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A SOFT TOPOLOGICAL MODEL FOR SPATIAL OBJECTS WITH UNCERTAIN BOUNDARIES

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Abstract: GIS analyzes the geometric relationships among arbitrary spatial objects. The topological spatial relations between spatial objects had been discussed in the literature for several years. Recently Khalil et.al. introduced Spatial object modeling in soft topology. In this paper soft 4 intersection model and soft 9-intersection model for soft regions with sharp soft boundary and broad soft boundary have been discussed.

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1. Introduction

In 1999, Molodtsov [11] initiated the concept of soft set theory as a tool for dealing with uncertainties. Following this, Maji et al. [10] defined operations on soft set theory. In 2009, Ali [1] presented algebraic operations for soft sets. Recently, Shabir et al. [12] initiated the study of soft topological spaces. Later soft topological spaces are studied in [2], [3], [8], [13], [15], [16] and [17].

In GIS, the term spatial data model refers to the representation and organization of spatial data. This model represent various kinds of spatial objects in the real world. Topological relationships are invariant under topological trans-

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formations such as translation, scaling and rotation. Topological relationships form a subset of spatial relationships. One of the major issue in data modeling for GIS is the representation of the topological relationships between spatial objects.

Topological relationships between spatial objects have gained much attention by the researchers during last two decades. Egenhofer and Franzosa [6] provided the 4-intersection model and Egenhofer and Herring [7] proposed the 9-intersection model for topological relations between two simple regions which are defined in the crisp topological spaces. These models are then generalized in different ways by different researchers. Most of these theoretical results applied in GIS software such as spatial reasoning , spatial querying and analysis.

Research had been conducted for representing topological relationships between regions with indeterminate boundaries by extending the 9-intersection model. Clementite and Felice [4] proposed an model by extending a region with a broad boundary. Cohn and Gotts [5] framed "egg-yolk" representation model and constructed a set with 46 relationships. This approach is similar as the relationships between simple regions with sharp boundaries to describe topological predicates between the objects with undetermined boundary by egg-yolk pairs.

In 2015, spatial object modeling in soft topology was coined by Khalil et al. [9]. In line the soft topological relations between spatial soft regions are framed. The soft region in this model has a sharp soft boundary. In this paper, we propose a model for spatial soft regions with uncertain soft boundaries. This model is an extension of an existing model of representing simple spatial soft regions with sharp soft boundaries.

Section 2 recalls the definitions of soft sets and soft topological spaces. Section 3 introduces the soft regions with broad soft boundary. Then in Section 4 we investigate the soft 4-intersection model for simple spatial soft regions with sharp boundaries and also for soft regions with broad soft boundaries. Section 5 discusses the soft 9-intersection model for the soft regions with sharp and broad soft boundaries.

2. Preliminaries

Throughout this paper X be an universal set and E be a parameter space.

A pair (F, E) is called a soft set [11] over X where $F : E \to 2^X$ is a mapping. $S(X, E)$ denotes the collection of all soft sets over X with parameter space E. We denote (F, E) by $\widetilde{F}$ in which case we write $\widetilde{F} = \{(e, F(e)) : e \in E\}$. In

[10] the following terms are defined, For any two soft sets $\widetilde{F}$ and $\widetilde{G}$ in $S(X,E)$, $\widetilde{F}$ is a soft subset of $\widetilde{G}$ (in brief $\widetilde{F} \subseteq \widetilde{G}$) if $F(e) \subseteq G(e)$ for all $e \in E$ and $\widetilde{F}$ and $\widetilde{G}$ are soft equal if and only if $F(e) = G(e)$ for all $e \in E$. That is $\widetilde{F} = \widetilde{G}$ if $\widetilde{F} \subseteq \widetilde{G}$ and $\widetilde{G} \subseteq \widetilde{F}$. The soft null and soft absolute sets are defined as $\widetilde{\Phi} = \{(e,\phi) : e \in E\} = \{(e,\Phi(e)) : e \in E\} = (\Phi, E)$. $\widetilde{X} = \{(e,X) : e \in E\} = \{(e,X(e)) : e \in E\} = (X,E)$. The union of two soft sets $\widetilde{F}$ and $\widetilde{G}$ is $\widetilde{F} \cup \widetilde{G} = (F \cup G, E)$ where $(F \cup G)(e) = F(e) \cup G(e)$ for all $e \in E$ and the intersection of two soft sets $\widetilde{F}$ and $\widetilde{G}$ is $\widetilde{F} \cap \widetilde{G} = (F \cap G, E)$ where $(F \cap G)(e) = F(e) \cap G(e)$ for all $e \in E$. If $\{\widetilde{F}_\alpha : \alpha \in \Delta\}$ is a collection of soft sets in $S(X,E)$ then the arbitrary union and the arbitrary intersection of soft sets are defined as $\bigcup\{\widetilde{F}_\alpha : \alpha \in \Delta\} = (\bigcup\{F_\alpha : \alpha \in \Delta\}, E)$ and $\bigcap\{\widetilde{F}_\alpha : \alpha \in \Delta\} = (\bigcap\{F_\alpha : \alpha \in \Delta\}, E)$ where $(\bigcup\{F_\alpha : \alpha \in \Delta\})(e) = \bigcup\{F_\alpha(e) : \alpha \in \Delta\}$ and $(\bigcap\{F_\alpha : \alpha \in \Delta\})(e) = \bigcap\{F_\alpha(e) : \alpha \in \Delta\}$, for all $e \in E$. The complement of a soft set $\widetilde{F}$ is denoted by $(\widetilde{F})' = (F', E)$ (relative complement in the sense of Ifran Ali et al. [1]), where $F' : E \to 2^X$ is a mapping given by $F'(e) = X - F(e)$ for all $e \in E$.

Definition 1. ([12]) Let $\widetilde{\tau}$ be a collection of soft subset of $\widetilde{X}$. Then $\widetilde{\tau}$ is said to be a soft topology on X with parameter space E if (i) $\widetilde{\Phi}, \widetilde{X} \in \widetilde{\tau}$, (ii) $\widetilde{\tau}$ is closed under arbitrary union, and (iii) $\widetilde{\tau}$ is closed under finite intersection.

If $\widetilde{\tau}$ is a soft topology on X with a parameter space E then the triplet $(X, E, \widetilde{\tau})$ is called a soft topological space over X with parameter space E. Identifying (X,E) with $\widetilde{X}$, $(\widetilde{X}, \widetilde{\tau})$ is a soft topological space.

The members of $\widetilde{\tau}$ are called soft open sets in $(X, E, \widetilde{\tau})$. A soft set $\widetilde{F}$ in $S(X,E)$ is soft closed in $(X, E, \widetilde{\tau})$, if its complement $(\widetilde{F})'$ belongs to $\widetilde{\tau}$. $(\widetilde{\tau})'$ denotes the collection of all soft closed sets in $(X, E, \widetilde{\tau})$. The soft closure of $\widetilde{F}$ is the soft set, $\overline{(\widetilde{F})} = \cap\{\widetilde{G} : \widetilde{G}$ is soft closed and $\widetilde{F} \subseteq \widetilde{G}\}$. The soft interior [13] of $\widetilde{F}$ is the soft set, $(\widetilde{F})^o = \cup\{\widetilde{O} : \widetilde{O}$ is soft open and $\widetilde{O} \subseteq \widetilde{F}\}$.

Definition 2. Let $(X, E, \widetilde{\tau})$ be a soft topological space over X with parameter space E. Then

(i) The soft exterior [13] of the soft set $\widetilde{F}$(denoted by $\widetilde{F}_o$) is defined by $\widetilde{F}_o = \left((\widetilde{F})'\right)^o$.

(ii) The soft boundary [13] of a soft set $\widetilde{F}$(denoted by $\partial\widetilde{F}$) is defined as $\partial\widetilde{F} = \overline{\widetilde{F}} \cap \overline{(\widetilde{F})'}$.

Definition 3. ([14]) Let $(X, E, \widetilde{\tau})$ be a soft topological space over X. Then $(X, E, \widetilde{\tau})$ is said to be soft connected, if there does not exist a pair $\widetilde{F}$ and $\widetilde{G}$ of nonempty disjoint soft open subsets of $(X, E, \widetilde{\tau})$ such that $\widetilde{X} = \widetilde{F} \cup \widetilde{G}$. Otherwise $(X, E, \widetilde{\tau})$ is said to be soft disconnected. In this case, the pair $\widetilde{F}$ and $\widetilde{G}$ is called the soft disconnection of X.

Definition 4. Let $(X, E, \widetilde{\tau})$ be a soft connected topological space over X. A spatial soft region[9] in X is a non-null soft regular closed set $\widetilde{F}$ whose interior is soft connected.

Egenhofer et al. [6] and [7] and Clementini et al. [4] introduced 4-intersection and 9-intersection models for spatial regions with sharp and broad boundaries using topological relations. Khalil et al. [9] introduced the model that describes the soft topological relations between two soft sets using four soft intersections of the soft boundary and soft interior.

3. The spatial data model for soft regions with broad boundaries

In this section we introduce soft regions with broad boundaries.

Definition 5. A soft region with a broad soft boundary $\widetilde{F}$ is made up of two soft regions $\widetilde{F}_1$ and $\widetilde{F}_2$ with $\widetilde{F}_1 \subseteq \widetilde{F}_2$, where $\partial\widetilde{F}_1$ is the inner soft boundary of $\widetilde{F}$ and $\partial\widetilde{F}_2$ is the outer soft boundary of $\widetilde{F}$.

Definition 6. The broad soft boundary $\Delta\widetilde{F}$ of a soft region with a broad soft boundary $\widetilde{F}$ is a soft closed connected subset of $\widetilde{X}$ with a soft hole.

$\Delta\widetilde{F}$ comprises the soft area between the inner soft boundary and the outer soft boundary of $\widetilde{F}$, such that $\Delta\widetilde{F} = \overline{\widetilde{F}_2 - \widetilde{F}_1}$ or $\Delta\widetilde{F} = \widetilde{F}_2 - \widetilde{F}_1^o$.

Definition 7. A soft hole of $\widetilde{F}$ denoted by $\widetilde{F}_H$ is the soft closure of an inner soft exterior. That is $\widetilde{F}_H = \overline{\widetilde{F}_o}$

Definition 8. The soft interior of a soft region with a broad soft boundary $\widetilde{F}$ is defined as: $\widetilde{F}^o = \widetilde{F}_2 - \Delta\widetilde{F}$.

Definition 9. The soft exterior of a soft region with a broad soft boundary $\widetilde{F}$ is defined as: $\widetilde{F}_o = \widetilde{X} - \widetilde{F}_2$.

Definition 10. The soft closure of a soft region with a broad soft boundary $\widetilde{F}$ is defined as: $\overline{\widetilde{F}} = \widetilde{F}^o \cup \Delta\widetilde{F}$.

4. Geometric conditions for Soft topological spatial relations

The following are the set of geometric conditions that are valid for simple soft regions.

4.1. Soft regions with sharp soft boundary

- S-(I) The soft exteriors of two soft regions intersect with each other;
- S-(II) Soft boundary of $\widetilde{F}$ intersect with atleast one part of $\widetilde{G}$, and vice versa;
- S-(III) If soft interior of $\widetilde{F}$ intersects with soft interior and soft exterior of $\widetilde{G}$, then it must also intersect with soft boundary of $\widetilde{G}$ and vice versa;
- S-(IV) If both soft boundaries do not intersect with each other then atleast one soft boundary must intersect with the opposite soft exterior;
- S-(V) If both soft boundaries intersect with the opposite soft interiors then the soft boundaries must also intersect with each other;
- S-(VI) If soft interior of $\widetilde{F}$ intersects with soft exterior of $\widetilde{G}$ then soft boundary of $\widetilde{F}$ must also intersect with soft exterior of $\widetilde{G}$;
- S-(VII) If both soft interiors are disjoint then soft interior of $\widetilde{F}$ intersects with soft exterior of $\widetilde{G}$ and vice versa;
- S-(VIII) If soft interiors of $\widetilde{F}$ is a subset of the soft closure of $\widetilde{G}$ then soft boundary of $\widetilde{F}$ must be a subset of soft closure of $\widetilde{G}$ as well, and vice versa;
- S-(IX) If soft exterior of $\widetilde{F}$ intersects with soft boundary of $\widetilde{G}$ then it must also intersect with soft exterior of $\widetilde{G}$, and vice versa;
- S-(X) If both soft interiors are disjoint then soft boundary of $\widetilde{F}$ cannot intersect with soft interior of $\widetilde{G}$ and vice versa;
- S-(XI) If the soft interiors do not intersect with each other then soft boundary of $\widetilde{F}$ must intersect with soft exterior of $\widetilde{G}$ and vice versa;

S-(XII) If both soft interiors do not intersect with each other then atleast one soft boundary must intersect with its opposite soft exterior.

4.2. Soft regions with broad soft boundary

The geometric conditions that are valid for simple soft regions with sharp soft boundaries, can be extended to soft regions with broad soft boundaries simply by replacing the soft boundary by broad soft boundary. But some of the geometric conditions that are valid for simple soft regions with sharp soft boundaries are not reliable for soft regions with broad soft boundaries. These conditions are replaced by the conditions that is reliable for broad soft boundary. The following are the geometric conditions for soft regions with broad soft boundary.

B-(I) The soft exteriors of two soft regions intersect with each other;

B-(II) The broad soft boundary of $\widetilde{F}$ intersect with atleast one part of $\widetilde{G}$, and vice versa;

B-(III) If soft interior of $\widetilde{F}$ intersects with soft interior and soft exterior of $\widetilde{G}$, then it must also intersect with broad soft boundary of $\widetilde{G}$ and vice versa;

B-(IV) If both broad soft boundaries do not intersect with each other then atleast one broad soft boundary must intersect with the opposite soft exterior;

B-(V) If both broad soft boundaries intersect with the opposite soft interiors then the broad soft boundaries must also intersect with each other;

B-(VI) If soft interior of $\widetilde{F}$ intersects with soft exterior of $\widetilde{G}$ then broad soft boundary of $\widetilde{F}$ must also intersect with soft exterior of $\widetilde{G}$;

B-(VII) If both soft interiors are disjoint and broad soft boundary of $\widetilde{F}$ intersects with soft interior of $\widetilde{G}$ then the two broad soft boundaries must intersect with each other, and vice versa;

B-(VIII) If soft interiors of $\widetilde{F}$ is a subset of the soft closure of $\widetilde{G}$ then broad soft boundary of $\widetilde{F}$ must intersect with soft closure of $\widetilde{G}$, and vice versa;

B-(IX) If both soft interiors are disjoint, then soft interior of $\widetilde{F}$ intersects either with broad boundary of $\widetilde{G}$ or with soft exterior of $\widetilde{G}$, and vice versa;

B-(X) If soft interior of $\widetilde{F}$ does not intersect with soft closure of $\widetilde{G}$, then broad soft boundary of $\widetilde{F}$ must intersect with soft exterior of $\widetilde{G}$, and vice versa;

B-(XI) If broad soft boundary of $\widetilde{F}$ intersects with soft interior and soft exterior of $\widetilde{G}$, then it must also intersect with broad soft boundary of $\widetilde{G}$, and vice versa;

B-(XII) If soft closure of $\widetilde{F}$ is a subset of soft interior of $\widetilde{G}$, then soft exterior of $\widetilde{F}$ must intersect with soft interior of $\widetilde{G}$, and vice versa.

5. The soft 4-intersection model for soft regions

In this section we discuss the soft 4-intersection models for two simple soft regions in the soft topological spaces.

5.1. The soft 4-intersection for sharp soft boundary

The soft topological relations between two spatial regions $\widetilde{F}$ and $\widetilde{G}$ of the soft topological space $(X, E, \widetilde{\tau})$ based on the four intersections of the soft interior and soft boundary of $\widetilde{F}$ and $\widetilde{G}$. These four soft region parts describe a soft topological relation and can be represented in a 2×2 matrix called the four intersection matrix. Assigning the values null $(\tilde{\Phi})$ and non-null$(\neg\Phi)$ to the entries in the 2×2 matrix we can distinguish between $2^4 = 16$ soft topological relations,

$$\begin{pmatrix} \widetilde{F}^o \cap \widetilde{G}^o & \widetilde{F}^o \cap \partial\widetilde{G} \\ \partial\widetilde{F} \cap \widetilde{G}^o & \partial\widetilde{F} \cap \partial\widetilde{G} \end{pmatrix}$$

The 16 possible 2×2 matrix are:

$$\underset{1}{\begin{pmatrix} \tilde{\Phi} & \tilde{\Phi} \\ \tilde{\Phi} & \tilde{\Phi} \end{pmatrix}} \quad \underset{2}{\begin{pmatrix} \neg\tilde{\Phi} & \tilde{\Phi} \\ \tilde{\Phi} & \tilde{\Phi} \end{pmatrix}} \quad \underset{3}{\begin{pmatrix} \neg\tilde{\Phi} & \neg\tilde{\Phi} \\ \tilde{\Phi} & \tilde{\Phi} \end{pmatrix}} \quad \underset{4}{\begin{pmatrix} \neg\tilde{\Phi} & \neg\tilde{\Phi} \\ \neg\tilde{\Phi} & \tilde{\Phi} \end{pmatrix}}$$

$$\underset{5}{\begin{pmatrix} \neg\tilde{\Phi} & \neg\tilde{\Phi} \\ \neg\tilde{\Phi} & \neg\tilde{\Phi} \end{pmatrix}} \quad \underset{6}{\begin{pmatrix} \tilde{\Phi} & \neg\tilde{\Phi} \\ \tilde{\Phi} & \tilde{\Phi} \end{pmatrix}} \quad \underset{7}{\begin{pmatrix} \tilde{\Phi} & \tilde{\Phi} \\ \neg\tilde{\Phi} & \tilde{\Phi} \end{pmatrix}} \quad \underset{8}{\begin{pmatrix} \tilde{\Phi} & \tilde{\Phi} \\ \tilde{\Phi} & \neg\tilde{\Phi} \end{pmatrix}}$$

$$\underset{9}{\begin{pmatrix} \tilde{\Phi} & \tilde{\Phi} \\ \neg\tilde{\Phi} & \neg\tilde{\Phi} \end{pmatrix}} \quad \underset{10}{\begin{pmatrix} \neg\tilde{\Phi} & \tilde{\Phi} \\ \neg\tilde{\Phi} & \tilde{\Phi} \end{pmatrix}} \quad \underset{11}{\begin{pmatrix} \tilde{\Phi} & \neg\tilde{\Phi} \\ \tilde{\Phi} & \neg\tilde{\Phi} \end{pmatrix}} \quad \underset{12}{\begin{pmatrix} \neg\tilde{\Phi} & \neg\tilde{\Phi} \\ \tilde{\Phi} & \neg\tilde{\Phi} \end{pmatrix}}$$

$$\begin{pmatrix} \tilde{\Phi} & \neg\tilde{\Phi} \\ \neg\tilde{\Phi} & \neg\tilde{\Phi} \end{pmatrix} \quad \begin{pmatrix} \neg\tilde{\Phi} & \tilde{\Phi} \\ \neg\tilde{\Phi} & \neg\tilde{\Phi} \end{pmatrix} \quad \begin{pmatrix} \tilde{\Phi} & \neg\tilde{\Phi} \\ \neg\tilde{\Phi} & \tilde{\Phi} \end{pmatrix} \quad \begin{pmatrix} \neg\tilde{\Phi} & \tilde{\Phi} \\ \tilde{\Phi} & \neg\tilde{\Phi} \end{pmatrix}$$

13 14 15 16

For two simple regions with well defined soft boundaries only eight of them can be realized by applying the geometric conditions S-I to S-XII.

$$\begin{pmatrix} \tilde{\Phi} & \tilde{\Phi} \\ \tilde{\Phi} & \tilde{\Phi} \end{pmatrix} \quad \begin{pmatrix} \tilde{\Phi} & \tilde{\Phi} \\ \tilde{\Phi} & \neg\tilde{\Phi} \end{pmatrix} \quad \begin{pmatrix} \neg\tilde{\Phi} & \tilde{\Phi} \\ \tilde{\Phi} & \neg\tilde{\Phi} \end{pmatrix} \quad \begin{pmatrix} \neg\tilde{\Phi} & \tilde{\Phi} \\ \neg\tilde{\Phi} & \tilde{\Phi} \end{pmatrix}$$

case 1 *case* 2 *case* 3 *case* 4

$$\begin{pmatrix} \neg\tilde{\Phi} & \tilde{\Phi} \\ \neg\tilde{\Phi} & \neg\tilde{\Phi} \end{pmatrix} \quad \begin{pmatrix} \neg\tilde{\Phi} & \neg\tilde{\Phi} \\ \tilde{\Phi} & \tilde{\Phi} \end{pmatrix} \quad \begin{pmatrix} \neg\tilde{\Phi} & \neg\tilde{\Phi} \\ \tilde{\Phi} & \neg\tilde{\Phi} \end{pmatrix} \quad \begin{pmatrix} \neg\tilde{\Phi} & \neg\tilde{\Phi} \\ \neg\tilde{\Phi} & \neg\tilde{\Phi} \end{pmatrix}$$

case 5 *case* 6 *case* 7 *case* 8

The 4-soft intersection matrix 2, 4, 6, 7, 9, 11, 13, 15 are not reliable. The matrix 2 is not reliable due to the geometric conditions S-II and S-III. Due to the geomertic condition S-V the matrix 4 and 15 are not reliable. The matrix 6 is not reliable due to the geometric condition S-II. By applying the geometric condition S-X the intersection matrix 7, 9 and 11 are not valid. The matrix 13 is not reliable due to the geometric condition S-III.

5.2. The soft 4-intersection model for soft regions with broad soft boundary

In this section, we determine all the reliable soft 4-intersection matrices for two soft regions with broad soft boundaries. These soft regions differ from simple soft region with regard to the definition of soft boundary. Simple soft regions can be seen as special cases of soft regions with broad soft boundaries in which the inner and outer soft boundaries coincide that is $\Delta\widetilde{F} = \partial\widetilde{F}$. The four intersection matrices that we consider have a broad soft boundary in place of the sharp soft boundary of the soft region. Therefore, we redefine the soft 4-intersection for soft regions with broad soft boundaries as follows:

$$\begin{pmatrix} \widetilde{F}^o \cap \widetilde{G}^o & \widetilde{F}^o \cap \Delta\widetilde{G} \\ \Delta\widetilde{F} \cap \widetilde{G}^o & \Delta\widetilde{F} \cap \Delta\widetilde{G} \end{pmatrix}$$

By applying the geometric conditions B-I to B-XII, it is possible to reduce the 4^2 matrices to 11 matrices, for soft regions with broad soft boundaries exists. These conditions are extended from the conditions for the soft regions with sharp soft boundary by changing the occurrence of soft boundary with broad soft boundary. The 4-intersection matrices with $\tilde{\Phi}$ and $\neg\tilde{\Phi}$ values are 16. The 11 realizable soft 4- intersection matrices for soft regions with broad soft boundaries are as follows:

$$\underset{case\ 1}{\begin{pmatrix}\tilde{\Phi} & \tilde{\Phi}\\ \tilde{\Phi} & \tilde{\Phi}\end{pmatrix}} \quad \underset{case\ 2}{\begin{pmatrix}\tilde{\Phi} & \tilde{\Phi}\\ \tilde{\Phi} & \neg\tilde{\Phi}\end{pmatrix}} \quad \underset{case\ 3}{\begin{pmatrix}\tilde{\Phi} & \neg\tilde{\Phi}\\ \tilde{\Phi} & \neg\tilde{\Phi}\end{pmatrix}} \quad \underset{case\ 4}{\begin{pmatrix}\tilde{\Phi} & \tilde{\Phi}\\ \neg\tilde{\Phi} & \neg\tilde{\Phi}\end{pmatrix}}$$

$$\underset{case\ 5}{\begin{pmatrix}\tilde{\Phi} & \neg\tilde{\Phi}\\ \neg\tilde{\Phi} & \neg\tilde{\Phi}\end{pmatrix}} \quad \underset{case\ 6}{\begin{pmatrix}\neg\tilde{\Phi} & \neg\tilde{\Phi}\\ \neg\tilde{\Phi} & \neg\tilde{\Phi}\end{pmatrix}} \quad \underset{case\ 7}{\begin{pmatrix}\neg\tilde{\Phi} & \tilde{\Phi}\\ \neg\tilde{\Phi} & \neg\tilde{\Phi}\end{pmatrix}} \quad \underset{case\ 8}{\begin{pmatrix}\neg\tilde{\Phi} & \neg\tilde{\Phi}\\ \tilde{\Phi} & \neg\tilde{\Phi}\end{pmatrix}}$$

$$\underset{case\ 9}{\begin{pmatrix}\neg\tilde{\Phi} & \tilde{\Phi}\\ \neg\tilde{\Phi} & \tilde{\Phi}\end{pmatrix}} \quad \underset{case\ 10}{\begin{pmatrix}\neg\tilde{\Phi} & \neg\tilde{\Phi}\\ \tilde{\Phi} & \tilde{\Phi}\end{pmatrix}} \quad \underset{case\ 11}{\begin{pmatrix}\neg\tilde{\Phi} & \tilde{\Phi}\\ \tilde{\Phi} & \neg\tilde{\Phi}\end{pmatrix}}$$

These matrices corresponds to the eight soft topological relations that also hold for soft regions with sharp soft boundaries in addition to that three more matrices. The five matrices that are not reliable are 2, 4, 6, 7, 15. The matrix 2 is not valid due to the geometric condition B-III and B-VII. Due to the geometric condition B-VII the matrix 6 and 7 are not reliable. The matrix 4 and 15 are impossible from the condition B-V.

6. The soft 9-intersection model for soft regions

In this section we discuss the soft 9-intersection models for two soft regions in soft topological spaces.

6.1. The soft 9-intersection model for soft regions with sharp soft boundary

The soft topological relations between two soft regions, $\widetilde{F}$ and $\widetilde{G}$ of the soft topological space $(X, E, \widetilde{\tau})$ is based on the intersection of $\widetilde{F}$'s soft interior, soft boundary and soft exterior with $\widetilde{G}$'s soft interior, soft soft boundary and soft exterior. The nine intersections between the six soft region parts describe a soft topological relation and can be represented in a 3×3 matrix called the soft 9-intersection matrix,

$$\begin{pmatrix}\widetilde{F}^o \cap \widetilde{G}^o & \widetilde{F}^o \cap \partial\widetilde{G} & \widetilde{F}^o \cap \widetilde{G}_o\\ \partial\widetilde{F} \cap \widetilde{G}^o & \partial\widetilde{F} \cap \partial\widetilde{G} & \partial\widetilde{F} \cap \widetilde{G}_o\\ \widetilde{F}_o \cap \widetilde{G}^o & \widetilde{F}_o \cap \partial\widetilde{G} & \widetilde{F}_o \cap \widetilde{G}_o.\end{pmatrix}$$

By assigning the values of null($\tilde{\Phi}$) and non-null ($\neg\tilde{\Phi}$) to the entries in the 3×3 matrix we can distinguish between $2^9 = 512$ soft topological relations. By applying the geometric conditions S-I to S-XII, for two simple soft regions with sharp soft boundaries it is possible to reduce 2^9 matrices to 8 matrices. The eight matrices corresponding to the soft topological relations between simple soft regions with sharp soft boundaries are:

$$\underset{1}{\begin{pmatrix} \tilde{\Phi} & \tilde{\Phi} & \neg\tilde{\Phi} \\ \tilde{\Phi} & \tilde{\Phi} & \neg\tilde{\Phi} \\ \neg\tilde{\Phi} & \neg\tilde{\Phi} & \neg\tilde{\Phi} \end{pmatrix}} \quad \underset{2}{\begin{pmatrix} \neg\tilde{\Phi} & \neg\tilde{\Phi} & \neg\tilde{\Phi} \\ \tilde{\Phi} & \tilde{\Phi} & \neg\tilde{\Phi} \\ \tilde{\Phi} & \tilde{\Phi} & \neg\tilde{\Phi} \end{pmatrix}} \quad \underset{3}{\begin{pmatrix} \neg\tilde{\Phi} & \tilde{\Phi} & \tilde{\Phi} \\ \neg\tilde{\Phi} & \tilde{\Phi} & \tilde{\Phi} \\ \neg\tilde{\Phi} & \neg\tilde{\Phi} & \neg\tilde{\Phi} \end{pmatrix}}$$

$$\underset{4}{\begin{pmatrix} \neg\tilde{\Phi} & \tilde{\Phi} & \tilde{\Phi} \\ \tilde{\Phi} & \neg\tilde{\Phi} & \tilde{\Phi} \\ \tilde{\Phi} & \tilde{\Phi} & \neg\tilde{\Phi} \end{pmatrix}} \quad \underset{5}{\begin{pmatrix} \tilde{\Phi} & \tilde{\Phi} & \neg\tilde{\Phi} \\ \tilde{\Phi} & \neg\tilde{\Phi} & \neg\tilde{\Phi} \\ \neg\tilde{\Phi} & \neg\tilde{\Phi} & \neg\tilde{\Phi} \end{pmatrix}} \quad \underset{6}{\begin{pmatrix} \neg\tilde{\Phi} & \neg\tilde{\Phi} & \neg\tilde{\Phi} \\ \tilde{\Phi} & \neg\tilde{\Phi} & \neg\tilde{\Phi} \\ \tilde{\Phi} & \tilde{\Phi} & \neg\tilde{\Phi} \end{pmatrix}}$$

$$\underset{7}{\begin{pmatrix} \neg\tilde{\Phi} & \tilde{\Phi} & \tilde{\Phi} \\ \neg\tilde{\Phi} & \neg\tilde{\Phi} & \tilde{\Phi} \\ \neg\tilde{\Phi} & \neg\tilde{\Phi} & \neg\tilde{\Phi} \end{pmatrix}} \quad \underset{8}{\begin{pmatrix} \neg\tilde{\Phi} & \neg\tilde{\Phi} & \neg\tilde{\Phi} \\ \neg\tilde{\Phi} & \neg\tilde{\Phi} & \neg\tilde{\Phi} \\ \neg\tilde{\Phi} & \neg\tilde{\Phi} & \neg\tilde{\Phi} \end{pmatrix}} \; .$$

6.2. The soft 9- intersection model for soft regions with broad soft boundaries

In this section, we determine all the reliable soft 9-intersection matrices for two soft regions with broad soft boundaries. The soft 9-intersection for soft regions with broad soft boundaries are redefined as:

$$\begin{pmatrix} \widetilde{F}^o \cap \widetilde{G}^o & \widetilde{F}^o \cap \Delta\widetilde{G} & \widetilde{F}^o \cap \widetilde{G}_o \\ \Delta\widetilde{F} \cap \widetilde{G}^o & \Delta\widetilde{F} \cap \Delta\widetilde{G} & \Delta\widetilde{F} \cap \widetilde{G}_o \\ \widetilde{F}_o \cap \widetilde{G}^o & \widetilde{F}_o \cap \Delta\widetilde{G} & \widetilde{F}_o \cap \widetilde{G}_o \end{pmatrix}.$$

By applying the geometric conditions B-I to B-XII it is possible to reduce the 2^9 matrices to 44 matrices. The soft 9-intersection matrices with $\tilde{\Phi}$ and $\neg\tilde{\Phi}$ values are 512. The 44 reliable soft 9-intersection matrices for soft regions with broad soft boundaries are as follows:

$$\underset{1}{\begin{pmatrix} \tilde{\Phi} & \tilde{\Phi} & \neg\tilde{\Phi} \\ \tilde{\Phi} & \tilde{\Phi} & \neg\tilde{\Phi} \\ \neg\tilde{\Phi} & \neg\tilde{\Phi} & \neg\tilde{\Phi} \end{pmatrix}} \quad \underset{2}{\begin{pmatrix} \tilde{\Phi} & \tilde{\Phi} & \neg\tilde{\Phi} \\ \tilde{\Phi} & \neg\tilde{\Phi} & \neg\tilde{\Phi} \\ \neg\tilde{\Phi} & \neg\tilde{\Phi} & \neg\tilde{\Phi} \end{pmatrix}} \quad \underset{3}{\begin{pmatrix} \tilde{\Phi} & \neg\tilde{\Phi} & \neg\tilde{\Phi} \\ \tilde{\Phi} & \neg\tilde{\Phi} & \neg\tilde{\Phi} \\ \neg\tilde{\Phi} & \neg\tilde{\Phi} & \neg\tilde{\Phi} \end{pmatrix}}$$

$$\underset{4}{\begin{pmatrix} \tilde{\Phi} & \neg\tilde{\Phi} & \tilde{\Phi} \\ \tilde{\Phi} & \neg\tilde{\Phi} & \neg\tilde{\Phi} \\ \neg\tilde{\Phi} & \neg\tilde{\Phi} & \neg\tilde{\Phi} \end{pmatrix}} \qquad \underset{5}{\begin{pmatrix} \tilde{\Phi} & \neg\tilde{\Phi} & \tilde{\Phi} \\ \tilde{\Phi} & \neg\tilde{\Phi} & \tilde{\Phi} \\ \neg\tilde{\Phi} & \neg\tilde{\Phi} & \neg\tilde{\Phi} \end{pmatrix}} \qquad \underset{6}{\begin{pmatrix} \tilde{\Phi} & \tilde{\Phi} & \neg\tilde{\Phi} \\ \neg\tilde{\Phi} & \neg\tilde{\Phi} & \neg\tilde{\Phi} \\ \neg\tilde{\Phi} & \neg\tilde{\Phi} & \neg\tilde{\Phi} \end{pmatrix}}$$

$$\underset{7}{\begin{pmatrix} \tilde{\Phi} & \tilde{\Phi} & \neg\tilde{\Phi} \\ \neg\tilde{\Phi} & \neg\tilde{\Phi} & \neg\tilde{\Phi} \\ \tilde{\Phi} & \neg\tilde{\Phi} & \neg\tilde{\Phi} \end{pmatrix}} \qquad \underset{8}{\begin{pmatrix} \tilde{\Phi} & \tilde{\Phi} & \neg\tilde{\Phi} \\ \neg\tilde{\Phi} & \neg\tilde{\Phi} & \tilde{\Phi} \\ \tilde{\Phi} & \tilde{\Phi} & \neg\tilde{\Phi} \end{pmatrix}} \qquad \underset{9}{\begin{pmatrix} \tilde{\Phi} & \neg\tilde{\Phi} & \neg\tilde{\Phi} \\ \neg\tilde{\Phi} & \neg\tilde{\Phi} & \neg\tilde{\Phi} \\ \neg\tilde{\Phi} & \neg\tilde{\Phi} & \neg\tilde{\Phi} \end{pmatrix}}$$

$$\underset{10}{\begin{pmatrix} \tilde{\Phi} & \neg\tilde{\Phi} & \tilde{\Phi} \\ \neg\tilde{\Phi} & \neg\tilde{\Phi} & \neg\tilde{\Phi} \\ \neg\tilde{\Phi} & \neg\tilde{\Phi} & \neg\tilde{\Phi} \end{pmatrix}} \qquad \underset{11}{\begin{pmatrix} \tilde{\Phi} & \neg\tilde{\Phi} & \tilde{\Phi} \\ \neg\tilde{\Phi} & \neg\tilde{\Phi} & \tilde{\Phi} \\ \neg\tilde{\Phi} & \neg\tilde{\Phi} & \neg\tilde{\Phi} \end{pmatrix}} \qquad \underset{12}{\begin{pmatrix} \tilde{\Phi} & \neg\tilde{\Phi} & \neg\tilde{\Phi} \\ \neg\tilde{\Phi} & \neg\tilde{\Phi} & \neg\tilde{\Phi} \\ \tilde{\Phi} & \neg\tilde{\Phi} & \neg\tilde{\Phi} \end{pmatrix}}$$

$$\underset{13}{\begin{pmatrix} \tilde{\Phi} & \neg\tilde{\Phi} & \neg\tilde{\Phi} \\ \neg\tilde{\Phi} & \neg\tilde{\Phi} & \neg\tilde{\Phi} \\ \tilde{\Phi} & \tilde{\Phi} & \neg\tilde{\Phi} \end{pmatrix}} \qquad \underset{14}{\begin{pmatrix} \tilde{\Phi} & \neg\tilde{\Phi} & \tilde{\Phi} \\ \neg\tilde{\Phi} & \neg\tilde{\Phi} & \neg\tilde{\Phi} \\ \tilde{\Phi} & \neg\tilde{\Phi} & \neg\tilde{\Phi} \end{pmatrix}} \qquad \underset{15}{\begin{pmatrix} \tilde{\Phi} & \neg\tilde{\Phi} & \tilde{\Phi} \\ \neg\tilde{\Phi} & \neg\tilde{\Phi} & \neg\tilde{\Phi} \\ \tilde{\Phi} & \tilde{\Phi} & \neg\tilde{\Phi} \end{pmatrix}}$$

$$\underset{16}{\begin{pmatrix} \tilde{\Phi} & \neg\tilde{\Phi} & \tilde{\Phi} \\ \tilde{\Phi} & \tilde{\Phi} & \neg\tilde{\Phi} \\ \tilde{\Phi} & \neg\tilde{\Phi} & \neg\tilde{\Phi} \end{pmatrix}} \qquad \underset{17}{\begin{pmatrix} \tilde{\Phi} & \neg\tilde{\Phi} & \tilde{\Phi} \\ \neg\tilde{\Phi} & \neg\tilde{\Phi} & \tilde{\Phi} \\ \tilde{\Phi} & \tilde{\Phi} & \neg\tilde{\Phi} \end{pmatrix}} \qquad \underset{18}{\begin{pmatrix} \neg\tilde{\Phi} & \neg\tilde{\Phi} & \neg\tilde{\Phi} \\ \neg\tilde{\Phi} & \neg\tilde{\Phi} & \neg\tilde{\Phi} \\ \neg\tilde{\Phi} & \neg\tilde{\Phi} & \neg\tilde{\Phi} \end{pmatrix}}$$

$$\underset{19}{\begin{pmatrix} \neg\tilde{\Phi} & \neg\tilde{\Phi} & \tilde{\Phi} \\ \neg\tilde{\Phi} & \neg\tilde{\Phi} & \neg\tilde{\Phi} \\ \tilde{\Phi} & \neg\tilde{\Phi} & \neg\tilde{\Phi} \end{pmatrix}} \qquad \underset{20}{\begin{pmatrix} \neg\tilde{\Phi} & \neg\tilde{\Phi} & \tilde{\Phi} \\ \neg\tilde{\Phi} & \neg\tilde{\Phi} & \tilde{\Phi} \\ \neg\tilde{\Phi} & \neg\tilde{\Phi} & \neg\tilde{\Phi} \end{pmatrix}} \qquad \underset{21}{\begin{pmatrix} \neg\tilde{\Phi} & \neg\tilde{\Phi} & \neg\tilde{\Phi} \\ \neg\tilde{\Phi} & \neg\tilde{\Phi} & \neg\tilde{\Phi} \\ \tilde{\Phi} & \neg\tilde{\Phi} & \neg\tilde{\Phi} \end{pmatrix}}$$

$$\underset{22}{\begin{pmatrix} \neg\tilde{\Phi} & \neg\tilde{\Phi} & \neg\tilde{\Phi} \\ \neg\tilde{\Phi} & \neg\tilde{\Phi} & \neg\tilde{\Phi} \\ \tilde{\Phi} & \tilde{\Phi} & \neg\tilde{\Phi} \end{pmatrix}} \qquad \underset{23}{\begin{pmatrix} \neg\tilde{\Phi} & \neg\tilde{\Phi} & \tilde{\Phi} \\ \neg\tilde{\Phi} & \neg\tilde{\Phi} & \neg\tilde{\Phi} \\ \tilde{\Phi} & \neg\tilde{\Phi} & \neg\tilde{\Phi} \end{pmatrix}} \qquad \underset{24}{\begin{pmatrix} \neg\tilde{\Phi} & \neg\tilde{\Phi} & \tilde{\Phi} \\ \neg\tilde{\Phi} & \neg\tilde{\Phi} & \tilde{\Phi} \\ \tilde{\Phi} & \neg\tilde{\Phi} & \neg\tilde{\Phi} \end{pmatrix}}$$

$$\underset{25}{\begin{pmatrix} \neg\tilde{\Phi} & \neg\tilde{\Phi} & \tilde{\Phi} \\ \neg\tilde{\Phi} & \neg\tilde{\Phi} & \neg\tilde{\Phi} \\ \tilde{\Phi} & \tilde{\Phi} & \neg\tilde{\Phi} \end{pmatrix}} \qquad \underset{26}{\begin{pmatrix} \neg\tilde{\Phi} & \neg\tilde{\Phi} & \tilde{\Phi} \\ \neg\tilde{\Phi} & \neg\tilde{\Phi} & \tilde{\Phi} \\ \tilde{\Phi} & \tilde{\Phi} & \neg\tilde{\Phi} \end{pmatrix}} \qquad \underset{27}{\begin{pmatrix} \neg\tilde{\Phi} & \tilde{\Phi} & \tilde{\Phi} \\ \neg\tilde{\Phi} & \neg\tilde{\Phi} & \neg\tilde{\Phi} \\ \neg\tilde{\Phi} & \neg\tilde{\Phi} & \neg\tilde{\Phi} \end{pmatrix}}$$

$$\underset{28}{\begin{pmatrix}\neg\tilde{\Phi} & \tilde{\Phi} & \tilde{\Phi}\\ \neg\tilde{\Phi} & \neg\tilde{\Phi} & \neg\tilde{\Phi}\\ \neg\tilde{\Phi} & \neg\tilde{\Phi} & \neg\tilde{\Phi}\end{pmatrix}} \quad \underset{29}{\begin{pmatrix}\neg\tilde{\Phi} & \tilde{\Phi} & \tilde{\Phi}\\ \neg\tilde{\Phi} & \neg\tilde{\Phi} & \neg\tilde{\Phi}\\ \tilde{\Phi} & \neg\tilde{\Phi} & \neg\tilde{\Phi}\end{pmatrix}} \quad \underset{30}{\begin{pmatrix}\neg\tilde{\Phi} & \tilde{\Phi} & \tilde{\Phi}\\ \neg\tilde{\Phi} & \neg\tilde{\Phi} & \neg\tilde{\Phi}\\ \tilde{\Phi} & \neg\tilde{\Phi} & \neg\tilde{\Phi}\end{pmatrix}}$$

$$\underset{31}{\begin{pmatrix}\neg\tilde{\Phi} & \tilde{\Phi} & \tilde{\Phi}\\ \neg\tilde{\Phi} & \neg\tilde{\Phi} & \neg\tilde{\Phi}\\ \tilde{\Phi} & \tilde{\Phi} & \neg\tilde{\Phi}\end{pmatrix}} \quad \underset{32}{\begin{pmatrix}\neg\tilde{\Phi} & \tilde{\Phi} & \tilde{\Phi}\\ \neg\tilde{\Phi} & \tilde{\Phi} & \tilde{\Phi}\\ \tilde{\Phi} & \tilde{\Phi} & \neg\tilde{\Phi}\end{pmatrix}} \quad \underset{33}{\begin{pmatrix}\neg\tilde{\Phi} & \neg\tilde{\Phi} & \neg\tilde{\Phi}\\ \tilde{\Phi} & \neg\tilde{\Phi} & \neg\tilde{\Phi}\\ \tilde{\Phi} & \tilde{\Phi} & \neg\tilde{\Phi}\end{pmatrix}}$$

$$\underset{34}{\begin{pmatrix}\neg\tilde{\Phi} & \neg\tilde{\Phi} & \neg\tilde{\Phi}\\ \tilde{\Phi} & \neg\tilde{\Phi} & \neg\tilde{\Phi}\\ \tilde{\Phi} & \neg\tilde{\Phi} & \neg\tilde{\Phi}\end{pmatrix}} \quad \underset{35}{\begin{pmatrix}\neg\tilde{\Phi} & \neg\tilde{\Phi} & \tilde{\Phi}\\ \tilde{\Phi} & \neg\tilde{\Phi} & \neg\tilde{\Phi}\\ \tilde{\Phi} & \neg\tilde{\Phi} & \neg\tilde{\Phi}\end{pmatrix}} \quad \underset{36}{\begin{pmatrix}\neg\tilde{\Phi} & \neg\tilde{\Phi} & \tilde{\Phi}\\ \tilde{\Phi} & \neg\tilde{\Phi} & \neg\tilde{\Phi}\\ \tilde{\Phi} & \tilde{\Phi} & \neg\tilde{\Phi}\end{pmatrix}}$$

$$\underset{37}{\begin{pmatrix}\neg\tilde{\Phi} & \neg\tilde{\Phi} & \tilde{\Phi}\\ \tilde{\Phi} & \neg\tilde{\Phi} & \tilde{\Phi}\\ \tilde{\Phi} & \neg\tilde{\Phi} & \neg\tilde{\Phi}\end{pmatrix}} \quad \underset{38}{\begin{pmatrix}\neg\tilde{\Phi} & \neg\tilde{\Phi} & \tilde{\Phi}\\ \tilde{\Phi} & \neg\tilde{\Phi} & \tilde{\Phi}\\ \tilde{\Phi} & \tilde{\Phi} & \neg\tilde{\Phi}\end{pmatrix}} \quad \underset{39}{\begin{pmatrix}\neg\tilde{\Phi} & \tilde{\Phi} & \tilde{\Phi}\\ \neg\tilde{\Phi} & \tilde{\Phi} & \tilde{\Phi}\\ \neg\tilde{\Phi} & \neg\tilde{\Phi} & \neg\tilde{\Phi}\end{pmatrix}}$$

$$\underset{40}{\begin{pmatrix}\neg\tilde{\Phi} & \neg\tilde{\Phi} & \neg\tilde{\Phi}\\ \tilde{\Phi} & \tilde{\Phi} & \neg\tilde{\Phi}\\ \tilde{\Phi} & \tilde{\Phi} & \neg\tilde{\Phi}\end{pmatrix}} \quad \underset{41}{\begin{pmatrix}\neg\tilde{\Phi} & \tilde{\Phi} & \tilde{\Phi}\\ \tilde{\Phi} & \neg\tilde{\Phi} & \tilde{\Phi}\\ \tilde{\Phi} & \tilde{\Phi} & \neg\tilde{\Phi}\end{pmatrix}} \quad \underset{42}{\begin{pmatrix}\neg\tilde{\Phi} & \tilde{\Phi} & \tilde{\Phi}\\ \tilde{\Phi} & \neg\tilde{\Phi} & \tilde{\Phi}\\ \tilde{\Phi} & \neg\tilde{\Phi} & \neg\tilde{\Phi}\end{pmatrix}}$$

$$\underset{43}{\begin{pmatrix}\neg\tilde{\Phi} & \tilde{\Phi} & \tilde{\Phi}\\ \tilde{\Phi} & \neg\tilde{\Phi} & \neg\tilde{\Phi}\\ \tilde{\Phi} & \tilde{\Phi} & \neg\tilde{\Phi}\end{pmatrix}} \quad \underset{44}{\begin{pmatrix}\neg\tilde{\Phi} & \tilde{\Phi} & \tilde{\Phi}\\ \tilde{\Phi} & \neg\tilde{\Phi} & \neg\tilde{\Phi}\\ \tilde{\Phi} & \neg\tilde{\Phi} & \neg\tilde{\Phi}\end{pmatrix}}$$

7. Conclusion

In this paper, we have investigated the soft 4-intersection model for soft topological relations between soft regions with sharp soft boundary and for soft regions with broad soft boundary. Also soft 9-intersection model for soft regions with sharp soft boundary and broad soft boundary have been studied. It was seen that the soft 4 intersection is a subset of the soft 9-intersection and soft 9-intersection gives more details than soft 4-intersection. For soft regions with sharp soft boundaries both the soft 4 and 9-intersection provide the same eight soft topological relations. But it varies for the soft regions with broad soft boundaries. These models can be used to formulate consistency constraints for spatial database and also used in information system such as mobile robots and route navigation systems.

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