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PLATT NUMBER OF TOTAL GRAPHS

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Abstract: The degree of an edge uv is defined as the number of edges incident on vertices u and v other than itself. The Platt number of a graph is the sum of degrees of all its edges. In this paper, the concept of degree of an edge is analysed in social networks. The Platt number is investigated in certain classes of graphs and their total graphs. Also related bounds are proposed on connected graphs. An algorithm developed to determine the Platt number of any connected graph is presented.

AMS Subject Classification: 05C10
Key Words: central vertices, diameter, peripheral vertices, Platt number, total graph

1. Introduction

In a graph, the degree of a vertex $v(d(v))$ of a graph H is the number of edges incident on v. Also degree of an edge is number of edges incident on its end vertices other than itself. In social networks, degree of an edge can serve as an indicator of the strength of an edge in terms of its supporting edges. An analysis on this parameter by determining the lesser, similar and greater edge strengths in a graph could be helpful in various contexts. In contexts like designing, laying of roads or introducing a new technology, working on edges of lesser strength is optimal as impact is minimal. An application can be reused along edges of similar strength. In contexts where we can maximize the impact like distribution or communication, edges with greater strength can be examined.

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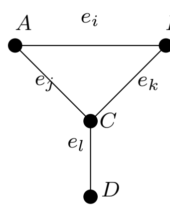
The definitions and results are in reference to [2]-[3]. The distance related parameter diameter is the maximum of the distances in a graph. The concept of diameter is analysed in networks in [1], [5]. The total graph $T(H)$ of a graph H as introduced by Harary [3] has vertex set $V(H)UE(H)$. There is an edge between two vertices in $T(H)$ if and only if there is edge-edge adjacency or edge-vertex incidency or vertex-vertex adjacency between them in H. So we get the number of vertices in $T(H)$ as $n+m$ where n are vertices of the original graph $H(n,m)$ and m are the new vertices representing edges of the original graph. The concepts of Platt number and zagreb indices are discussed in the context of molecular structures in Chemistry in [4], [6]. The degree sequences in total graphs are investigated by Thomas and Varghese [7]. The diameter related parameters in total graphs are analysed in literature [8]-[9].

2. Platt Number of a Graph

The degree $d(xy)$ of an edge is defined as number of edges incident on vertices x and y excluding itself. Hence $d(xy) = d(x) + d(y) - 2$, where xy is an edge and $d(x)$ is the degree of the vertex x. The Platt number of the graph is given by

$$I_{pl}(H) = \sum_{xy \in E(H)} d(xy) = \sum_{xy \in E(H)} d(x) + d(y) - 2.$$



$H(4,4)$

Example 2.1. Consider a graph $H(4,4)$. It can be noted that $d(e_i) = 2$, $d(e_j) = 3$, $d(e_k) = 3$ and $d(e_l) = 2$. Hence Platt number of H is

$$I_{pl}(H) = \sum_{uv \in E(H)} d(uv) = 2 + 3 + 3 + 2 = 10.$$

The following theorem relates degrees of edges to degrees of vertices.

Theorem 2.2. *Given a connected graph $H(V, E)$,*

$$I_{pl}(H) = \sum_{e \in E(H)} d(e) = \sum_{u \in V(H)} d(u)(d(u) - 1).$$

Proof. Consider a vertex u in H. There are $d(u)$ edges incident on u and $d(u) - 1$ edges contribute to the degree of each of those edges. Hence from each vertex u, $d(u)(d(u) - 1)$ is contributed to the sum of degrees of the edges incident on it. Hence sum of the degrees of all edges of H is

$$I_{pl}(H) = \sum_{e \in E(H)} d(e) = \sum_{u \in V(H)} d(u)(d(u) - 1).$$

□

The next section deals with the edges of $T(H)$ which will be categorized and their degrees are determined. Also Platt numbers are investigated in total graphs of certain classes of graphs.

3. Platt Number in Total Graphs

Let the degrees of edges e and e' in H and $T(H)$ be represented by $d^H(e)$ and $d^{T(H)}(e')$ respectively. Also let the degrees of vertices v and v' in H and $T(H)$ be represented by $d^H(v)$ and $d^{T(H)}(v')$ respectively. Total graphs have the vertex set $V(H)UE(H)$. Since $v_i \in V(H)$, will be adjacent with $d^H(v_i)$ vertices and edges incident on it in H, hence degree of a vertex v_i in $T(H)$ will be $2d^H(v_i)$. Also since $e_i \in E(H)$ is adjacent to $d^H(e_i)$ edges and its own end vertices of H, degree of a vertex e_i in $T(H)$ is $d^H(e_i) + 2$. The following results are on the three categories of the edges of total graphs.

Proposition 3.1. *If an edge e in $T(H)$ has end vertices v_i and v_j where v_i,$v_j \in V(H)$, then*

$$d^{T(H)}(e) = 2(d^H(v_i) + d^H(v_j) - 1).$$

Proof. Consider vertices v_i and v_j where v_i,$v_j \in V(H)$. The degree of v_i in $T(H)$ is $2d^H(v_i)$. Also degree of v_j in $T(H)$ is $2d^H(v_j)$. Hence the degree of edge $e = \{v_i, v_j\}$ in $T(H)$ is

$$d^{T(H)}(e) = 2(d^H(v_i)) + 2(d^H(v_j)) - 2 = 2(d^H(v_i) + d^H(v_j) - 1).$$

□

Proposition 3.2. *If an edge e in $T(H)$ has end vertices e_i and e_j where e_i,$e_j \in E(H)$, then*

$$d^{T(H)}(e) = d^H(e_i) + d^H(e_j) + 2.$$

Proof. Consider vertices e_i and e_j in $T(H)$ where e_i,$e_j \in E(H)$. The degree of e_i in $T(H)$ is $d^H(e_i) + 2$. Also degree of e_j in $T(H)$ is $d^H(e_j) + 2$.

Hence the degree of edge $e = \{e_i, e_j\}$ in $T(H)$ is

$$d^{T(H)}(e) = d^H(e_i) + 2 + d^H(v_j) + 2 - 2 = d^H(e_i) + d^H(e_j) + 2.$$

□

Proposition 3.3. *If an edge e in $T(H)$ has end vertices v_i and e_j where $v_i \in V(H)$ and $e_j \in E(H)$, then*

$$d^{T(H)}(e) = 2d^H(v_i) + d^H(e_j).$$

Proof. Consider vertices v_i and e_j of $T(H)$ where $v_i \in V(H)$ and $e_j \in E(H)$. Degree of a vertex v_i in $T(H)$ is $2d^H(v_i)$. Also degree of vertex e_j in $T(H)$ is $d^H(e_i) + 2$. Hence the degree of edge $e = \{v_i, e_j\}$ in $T(H)$ is

$$d^{T(H)}(e) = 2d^H(v_i) + d^H(e_j) + 2 - 2 = 2d^H(v_i) + d^H(e_j).$$

□

Further, we determine Platt number in total graphs of certain classes of graphs. These results can be verified by using the previous results on degrees of vertices of total graphs and by Theorem 2.2.

Class of Graphs	Platt number
Path P_n	2(n-2)
$T(P_n)$	$24n - 44$, $n \geq 3$
Cycle C_n	$2n$
$T(C_n)$	24n
Wheel W_n	$(n+4) * (n-1)$, $n \geq 4$
$T(W_n)$	$(n-1) * (n^2 + 7n + 56)$
Complete graph K_n	$n * (n-1) * (n-2)$, $n \geq 2$
$T(K_n)$	$2n^4 - 3n^3 - 2n^2 + 3n.$
Complete bipartite graph $K_{p,q}$	$pq * (p + q - 2)$
$T(K_{p,q})$	$pq * (p^2 + q^2 + 2pq + 3p + 3q - 4)$

In the next section, we propose bounds on Platt number in connected graphs and their total graphs.

4. Bounds on Platt Number

In the following section, bounds on Platt numbers of connected graphs and trees are proposed.

Proposition 4.1. *For a given connected graph H of order $n \geq 3$,*

$$1 \leq d(e) \leq 2 * (n-2),$$

where $e \in E(H)$.

Proof. Consider the given connected graph H of three or more vertices. Since every edge has at least one adjacent edge we get $1 \leq d(e)$. Since a vertex can have degree atmost n-1, the degree of an edge can be atmost $2*(n-1)-2$=$2(n-2)$, we get $d(e) \leq 2*(n-2)$. Hence

$$1 \leq d(e) \leq 2 * (n-2).$$

□

Proposition 4.2. *For a connected graph H of order $n \geq 3$, if $d(e) = 2*(n-2)$ for some $e \in E(H)$ then diameter $d \leq 2$.*

Proof. Since there is an edge $e = \{xy\}$ of degree $2(n-2)$, its end vertices x and y are adjacent to all the other vertices of the graph. Consider any two vertices in $V(H)$ which are non adjacent. There exists a path of length two between those vertices through x or y. Hence the distance can be atmost two between any pair of vertices. □

Proposition 4.3. *In a connected graph H of order $n \geq 3$,*

$$2(n-2) \leq I_{pl}(H) \leq n(n-1)(n-2).$$

Proof. Since the degree of an edge in a path P_n is atmost two, the Platt number takes the least value $2(n-2)$. Hence $2(n-2) \leq I_{pl}(H)$. Also complete graph has edges with maximmum degree $2(n-2)$. Hence Platt number takes the maximum value $n(n-1)(n-2)$. Hence $I_{pl}(H) \leq n(n-1)(n-2)$. In conclusion

$$2(n-2) \leq I_{pl}(H) \leq n(n-1)(n-2).$$

□

Proposition 4.4. *For a connected graph H of size $m \geq 2$,*

$$2(m-1) \leq I_{pl}(H) \leq m(m-1).$$

Proof. Since the degree of an edge in a path P_n with m edges is atmost two, Platt number takes the least value $2(m-1)$. Hence $2(m-1) \leq I_{pl}(H)$. Also in a star $K_{1,n}$ with m edges, each edge has degree $m-1$. Hence $I_{pl}(H) \leq m*(m-1)$. □

Proposition 4.5. *Let tree T be of order $n \geq 3$, Then*

$$2(n-2) \leq I_{pl}(T) \leq (n-1)(n-2).$$

Proof. By the previous proposition, $2(n-2) \leq I_{pl}(T)$ since path P_n is a tree. Also star $K_{1,n-1}$ which is a tree has all edges with maximmum degree $n-2$. Hence the Platt number takes the maximum value $(n-1)(n-2)$. Hence $I_{pl}(T) \leq (n-1)(n-2)$. □

We introduce M_3 as the sum of cubes of degrees of vertices. Hence

$$M_3 = \sum_{y \in V(H)} d(y)^3 = \sum_{yx \in E(H)} d(y)^2 + d(x)^2.$$

The following theorem relates the Platt number of $T(H)$ with topological indices M_1, M_2[[4]-[6]] and M_3 of H. Here M_1 and M_2 are Zagreb indices and are given by

$$M_1 = \sum_{y \in V(H)} d(y)^2 = \sum_{yx \in E(H)} d(y) + d(x)$$

and

$$M_2 = \sum_{yx \in E(H)} d(y)d(x).$$

Theorem 4.6.

$$I_{pl}(T(H)) = 3M_1 + 2M_2 + M_3 - 4m,$$

where M_1, M_2 and M_3 are topological indices of H and m the number of edges in H.

Proof. Consider vertex set of $T(H)$ to be $V_1 U V_2$, where $V_1 = \{v_i : v_i \in V(H)\}$ and $V_2 = \{e_j : e_j \in E(H)\}$. Let $\mid V_1 \mid = n$ and $\mid V_2 \mid = m$.

We know that by Theorem 2.2,

$$\begin{aligned} I_{pl}(T(H)) &= \sum_{w \in V(T(H))} d(w)(d(w)-1) \\ &= \sum_{v_i \in V_1} d(v_i)(d(v_i)-1) + \sum_{e_j \in V_2} d(e_j)(d(e_j)-1) \\ &= \sum_{v_i \in V_1} 2d^H(v_i)(2d^H(v_i)-1) + \sum_{e_j \in V_2} (d^H(e_j)+2)(d^H(e_j)+1) \\ &= S_1 + S_2. \end{aligned}$$

Since

$$\begin{aligned} S_1 &= \sum_{v_i \in V_1} 2d^H(v_i)(2d^H(v_i)-1) \\ &= 4 \sum_{v_i \in V(H)} (d^H(v_i))^2 - 2 \sum_{v_i \in V(H)} d^H(v_i) \\ &= 4M_1 - 2(2m) = 4M_1 - 4m \end{aligned}$$

and

$$\begin{aligned} S_2 &= \sum_{e_j \in V_2} (d^H(e_j)+2)(d^H(e_j)+1) \\ &= \sum_{e_j \in E(H)} (d^H(e_j)+2)(d^H(e_j)+1) \\ &= \sum_{yx \in E(H)} (d^H(y)+d^H(x)-2+2)(d^H(y)+d^H(x)-2+1) \\ &= \sum_{yx \in E(H)} (d^H(y)+d^H(x))(d^H(y)+d^H(x)-1) \\ &= 2\sum_{yx \in E(H)} d^H(y)d^H(x) + \sum_{yx \in E(H)} (d^H(y))^2 + (d^H(x))^2 - \sum_{yx \in E(H)} d^H(y) + d^H(x) \\ &= 2M_2 + M_3 - M_1, \end{aligned}$$

we get

$$I_{pl}(T(H)) = S_1 + S_2 = 3M_1 + 2M_2 + M_3 - 4m.$$

□

The following algorithm is developed to find the adjacency matrix $A = [a_{ij}]$ of Total graph of a given graph and determine its Platt number. This algorithm takes polynomial time.

Algorithm.

Consider $A = [a_{ij}]$ to be the adjacency matrix of any graph H.
Let num be the variable used to represent the number of vertices. Let val be the variable used to store number of edges.
Let $B = [b_{ij}]$ be the adjacency matrix of the total graph.

Initialize $k = 0$ val= 0

1. Run a loop through the adjacency matrix to calculate the number of edges.

for $i = 0$ to num
for $j = i$ to num
if $(A[i][j] == 1)$
val=val +1
endif
endfor j
endfor i
val= $val/2$

2. Create an empty matrix for the total graph and initialize all the entries to be 0.The variable val can now be reused to store the order of the matrix.
Assign val=val+num
Hence the order of the matrix is val $\times$ val for $i = 0$ to val
for $j = 0$ to val
$B[i][j] = 0$
endfor j
endfor i

3. Run loops through the adjacency matrix to fill the total graph matrix. This checks for vertex-vertex adjacency, vertex- edge adjacency and edge- edge adjacency. Initialize Platt number p to be 0.
for $i = 0$ to num
for $j = 0$ to num
if $(A[i][j] == 1)$
$B[i][j] = 1$
$B[j][i] = 1$
$B[i][num + k] = 1$
$B[num + k][i] = 1$
$B[j][num + k] = 1$

$B[num + k][j] = 1$
$k+ = 1$
endif
endfor j
for $l = val - 1$ downto num-1
if($B[i][l] == 1$)
for$m = l - 1$ downto num-1
if ($B[i][m]) == 1$
$B[m][l] = 1$
p=p+1 $B[l][m] = 1$
p=p+1 endif
endfor m
endif
endfor l
endfor i

4.If needed, run loops through the trial matrix to display the matrix
for $i = 0$ to val
for $j = 0$ to val
print($B[i][j]$)
endfor
endfor
print p

In order to obtain the Platt number of the Total graph we can use the obtained matrix as the input.

5. Conclusion

In this paper, the concept of Platt number is analysed in certain classes of graphs and their total graphs. Bounds on the Platt numbers of connected graphs are proposed. The focus of further research would be on identifying a few applications of this parameter in social networks.

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