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VIBRATION STABILITY OF COAXIAL CYLINDRICAL SHELLS

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Abstract: The paper considers the influence of a diamagnetic vacuum gap on the occurrence of instability in a dynamic system consisting of two coaxial shells with an azimuthal magnetic field in the gap.

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1. Introduction

In the works [1] – [3], some questions of the stability of oscillations of magnetoelastic systems - elastic conducting thin objects such as plates or shells, were studied.

Plane problems (models) are considered in [1], [3]: superconducting flat surfaces are separated by a vacuum gap filled with a constant uniform magnetic field and move with a relative velocity V_0 parallel to each other. The main regularities of such a modified tangential discontinuity were studied using the example of a model of two elastic half-spaces separated by a vacuum diamagnetic gap [1] and two parallel thin plates [3]. In [1], the stability of a tangential discontinuity with respect to small perturbations in the form of surface waves was studied. The influence of the diamagnetic gap leads to the existence of two types of waves with phase velocities $V_{1\Phi}$ and $V_{2\Phi}$. It is shown that such perturbations are unstable in the case when the value of the velocity of motion of the half-spaces is in the interval $(2V_{1\Phi}, 2V_{2\Phi})$.

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In [3] the authors studied stability of a dynamic system consisting of two parallel thin plates with respect to small perturbations such as bending waves propagating in the plates. It is shown that two types of waves are also possible in plates: slow and fast with phase velocities $V_{1\Phi}$ and $V_{2\Phi}$. And in this case, the system turns out to be unstable if the value of the velocity of the relative sliding of the plates V_0 also falls into the interval between $2V_{1\Phi}$ and $2V_{2\Phi}$.

In the same work, another model was considered, which made it possible to investigate the effect of a diamagnetic gap on the occurrence of instability in a plate flown by an incompressible fluid. Installed that in this case there are two regions of instability: the first limited region of instability is associated with the diamagnetic gap and is determined by the inequality $2V_{1\Phi} < V_0 < 2V_{2\Phi}$, where $V_{1\Phi}$ and $V_{2\Phi}$ - phase velocities of waves propagating in such a system; the second region of instability is determined by the inequality $V_0 > V_{kp}$ and corresponds to the instability in a solitary plate, streamlined by a fluid flow in the presence of a magnetic field.

In [2], the stability of oscillations of an ideally conducting shell with a conducting current, containing a flow of an ideal incompressible fluid, was studied with respect to radial perturbations of the shell shape. It is shown that perturbations of the form $\exp\left(i\left(\omega t - n\varphi\right)\right)$ are always stable, and the instability conditions for perturbations of the form $\exp\left(i\left(\omega t - kz\right)\right)$ are determined.

The studies showed that the presence of a vacuum diamagnetic gap leads to instability of surface perturbations during the relative motion of surfaces. This specific type of instability is analogous to the instability that occurs from a tangential discontinuity. However, in contrast to the latter, here the instability takes place in a finite interval of the velocities of motion of the boundaries.

The stability of coaxial current-carrying shells was considered in [4], [5] for the case with an azimuthal magnetic field and disturbances traveling along the generatrix of the shell. In this case, the vibrations of the shells turn out to be independent and the presence of the outer shell does not lead to new effects.

In this paper, we consider a cylindrical model of a tangential discontinuity formed by two rotating coaxial shells with an azimuthal magnetic field in the gap.

2. Dispersion equation

The system of rotation of coaxial shells with an azimuthal magnetic field in the gap leads to the model of a tangential discontinuity we need in the case of shell perturbations of the form $\exp\left(i\left(n\varphi - \omega t\right)\right)$, see [6].

Indeed, the solution of the Laplace equation for the magnetic potential Ψ in the gap has in this case the form

$$\Psi = \left(ar^{n} + br^{-n}\right)\exp\left(in\varphi\right). \tag{1}$$

Using the condition on the surface of superconducting shells, we obtain

$$a = \frac{iH_{0\varphi}}{\Delta}\left(\xi_{02}R_1^{-n} - \xi_{01}R_2^{-n}\right),$$

$$b = \frac{iH_{0\varphi}}{\Delta}\left(\xi_{02}R_1^{n} - \xi_{01}R_2^{n}\right),$$

where $H_{0\varphi}$ is the magnetic field in the gap in the absence of shells perturbations, R_2 and R_1 are the radii of the outer and inner shells and

$$\Delta = \left(\frac{R_2}{R_1}\right)^{n} - \left(\frac{R_1}{R_2}\right)^{n}.$$

Substituting the found values a and b into solution (1), after simple transformations, we obtain the following expressions for the magnetic pressure on the surface of the shells

$$P_{1m} = P_{0m}\left(1 + 2n\alpha\frac{\xi_1}{R_1} - 4n\beta\frac{\xi_2}{R_2}\right), \tag{2}$$

$$P_{2m} = P_{0m}\left(1 - 2n\alpha\frac{\xi_2}{R_2} + 4n\beta\frac{\xi_1}{R_1}\right), \tag{3}$$

where

$$\alpha = \frac{R_2^{2n} + R_1^{2n}}{R_2^{2n} - R_1^{2n}}, \quad \beta = \frac{R_2^{n}\cdot R_1^{n}}{R_2^{2n} - R_1^{2n}}, \quad P_{0m} = \frac{H_{0\varphi}^2}{8\pi}. \tag{4}$$

The equations of radial vibrations of shells in the case under consideration have the form

$$\rho h\left(\frac{\partial^2}{\partial t^2} + \omega_0\frac{\partial^2}{\partial t\partial\varphi}\right)\xi_1 + \frac{D}{R_1^4}\cdot\frac{\partial^4\xi_1}{\partial\varphi^4} = \rho h\omega_0^2\xi_1 - P_{1m}, \tag{5}$$

$$\rho h\frac{\partial^2\xi_2}{\partial t^2} + \frac{D}{R_2^4}\cdot\frac{\partial^4\xi_2}{\partial\varphi^4} = P_{2m}. \tag{6}$$

Substituting into the system of equations (5), (6) the expressions for the magnetic pressure on the surface of the shells (2), (3) and taking into account that

$$\xi_1 = \xi_{01}\exp\left(i\left(n\varphi - \omega t\right)\right),$$

$$\xi_2 = \xi_{02} \exp \left(i \left(n\varphi - \omega t \right) \right),$$

we obtain a system of homogeneous equations that determine possible oscillations in the system

$$\left(\omega^2 - n\omega\omega_0 - \Omega_1^2 + \omega_0^2 \right) \xi_1 + 2\beta\gamma_{1n}^2 \xi_2 = 0,$$

$$2\beta\gamma_{2n}^2 \xi_1 + \left(\omega^2 - \Omega_2^2 \right) \xi_2 = 0.$$

Here ω_0 is the angular velocity of shells rotation,

$$\Omega_1^2 = \omega_{1n}^2 + \alpha\gamma_{1n}^2, \;\; \Omega_2^2 = \omega_{2n}^2 + \alpha\gamma_{2n}^2, \tag{7}$$

$$\omega_{1n}^2 = \frac{Dn^4}{R_1^4} \cdot \frac{1}{\rho h}, \quad \omega_{2n}^2 = \frac{Dn^4}{R_2^4} \cdot \frac{1}{\rho h}$$

are eigenfrequencies of shell vibrations in the absence of a magnetic field, and

$$\gamma_{1n}^2 = n \frac{2P_{0m}}{\rho h} \cdot \frac{1}{R_1}, \;\; \gamma_{2n}^2 = n \frac{2P_{0m}}{\rho h} \cdot \frac{1}{R_2}$$

are parameters that determine the influence of the magnetic field of the gap.

The consistence condition of the obtained system gives the dispersion equation of the problem

$$\left(\omega^2 - n\omega_0\omega - \Omega_1^2 + \omega_0^2 \right) \cdot \left(\omega^2 - \Omega_2^2 \right) = 4\beta^2 \gamma_{1n}^2 \gamma_{2n}^2. \tag{8}$$

3. Vibration of shells in the absence of their rotation

First, consider a special case when there is no rotation of the shells $(\omega_0 = 0)$. In this case, dispersion equation (8) takes the form

$$\left(\omega^2 - \Omega_1^2 \right) \cdot \left(\omega^2 - \Omega_2^2 \right) = 4\beta^2 \gamma_{1n}^2 \gamma_{2n}^2,$$

or

$$\omega^4 - \left(\Omega_1^2 + \Omega_2^2 \right) \omega^2 + \Omega_1^2 \Omega_2^2 - 4\beta^2 \gamma_{1n}^2 \gamma_{2n}^2 = 0.$$

This equation determines the eigenfrequencies of the coupled oscillations of the system under consideration, which are equal to

$$\omega^2 = \frac{1}{2} \left(\Omega_1^2 + \Omega_2^2 \pm \sqrt{\left(\Omega_1^2 - \Omega_2^2 \right)^2 + 16\beta^2 \gamma_{1n}^2 \gamma_{2n}^2} \right). \tag{9}$$

It is easy to see that the eigenfrequencies (9) are real if the inequality holds

$$\Omega_1^2\Omega_2^2 > 4\beta^2\gamma_{1n}^2\gamma_{2n}^2.$$

Substituting into this inequality the values Ω_1^2 and Ω_2^2 from (7) and taking into account that

$$\alpha^2 - 4\beta^2 = 1$$

(this can be easily verified using (4)), we obtain the condition for the realness of frequencies

$$\omega_{1n}^2\omega_{2n}^2 + \alpha\omega_{1n}^2\gamma_{2n}^2 + \alpha\omega_{2n}^2\gamma_{1n}^2 + \gamma_{1n}^2\gamma_{2n}^2 > 0.$$

Obviously, this inequality holds identically. This means that in the absence of shell rotation, the oscillations of shells of the considered type are always stable.

4. Oscillation of rotating shells

Next, we consider the general case when the shells rotate. An analysis of the dispersion equation (8) in this case shows that with an increase in angular velocity ω_0, instability occurs. In this case, there are two different critical velocities ω_{1kr} and ω_{2kr}, the values of which depend on the magnitude of the magnetic field of the gap, the ratio of the shell radii and the harmonic number n. Instability occurs when the rotation speed ω_0 is in the interval between ω_{1kr} and ω_{2kr}, so $\omega_{1kr} < \omega_0 < \omega_{2kr}$.

The values ω_{1kr} and ω_{2kr} in the general case can be found numerically. However, in the particular case when $n = 2$, for a small gap, the instability conditions can be obtained in an analytical form. At $n = 2$ the perturbed shape of the shell has the form of an ellipse with a variable eccentricity.

Let $\frac{R_2}{R_1} = 1 + \varepsilon,\ \ \varepsilon \ll 1$. Then, if the field in the gap is not too small and satisfies the inequality [7], [8]

$$H_0^2 \gg \frac{8\pi}{3}\varepsilon\frac{E}{1-y^2}\left(\frac{h}{R_1}\right)^3, \quad \left(\varepsilon\frac{\omega_{1n}^2}{\gamma_{1n}^2} \ll 1\right),$$

then, in the zeroth-order approximation in ε, can be written

$$\Omega_1^2 = \Omega_2^2, \quad \gamma_{1n}^2 = \gamma_{2n}^2.$$

Under these conditions, dispersion equation (8) takes the form

$$\left((\omega-\omega_0)^2-\Omega_1^2\right)\cdot\left(\omega^2-\Omega_1^2\right)=4\beta^2\gamma_{1n}^4. \tag{10}$$

Assuming no rotation ($\omega_0=0$) equation (10) takes the form

$$\left(\omega^2-\Omega_1^2\right)^2=4\beta^2\gamma_{1n}^4,$$

and its roots are equal

$$\omega^2=\Omega_1^2\pm 2\beta\gamma_{1n}^2.$$

Consequently, the eigenfrequencies of the system oscillations have the form

$$\omega_1^2=\omega_{1n}^2+(\alpha-2\beta)\,\gamma_{1n}^2,$$

$$\omega_2^2=\omega_{2n}^2+(\alpha+2\beta)\,\gamma_{1n}^2.$$

In general, when the shells rotate ($\omega_0\neq 0$), the roots of the equation have the form

$$\omega_{1,2}=\frac{1}{2}\omega_0\pm\sqrt{\Omega_1^2+\left(\frac{1}{2}\omega_0\right)^2\pm\sqrt{\Omega_1^2\omega_0^2+4\beta^2\gamma_{1n}^2}}.$$

We are interested in the case when the second root of equation (10) is complex. Obviously, this takes place when for the angular velocity of shells rotation, the following inequality holds:

$$\Omega_1^2-2\beta\gamma_{1n}^2<\left(\frac{1}{2}\omega_0\right)^2<\Omega_1^2+2\beta\gamma_{1n}^2,$$

or, given the expression for Ω_1^2 from (7), we finally have the boundaries for this velocity of shells rotation

$$2\sqrt{\omega_{1n}^2+(\alpha-2\beta)\,\gamma_{1n}^2}<\omega_0<2\sqrt{\omega_{1n}^2+(\alpha+2\beta)\,\gamma_{1n}^2}.$$

5. Conclusion

The stability of the tangential discontinuity with respect to small radial perturbations of the shell shape proportional to $\exp(in\varphi)$ is studied. It is shown that the diamagnetic gap leads to the splitting of the eigenfrequency of oscillations into two frequencies ω_{1n} and ω_{2n}, and the values of the critical velocities of shell rotation turn out to be equal to these eigenfrequencies of the system oscillations. The instability condition for such a tangential discontinuity is obtained, which has the form similar to the instability condition for plane problems, i.e. $2\omega_{1n} < \omega_0 < 2\omega_{2n}$.

Thus, the stability conditions for the cylindrical model turned out to be identical to the stability conditions obtained for the plane models.

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