ and all $T-$sets, $esp_T(G) \leq sp_T(G)$. Which implies

$$c(v_{ij}) - c(v_{ab}) \ | \leq \min\{sp_T(G_1), sp_T(G_2)\}.$$

Therefore,

$$\Rightarrow sp_T(G_1 \circ G_2) \leq \min\{sp_T(G_1), sp_T(G_2)\}. \tag{9}$$

Therefore from equations (8) and (9),

$$\Rightarrow sp_T(G_1 \circ G_2) = \min\{esp_T(G_1), esp_T(G_2)\}. \tag{10}$$

□

**Corollary 8.** *Let $G_1$ be a subgraph of a graph $G_2$, then for any $T$-set, $esp_T(G_1 \circ G_2) = esp_T(G_1)$ and $sp_T(G_1 \circ G_2) = sp_T(G_1)$.*

*Proof.* Since $G_1$ is a subgraph of $G_2$, then by the subgraph property of $T$-coloring, $esp_T(G_1) \leq esp_T(G_2)$ and $sp_T(G_1) \leq sp_T(G_2)$. From Theorem 7, $esp_T(G_1 \circ G_2) = \min\{esp_T(G_1), esp_T(G_2)\}$. Hence, $esp_T(G_1 \circ G_2) = esp_T(G_1)$. Similarly, $sp_T(G_1 \circ G_2) = sp_T(G_1)$. □

Figures 1 and 2 represent $T$-coloring of Corona product and Edge Corona product of cycle graph $C_4$ with a path graph $P_3$ respectively for a $T$ set, $T = \{0, 1, 3\}$.

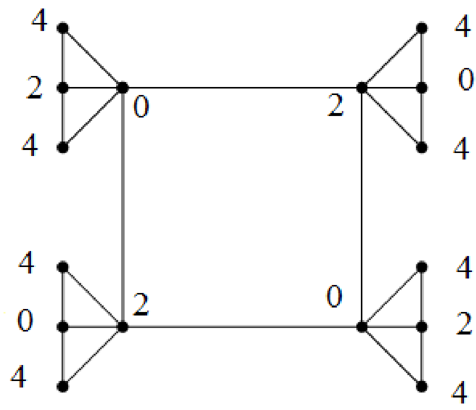


Figure 1: $\chi(C_4 \circ P_3) = \chi(C_4) + 1 = 3$ in which $\chi(C_4) = \chi(P_3) = 2$ and $sp_T(C_4 \circ P_3) = esp_T(C_4 \circ P_3) = 4$.

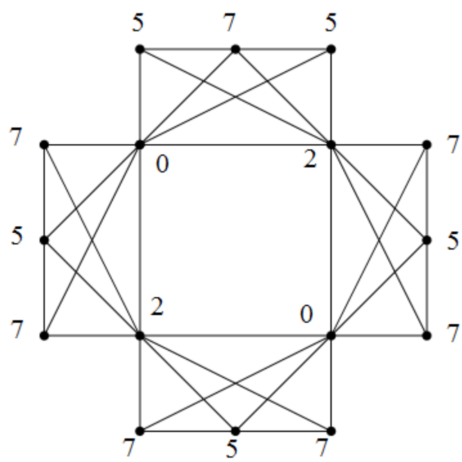


Figure 2: $\chi(C_4 \diamond P_3) = \chi(C_4) + 2 = 4$ in which $\chi(C_4) = \chi(P_3) = 2$ and $sp_T(C_4 \diamond P_3) = esp_T(C_4 \diamond P_3) = 7$.

### 4.2. $T$-coloring of Edge Corona Network

**Theorem 9.** *For any $T-$sets of positive integers that includes zero, as well as any two graphs $G_1$ and $G_2$,*

*i.* $esp_T(G_1 \diamond G_2) = \max\{esp_T(G_1), esp_T(G_2)\}$,

*ii.* $sp_T(G_1 \diamond G_2) = \max\{sp_T(G_1), sp_T(G_2)\}$.

*Proof.* $(i)$ Considering $G_1 \diamond G_2$ contains subgraphs that are isomorphic to both $G_1$ and $G_2$. Hence, by sub graph property of $T-$coloring, $esp_T(G_1) \leq esp_T(G_1 \diamond G_2)$ and $esp_T(G_2) \leq esp_T(G_1 \diamond G_2)$. Hence,

$$esp_T(G_1 \diamond G_2) \geq \max\{esp_T(G_1), esp_T(G_2)\}. \tag{11}$$

Let $g$ and $g'$ be two $T$-colorings of $G_1$ and $G_2$ respectively such that $esp_T^g(G_1) = sp_T(G_1)$ and $esp_T^{g'}(G_2) = esp_T(G_2)$. Let, $c$ be a coloring on $(G_1 \diamond G_2)$ defined as:

$$c(v_{ijk}) = \begin{cases} g(v_j), & \text{if } i = j = 0 \text{ and } k \in \{1, \ldots, m\} \\ g(v_i) - g'(v_k) - g'(v_{r+m}), & \text{if } i, j \in \{1, \ldots, m\} \text{ and } k \in \\ & \{m+1, \ldots, m+n\} \end{cases},$$

where $r \in \{1, 2, 3, ..., n\}$. Now, in $G_1 \diamond G_2$, the edges can be distributed in three disjoint and distinct sets as $\{(v_{00i}, v_{00j}) : i, j \in \{1, \ldots, m\}\}$, $\{(v_{ijk}, v_{ijl}) : i, j \in \{1, \ldots, m\}$ and $k, l \in \{m+1, \ldots, m+n\}\}$ and $\{(v_{00i}, v_{ijk}) : i, j \in \{1, \ldots, m\}$ and $k \in \{m+1, \ldots, m+n\}\} \bigcup \{(v_{00j}, v_{ijk}) : i, j \in \{1, \ldots, m\}$ and $k \in \{m+1, \ldots, m+n\}\}$. Now for the edges, $\{(v_{00i}, v_{00j}) : i, j \in \{1, \ldots, m\}\}$ for $(v_{00i}, v_{00j})\ \epsilon E(G_1 \diamond G_2)$. Then,

$$| c(v_{00i}) - c(v_{00j}) | = | g(v_i) - g(v_j) | \notin T.$$

Thus $c$ is a $T-$coloring of $G_1 \diamond G_2$. Hence,

$$| c(v_{00i}) - c(v_{00j}) | = | g(v_i) - g(v_j) | \leq esp_T(G_1). \tag{12}$$

Again for the edges $\{(v_{ijk}, v_{ijl}) : i, j \in \{1, \ldots, m\}$ and $k, l \in \{m+1, \ldots, m+n\}\}$

$$\begin{aligned} | c(v_{ijk}) - c(v_{ijl}) | &= | g(v_i) - g'(v_k) - g'(v_{r+m}) - g(v_i) - g'(v_l) \\ &\quad + g'(v_{r+m}) | = | g'(v_k) - g'(v_l) | \notin T. \end{aligned}$$

Thus $c$ is a $T-$coloring of $G_1 \diamond G_2$. Hence,

$$| c(v_{ijk}) - c(v_{ijl}) |=| g'(v_k) - g'(v_l) |\leq esp_T(G_2). \tag{13}$$

Similarly for the edges $\{(v_{00i}, v_{ijk}) : i,j \in \{1,\dots,m\}$ and $k \in \{m+1,\dots,m+n\}\}\bigcup\{(v_{00j}, v_{ijk}) : i,j \in \{1,\dots,m\}$ and $k \in \{m+1,\dots,m+n\}\}$, for $(v_{00i}, v_{ijk})\epsilon E(G_1 \diamond G_2)$

$$\begin{aligned} | c(v_{00i}) - c(v_{ijk}) | &=| g(v_i) - g(v_i) - g'(v_k) + g(v_{r+m}) | \\ &=| g'(v_{r+m}) - g'(v_k) |\notin T. \end{aligned}$$

Or for the edge $(v_{00j}, v_{ijk})\epsilon E(G_1 \diamond G_2)$,

$$\begin{aligned} | c(v_{00j}) - c(v_{ijk}) |=| c(v_{00j}) - c(v_{jik}) | &=| g(v_j) - g(v_j) - g'(v_k) \\ &\quad + g(v_{r+m}) |, \quad (\text{Since } v_{ijk} = v_{jik}) \\ &=| g'(v_{r+m}) - g'(v_k) |\notin T. \end{aligned}$$

Thus $c$ is a $T-$coloring of $G_1 \diamond G_2$. Hence,

$$| c(v_{00j}) - c(v_{ijk}) |=| g'(v_{r+m}) - g'(v_l) |\leq esp_T(G_2). \tag{14}$$

Now from equations (12), (13) and (14), for any edge $(v_{ijk}, v_{abc}) \in E(G_1 \diamond G_2)$

$$| c(v_{ijk}) - c(v_{abc}) |\leq \max\{esp_T(G_1), esp_T(G_2)\} \tag{15}$$

$$\Rightarrow esp_T(G_1 \diamond G_2) \leq esp_T^c(G_1 \diamond G_2) \leq \max\{esp_T(G_1), esp_T(G_2)\}. \tag{16}$$

Therefore from equations (11) and (16),

$$\Rightarrow esp_T(G_1 \diamond G_2) = \max\{esp_T(G_1), esp_T(G_2)\}. \tag{17}$$

$(ii)$ Considering $G_1 \diamond G_2$ contains sub graphs that are isomorphic to both $G_1$ and $G_2$. Hence, by sub graph property of $T-$coloring, $sp_T(G_1) \leq sp_T(G_1 \diamond G_2)$ and $sp_T(G_2) \leq sp_T(G_1 \diamond G_2)$. Hence,

$$sp_T(G_1 \diamond G_2) \geq \max\{sp_T(G_1), sp_T(G_2)\}. \tag{18}$$

Again, from equation (15), for any edge $(v_{ijk}, v_{abc}) \in E(G_1 \diamond G_2)$

$$| c(v_{ijk}) - c(v_{abc}) |\leq \max\{esp_T(G_1), esp_T(G_2)\}.$$

Since, for all graphs $G$ and all $T-$sets, $esp_T(G) \leq sp_T(G)$, this implies

$$c(v_{ijk}) - c(v_{abc}) |\leq \max\{sp_T(G_1), sp_T(G_2)\}.$$

Therefore,

$$\Rightarrow sp_T(G_1 \diamond G_2) \leq \max\{sp_T(G_1), sp_T(G_2)\}. \tag{19}$$

Therefore from equation 18 and 19,

$$\Rightarrow sp_T(G_1 \diamond G_2) = \max\{esp_T(G_1), esp_T(G_2)\}. \tag{20}$$

□

**Corollary 10.** *Let $G_1$ be a subgraph of a graph $G_2$, then for any $T$-set, $esp_T(G_1 \diamond G_2) = esp_T(G_2)$ and $sp_T(G_1 \diamond G_2) = sp_T(G_2)$.*

*Proof.* As, $G_1$ is a subgraph of $G_2$, hence by subgraph property of $T$-coloring, $esp_T(G_1) \leq esp_T(G_2)$ and $sp_T(G_1) \leq sp_T(G_2)$. From Theorem 9, $esp_T(G_1 \diamond G_2) = \max\{esp_T(G_1), esp_T(G_2)\}$. Hence, $esp_T(G_1 \diamond G_2) = esp_T(G_2)$. Similarly, $sp_T(G_1 \diamond G_2) = sp_T(G_2)$. □

Now, following are the generalized results of $T$-coloring of Corona Product and Edge Corona product of $n$ graphs, $G_1, G_2, G_3, ...., G_n$, which are derived from Theorem 7 and Theorem 9.

**Theorem 11.** *For any $T-$sets of positive integers that includes zero, as well as any two graphs $G_1$, $G_2$, $G_3$,....., and $G_n$,*

*i.* $esp_T(G_1 \circ G_2 \circ G_3 \circ ..... \circ G_n) = \min\{esp_T(G_1), esp_T(G_2), esp_T(G_3), ..., esp_T(G_n)\}$,

*ii.* $sp_T(G_1 \circ G_2 \circ G_3 \circ ..... \circ G_n) = \min\{sp_T(G_1), sp_T(G_2), sp_T(G_3), ..., sp_T(G_n)\}$,

*iii.* $esp_T(G_1 \diamond G_2 \diamond G_3 ..... \diamond G_n) = \max\{esp_T(G_1), esp_T(G_2), esp_T(G_3), ..., esp_T(G_n)\}$,

*iv.* $sp_T(G_1 \diamond G_2 \diamond G_3 ..... \diamond G_n) = \max\{sp_T(G_1), sp_T(G_2), sp_T(G_3), ..., sp_T(G_n)\}$.

The proofs of these results are immediate from Theorem 7 and Theorem 9.

## 5. $ST$-coloring of Corona Product and Edge Corona Network

In this section we consider $ST$-coloring of Corona Product and Edge Corona Network of two graphs $G_1$ and $G_2$.

### 5.1. $ST$-coloring of Corona Product of Graphs

**Theorem 12.** *For any $T-$sets of positive integers that includes zero, as well as any two graphs $G_1$ and $G_2$,*

*i.* $esp_{ST}(G_1 \circ G_2) = \min\{esp_{ST}(G_1), esp_{ST}(G_2)\}$,

*ii.* $sp_{ST}(G_1 \circ G_2) = \min\{sp_{ST}(G_1), sp_{ST}(G_2)\}$.

*Proof.* $(i)$ Considering $G_1 \circ G_2$ contains subgraphs that are isomorphic to both $G_1$ and $G_2$. Hence, by sub graph property of $ST-$coloring, $esp_{ST}(G_1) \leq esp_{ST}(G_1 \circ G_2)$ and $esp_{ST}(G_2) \leq esp_{ST}$ $(G_1 \circ G_2)$. Hence,

$$esp_{ST}(G_1 \circ G_2) \geq \min\{esp_{ST}(G_1), esp_{ST}(G_2)\}. \tag{21}$$

Let $f$ and $g$ be two $ST$-colorings of $G_1$ and $G_2$ respectively such that $esp^f_{ST}(G_1) = sp_{ST}(G_1)$ and $esp^g_{ST}(G_2) = esp_{ST}(G_2)$. Let $c$ be a coloring on $(G_1 \circ G_2)$ defined as:

$$c(v_{ij}) = \begin{cases} f(v_j), & \text{if } i = 0 \text{ and } j \in \{1, 2, 3, ..., m\} \\ f(v_i) - g(v_j) - g(v_{k+m}), & \text{if } i \in \{1, \dots, m\} \text{ and} \\ & j \in \{m+1, \dots, m+n\} \end{cases},$$

where $k \in \{1, 2, 3, ..., n\}$. Now, in $G_1 \circ G_2$, the edges can be distributed in three disjoint and distinct sets as $\{(v_{0j}, v_{0l}) : j, l \in \{1, 2, 3, ...., m\}\}$, $\{(v_{0j}, v_{jl}) : j \in \{1, 2, 3, ...., m\}$ and $l \in \{m+1, m+2, m+3, ...., m+n\}\}$ and $\{(v_{ij}, v_{il}) : i \in \{1, 2, 3, ...., m\}$ and $j, l \in \{m+1, m+2, m+3, ...., m+n\}\}$. Now, it is seen that for any two distinct edges $(v_{ij}, v_{lm})$ and $(v_{ab}, v_{cd})$

$$\mid c(v_{ij}) - c(v_{lm}) \mid \neq \mid c(v_{ab}) - c(v_{cd}) \mid . \tag{22}$$

Now for the edges, $\{(v_{0j}, v_{0l}) : j, l \in \{1, \dots, m\}\}$,

$$\mid c(v_{0j}) - c(v_{0l}) \mid = \mid f(v_j) - f(v_l) \mid \notin T. \tag{23}$$

Hence, from equation (22) and (23), $c$ is a $ST-$coloring of $G_1 \circ G_2$. Hence,

$$\mid c(v_{0j}) - c(v_{0l}) \mid = \mid f(v_j) - f(v_l) \mid \leq esp_{ST}(G_1). \tag{24}$$

Again for the edges $\{(v_{0j}, v_{jl}) : j \in \{1, \dots, m\}$ and $l \in \{m+1, \dots, m+n\}\}$

$$\mid c(v_{0j}) - c(v_{jl}) \mid = \mid f(v_j) - f(v_j) - g(v_l) + g(v_{k+m}) \mid = \mid g(v_{k+m}) - g(v_l) \mid$$

$$\Rightarrow \mid c(v_{0j}) - c(v_{jl}) \mid \notin T. \tag{25}$$

Hence, from equations (22) and (25), $c$ is a $ST-$coloring of $G_1 \circ G_2$. Hence,

$$\mid c(v_{0j}) - c(v_{jl}) \mid = \mid g(v_{k+m}) - g(v_l) \mid \leq esp_{ST}(G_2). \tag{26}$$

Similarly for the edges $\{(v_{ij}, v_{il}) : i \in \{1, \dots, m\}$ and $j, l \in \{m+1, \dots, m+n\}\}$

$$\mid c(v_{ij}) - c(v_{il}) \mid = \mid f(v_i) - g(v_j) - g(v_{k+m}) - f(v_i) - g(v_l) + g(v_{k+m}) \mid$$

$$\Rightarrow \mid c(v_{ij}) - c(v_{il}) \mid = \mid g(v_j) - g(v_l) \mid \notin T. \tag{27}$$

Thus, from equations (22) and (27), $c$ is a $ST-$coloring of $G_1 \circ G_2$. Hence,

$$\mid c(v_{ij}) - c(v_{il}) \mid = \mid g(v_j) - g(v_l) \mid \leq esp_{ST}(G_2). \tag{28}$$

Now from equations (24), (26) and (28), for any edge $(v_{ij}, v_{ab}) \in E(G_1 \circ G_2)$

$$\mid c(v_{ij}) - c(v_{ab}) \mid \leq \min\{esp_{ST}(G_1), esp_{ST}(G_2)\} \tag{29}$$

$$\Rightarrow esp_{ST}(G_1 \circ G_2) \leq esp^c_{ST}(G_1 \circ G_2) \leq \min\{esp_{ST}(G_1), esp_{ST}(G_2)\}. \tag{30}$$

Therefore from equations (21) and (30), we have

$$\Rightarrow esp_{ST}(G_1 \circ G_2) = \min\{esp_{ST}(G_1), esp_{ST}(G_2)\}. \tag{31}$$

$(ii)$ Considering $G_1 \circ G_2$ contains sub graphs that are isomorphic to both $G_1$ and $G_2$. Hence, by sub graph property of $ST-$coloring, $sp_{ST}(G_1) \leq sp_{ST}(G_1 \circ G_2)$ and $sp_{ST}(G_2) \leq sp_{ST}(G_1 \circ G_2)$. Hence,

$$sp_{ST}(G_1 \circ G_2) \geq \min\{sp_{ST}(G_1), sp_{ST}(G_2)\}. \tag{32}$$

Again, from equation (30), for any edge $(v_{ij}, v_{ab}) \in E(G_1 \circ G_2)$

$$\mid c(v_{ij}) - c(v_{ab}) \mid \leq \min\{esp_{ST}(G_1), esp_{ST}(G_2)\}.$$

Since, for all graphs $G$ and all $T-$sets, $esp_{ST}(G) \leq sp_{ST}(G)$, this implies

$$\mid c(v_{ij}) - c(v_{ab}) \mid \leq \min\{sp_{ST}(G_1), sp_{ST}(G_2)\}$$

$$\Rightarrow sp_{ST}(G_1 \circ G_2) \leq \min\{sp_{ST}(G_1), sp_{ST}(G_2)\}. \tag{33}$$

Hence from equations (32) and (33),

$$\Rightarrow sp_{ST}(G_1 \circ G_2) = \min\{esp_{ST}(G_1), esp_{ST}(G_2)\}. \tag{34}$$

$\square$

**Corollary 13.** *Let $G_1$ be a subgraph of a graph $G_2$, then for any $T$-set, $esp_T(G_1 \circ G_2) = esp_T(G_1)$ and $sp_T(G_1 \circ G_2) = sp_T(G_1)$.*

*Proof.* As, $G_1$ is a subgraph of $G_2$, hence by subgraph property of $ST$-coloring, $esp_{ST}(G_1) \leq esp_{ST}(G_2)$ and $sp_{ST}(G_1) \leq sp_{ST}(G_2)$. From Theorem 12, $esp_{ST}(G_1 \circ G_2) = \min\{esp_{ST}(G_1), esp_{ST}(G_2$
$)\}$. Hence, $esp_{ST}(G_1 \circ G_2) = esp_{ST}(G_1)$. Similarly, $sp_{ST}(G_1 \circ G_2) = sp_{ST}(G_1)$. □

### 5.2. $ST$-coloring of Edge Corona Network

**Theorem 14.** *For any $T-$sets of positive integers that includes zero, as well as for any two graphs $G_1$ and $G_2$,*

*i.* $esp_{ST}(G_1 \diamond G_2) = \max\{esp_{ST}(G_1), esp_{ST}(G_2)\}$,

*ii.* $sp_{ST}(G_1 \diamond G_2) = \max\{sp_{ST}(G_1), sp_{ST}(G_2)\}$.

*Proof.* $(i)$ Considering $G_1 \diamond G_2$ contains subgraphs that are isomorphic to both $G_1$ and $G_2$. Hence, by sub graph property of $T-$coloring, $esp_{ST}(G_1) \leq esp_{ST}(G_1 \diamond G_2)$ and $esp_{ST}(G_2) \leq esp_{ST}$ $(G_1 \diamond G_2)$. Hence,

$$esp_{ST}(G_1 \diamond G_2) \geq \max\{esp_{ST}(G_1), esp_{ST}(G_2)\}. \tag{35}$$

Let $g$ and $g'$ be two $T$-colorings of $G_1$ and $G_2$ respectively such that $esp^g_{ST}(G_1) = sp_{ST}(G_1)$ and $esp^{g'}_{ST}(G_2) = esp_{ST}(G_2)$. Let, $c$ be a coloring on $(G_1 \diamond G_2)$ defined as:

$$c(v_{ijk}) = \begin{cases} g(v_j), & \text{if } i = j = 0 \text{ and} \\ & k \in \{1, 2, 3, ..., m\} \\ g(v_i) - g'(v_k) - g'(v_{r+m}), & \text{if } i, j \in \{1, \ldots, m\} \text{ and} \\ & k \in \{m+1, \ldots, m+n\} \end{cases},$$

where $r \in \{1, 2, 3, ..., n\}$. Now, in $G_1 \diamond G_2$, the edges can be distributed in three disjoint and distinct sets as $\{(v_{00i}, v_{00j}) : i, j \in \{1, 2, 3, ...., m\}\}$, $\{(v_{ijk}, v_{ijl}) : i, j \in \{1, 2, 3, ...., m\}$ and $k, l \in \{m+1, m+2, m+3, ...., m+n\}\}$ and $\{(v_{00i}, v_{ijk}) : i, j \in \{1, 2, 3, ...., m\}$ and $k \in \{m+1, m+2, m+3, ...., m+n\}\} \bigcup \{(v_{00j}, v_{ijk}) : i, j \in \{1, 2, 3, ...., m\}$ $k \in \{m+1, m+2, m+3, ...., m+n\}\}$. Now, it is seen that for any two distinct edges $(v_{ijk}, v_{lmn})$ and $(v_{abc}, v_{deh})$

$$\mid c(v_{ijk}) - c(v_{lmn}) \mid \neq \mid c(v_{abc}) - c(v_{deh}) \mid . \tag{36}$$

Now for the edges, $\{(v_{00i}, v_{00j}) : i, j \in \{1, 2, 3, ...., m\}\}$, for $(v_{00i}, v_{00j}) \in E(G_1 \diamond G_2)$, we have

$$| c(v_{00i}) - c(v_{00j}) |=| g(v_i) - g'(v_j) |\notin T. \tag{37}$$

Thus, from equations (36) and (37), $c$ is a $ST-$coloring of $G_1 \diamond G_2$. Hence,

$$| c(v_{00i}) - c(v_{00j}) |=| g(v_i) - g(v_j) |\leq esp_{ST}(G_1). \tag{38}$$

Again for the edges $\{(v_{ijk}, v_{ijl}) : i, j \in \{1, \ldots, m\}$ and $k, l \in \{m+1, \ldots, m+n\}\}$

$$| c(v_{ijk}) - c(v_{ijl}) |=| g(v_i) - g'(v_k) - g'(v_{r+m}) - g(v_i) - g'(v_l) + g'(v_{r+m}) |$$

$$\Rightarrow| c(v_{ijk}) - c(v_{ijl}) |\notin T. \tag{39}$$

Thus, from equation (36) and (38), $c$ is a $ST-$coloring of $G_1 \diamond G_2$. Hence,

$$| c(v_{ijk}) - c(v_{ijl}) |=| g'(v_k) - g'(v_l) |\leq esp_{ST}(G_2). \tag{40}$$

Similarly for the edges $\{(v_{00i}, v_{ijk}) : i, j \in \{1, 2, 3, ...., m\}$ and $k \in \{m + 1, m + 2, m + 3, ...., m + n\}\} \bigcup \{(v_{00j}, v_{ijk}) : i, j \in \{1, 2, 3, ...., m\}$ and $k \in \{m + 1, m + 2, m + 3, ...., m + n\}\}$, for $(v_{00i}, v_{ijk}) \epsilon E(G_1 \diamond G_2)$

$$| c(v_{00i}) - c(v_{ijk}) |=| g(v_i) - g(v_i) - g'(v_k) + g(v_{r+m}) |$$

$$\Rightarrow| c(v_{00i}) - c(v_{ijk}) |=| g'(v_{r+m}) - g'(v_k) |\notin T. \tag{41}$$

Since $v_{ijk} = v_{jik}$, therefore the edge $(v_{00j}, v_{ijk}) \in E(G_1 \diamond G_2)$,

$$\begin{aligned} | c(v_{00j}) - c(v_{ijk}) |=| c(v_{00j}) - c(v_{jik}) | =| g(v_j) - g(v_j) - g'(v_k) \\ + g(v_{r+m}) | . \end{aligned}$$

So,

$$| c(v_{00j}) - c(v_{ijk}) |=| g'(v_{r+m}) - g'(v_k) |\notin T.$$

Thus, from equation 36 and 41, $c$ is a $ST-$coloring of $G_1 \diamond G_2$. Hence,

$$| c(v_{00j}) - c(v_{ijk}) |=| g'(v_{r+m}) - g'(v_l) |\leq esp_{ST}(G_2). \tag{42}$$

Now from equations 38, 40 and 42, for any edge $(v_{ijk}, v_{abc}) \in E(G_1 \diamond G_2)$,

$$| c(v_{ijk}) - c(v_{abc}) |\leq \max\{esp_{ST}(G_1), esp_{ST}(G_2)\} \tag{43}$$

$$\Rightarrow esp_{ST}(G_1 \diamond G_2) \leq esp^c_{ST}(G_1 \diamond G_2) \leq \max\{esp_{ST}(G_1), esp_{ST}(G_2)\}. \tag{44}$$

Therefore from equations (35) and (44),

$$\Rightarrow esp_{ST}(G_1 \diamond G_2) = \max\{esp_{ST}(G_1), esp_{ST}(G_2)\}. \tag{45}$$

$(ii)$ Considering $G_1 \diamond G_2$ contains sub graphs that are isomorphic to both $G_1$ and $G_2$. Hence, by sub graph property of $ST-$coloring, $sp_{ST}(G_1) \leq sp_{ST}(G_1 \diamond G_2)$ and $sp_{ST}(G_2) \leq sp_{ST}(G_1 \diamond G_2)$. Hence,

$$sp_{ST}(G_1 \diamond G_2) \geq \max\{sp_{ST}(G_1), sp_{ST}(G_2)\}. \tag{46}$$

Again, from equation 43, for any edge $(v_{ijk}, v_{abc}) \in E(G_1 \diamond G_2)$

$$| \; c(v_{ijk}) - c(v_{abc}) \; | \leq \max\{esp_{ST}(G_1), esp_{ST}(G_2)\}.$$

Since, for all graphs $G$ and all $T-$sets, $esp_{ST}(G) \leq sp_{ST}(G)$, this implies

$$c(v_{ijk}) - c(v_{abc}) \; | \leq \max\{sp_{ST}(G_1), sp_{ST}(G_2)\}.$$

Therefore,

$$\Rightarrow sp_{ST}(G_1 \diamond G_2) \leq \max\{sp_{ST}(G_1), sp_{ST}(G_2)\}. \tag{47}$$

Therefore from equations (46) and (47),

$$\Rightarrow sp_{ST}(G_1 \diamond G_2) = \max\{esp_{ST}(G_1), esp_{ST}(G_2)\}. \tag{48}$$

□

**Corollary 15.** *Let $G_1$ be a subgraph of a graph $G_2$, then for any $T$-set, $esp_{ST}(G_1 \diamond G_2) = esp_{ST}(G_2)$ and $sp_{ST}(G_1 \diamond G_2) = sp_{ST}(G_2)$.*

*Proof.* As, $G_1$ is a subgraph of $G_2$, hence by subgraph property of $T$-coloring, $esp_{ST}(G_1) \leq esp_{ST}(G_2)$ and $sp_{ST}(G_1) \leq sp_{ST}(G_2)$. From Theorem 14, $esp_{ST}(G_1 \diamond G_2) = \max\{esp_{ST}(G_1), esp_{ST}(G_2)\}$. Hence, $esp_{ST}(G_1 \diamond G_2) = esp_{ST}(G_2)$. Similarly, $sp_{ST}(G_1 \diamond G_2) = sp_{ST}(G_2)$. □

Now, they follow the generalized results of $ST$-coloring of Corona Product and Edge Corona product of $n$ graphs, $G_1, G_2, G_3, ..., G_n$, which are derived from Theorem 12 and Theorem 14.

**Theorem 16.** *For any $T-$sets of positive integers that includes zero, as well as for any two graphs $G_1$, $G_2$, $G_3$,....., and $G_n$,* *i.* $esp_{ST}(G_1 \circ G_2 \circ G_3 \circ ..... \circ G_n) = \min\{esp_{ST}(G_1), esp_{ST}(G_2), esp_{ST}(G_3), ..., esp_{ST}(G_n)\}$,

*ii.* $sp_{ST}(G_1 \circ G_2 \circ G_3 \circ ..... \circ G_n) = \min\{sp_{ST}(G_1), sp_{ST}(G_2), sp_{ST}(G_3), ..., sp_{ST}(G_n)\}$,

*iii.* $esp_{ST}(G_1 \diamond G_2 \diamond G_3 ..... \diamond G_n) = \max\{esp_{ST}(G_1), esp_{ST}(G_2), esp_{ST}(G_3), ..., esp_{ST}(G_n)\}$,

*iv.* $sp_{ST}(G_1 \diamond G_2 \diamond G_3 ..... \diamond G_n) = \max\{sp_{ST}(G_1), sp_{ST}(G_2), sp_{ST}(G_3), ..., sp_{ST}(G_n)\}$.

The proofs of these results are immediate from (12) and (14).

## 6. Conclusions

In this paper, we took into account the Corona product and Edge Corona product and explored $T$ and $ST$-coloring for each of the product. With regard to $T$ and $ST$-coloring, we arrived at some conclusions on the parameters span and edge span. We proved that for both $T$ and $ST$-coloring, span and edge span of Corona product of two graphs or more are equals to the minimum of the corresponding spans and edge spans of the Graphs respectively, whereas, the span and edge span of Edge Corona product of two Graphs or more are equals to the minimum of the corresponding spans and edge spans of the graphs respectively. Here, we took a look at a few generalised Graphs. It is worth mentioning that the study of $T$ and $ST$-colorings on these products of certain particular Graphs might be a challenging task.

## References

[1] S. Aisyah, R. Alfarisi, R.M. Prihandini, A.I. Kristiana and R.D. Christyanti, On the local edge antimagic coloring of corona product of path and cycle, *Cauchy-Journal Matematika Murni dan Aplikasi*, **6**, No 1 (2019), 40-48.

[2] S. Barik and G. Sahoo, On the Laplacian spectra of some varients of corona, *Linear Algebra and its Applications*, **512** (2017), 32-47.

[3] M.B. Cozzens and D.I. Wang, The general channel assignment problem, *Congressus Numerantium*, **41** (1984), 115-129.

[4] M.B. Cozzens and F.S. Roberts, $T$-colorings of graphs and the channel assignment problem, *Congressus Numerantium*, **35** (1982), 191-208.

[5] R. Frucht and F. Harary, On the corona of two graphs, *Aequationes Mathematicae*, **4**, No 3 (1970), 322-325.

[6] W.K. Hale, Frequency assignment: Theory and applications, In: *Proceedings of the IEEE*, **68** (1980), 1497-1514.

[7] R. Hammack, W. Imrich and S. Klavžar, *Handbook of Product Graphs*, CRC Press, Taylor & Francis Group, New York (2011).

[8] T.W. Haynes and L.M. Lawson, Applications of E-graphs in network design, *Networks*, **23**, No 5 (1993), 473-479.

[9] K. Kaliraj, R. Sivakami and J.V. Vernold, Star edge coloring of corona product of path with some graphs, *International Journal of Mathematical Combinatorics*, **3** (2016), 115-122.

[10] Q. Lu, S. Tian and L. Peng, The acyclic coloring of the Corona product and Edge corona product of special graphs, In: *International Conference on Computer Science and Management Technology (ICCSMT)*, Shanghai, China, (2020), 69-72; DOI: 10.1109/ICCSMT51754.2020.00021.

[11] S. Mohan, J. Geetha and K. Somasundaram, Total coloring of the corona product of two graphs, *Australian Journal of Combinatorics*, **68**, No 1 (2017), 15-22.

[12] R. Moran, N. Bora, A.K. Baruah and A. Bharali, $ST$-coloring of join and disjoint union of graphs, *Advances in Mathematics: Scientific Journal*, **9**, No 11 (2020), 9393-9399.

[13] R. Moran, N. Bora, A.K. Baruah and A. Bharali, $ST$-coloring of some products of graphs, *Journal of Mathematical and Computational Science*, **11**, No 1 (2020), 337-347.

[14] S.J. Roselin, $T$-coloring of product graphs, *Discrete Mathematics, Algorithms and Applications*, **14**, No 2 (2022), Article No 2150103.

[15] S.J. Roselin and L.B. M. Raj, $T-$Coloring of certain non perfect graphs,*Journal of Applied Science and Computations*, **6**, No 2 (2019), 1456-1468.

[16] S.J. Roselin and L.B.M. Raj, $T-$Coloring of wheel graphs, *International Journal of Information and Computing Science*, **6**, No 3 (2019), 11-18.

[17] S.J. Roselin, L.B.M. Raj and K.A. Germina, Strong $T$-coloring of graphs, *International Journal of Innovative Technology and Exploring Engineering*, **8**, No 12 (2019), 4677-4681.

[18] G.P. Singh, A. Borah and S. Ray, A review paper on corona product of graphs, *Advances and Applications in Mathematical Sciences*, **19**, No 10 (2020), 1047-1054.

[19] P. Sivagami, $T-$coloring of certain graphs, *International Journal of Pure and Applied Mathematics*, **120**, No 8 (2018), 119-127.

[20] P. Sivagami and I. Rajasingh, $T-$coloring of certain networks, *Mathematics in Computer Science*, **10**, No 2 (2016), 239-248.

[21] B.A. Tesman, $T$-colorings, list $T$-colorings, and set $T$-colorings of graphs, *RUTCOR Res. Rept. RRR* Rutgers University, New Brunswick, NJ (1989), 57-89.

[22] Y. Wang, Y. Yi, W. Xu and Z. Zhang, Edge corona product as an approach to modeling complex simplical networks, *Preprint* (2020); DOI:10.48550/arXiv.2002.12219.