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DELVING INTO COMBINED GRAPHS AND MATRICES IN SOFT SEMIGRAPHS

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Abstract

Molodtsov pioneered the concept of soft sets, providing a way to classify elements of a universe based on specific parameters, effectively modelling vagueness and uncertainty. Semigraphs, a generalized form of graphs, were

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introduced by Sampathkumar. The incorporation of soft set theory into semigraphs resulted in the development of soft semigraphs. Due to its proficiency in managing parameterization, the field of soft semigraph theory is rapidly advancing. In this study, we introduce various types of combined graphs and matrices related to soft semigraphs and examine some of their characteristics.

MSC 2020: 05C99

Key Words and Phrases: soft set; soft graph; semigraph; soft semigraph

1. Introduction

Molodtsov [9] introduced the concept of soft sets, providing a way to categorise elements of a universe according to a defined set of parameters. This method is used to represent vagueness and uncertainty. Authors such as Maji, Biswas, and Roy [8] have expanded on soft set theory, employing it to resolve decision-making problems. The notion of soft graphs was introduced by Thumbakara and George [14]. In 2015, Akram and Nawas [1] modified the definition of soft graphs. Contributions to the study of soft graphs have been made by Thenge, Jain, and Reddy [12], [13]. Soft graphs, owing to their utility in handling parameterisation, represent a growing domain within graph theory. The concept of semigraphs, a broader version of graphs, was first introduced by Sampathkumar [10, 11]. Unlike hypergraphs, semigraphs maintain a specific order of vertices within their edges. When represented on a plane, semigraphs resemble conventional graphs. In 2022, George, Thumbakara, and Jose [2], [3] introduced soft semigraphs by applying soft set principles to semigraphs and defined some soft semigraph operations. Moreover, they introduced connectedness [4] and various degrees, graphs, and matrices associated with soft semigraphs [5], [3]. George, Jose, and Thumbakara [6], [7] also presented soft semigraph isomorphisms and Eulerian and Hamiltonian soft semigraphs. In this study, we introduce different types of combined graphs and matrices associated with soft semigraphs and explore some of their characteristics.

2. Preliminaries

In this preliminary section, we lay the foundation for comprehending soft sets, semigraphs, and soft semigraphs. We define fundamental concepts such as partial edges and p-part, which are crucial to the structure of soft semigraphs.

2.1. **Semigraph.** The notion of semigraph was introduced by Sampathkumar [10], [11] as follows. "A *semigraph* S is a pair (T, D) where T is a nonempty set whose elements are called vertices of S, and D is a set of n-tuples, called edges of S, of distinct vertices, for various $n \geq 2$, satisfying the following conditions.

(1) Any two edges have at most one vertex in common

(2) Two edges $(u_1, u_2, \ldots, u_n)$ and $(v_1, v_2, \ldots, v_m)$ are considered to be equal if and only if
   (a) $m = n$ and
   (b) either $u_i = v_i$ for $1 \leq i \leq n$, or $u_i = v_{n-i+1}$ for $1 \leq i \leq n$.

Let $S = (T, D)$ be a semigraph and $E = (v_1, v_2, \ldots, v_n)$ be an edge of S. Then v_1 and v_n are the *end vertices* of E and $v_i, 2 \leq i \leq n-1$ are the *middle vertices(or m-vertices)* of E. If a vertex v of a semigraph S appears only as an end vertex then it is called an *end vertex*. If a vertex v is only a middle vertex then it is a *middle vertex* or *m-vertex* while a vertex v is called *middle-cum-end vertex or* (m, e)*-vertex* if it is a middle vertex of some edge and an end vertex of some other edge. A *subedge* of an edge $E = (v_1, v_2, \ldots, v_n)$ is a k-tuple $E' = (v_{i_1}, v_{i_2}, \ldots, v_{i_k})$, where $1 \leq i_1 < i_2 < \cdots < i_k \leq n$ or $1 \leq i_k < i_{k-1} < \cdots < i_1 \leq n$. We say that the subedge E' is *induced* by the set of vertices $\{v_{i_1}, v_{i_2}, \ldots v_{i_k}\}$. A *partial edge* of $E = (v_1, v_2, \ldots, v_n)$ is a $(j-i+1)$-tuple $E(v_i, v_j) = (v_i, v_{i+1}, \ldots, v_j)$, where $1 \leq i < j \leq n$. $S' = (T', D')$ is a *partial semigraph* of a semigraph S if the edges of S' are partial edges of S. Two vertices u and v in a semigraph S are said to be *adjacent* if they belong to the same edge. If u and v are adjacent and consecutive in order then they are said to be *consecutively adjacent*. u and v are said to be *e-adjacent* if they are the end vertices of an edge and *1e-adjacent* if both the vertices u and v belong to the same edge and at least one of them, is an end vertex of that edge".

2.2. **Soft Set.** In 1999 Molodtsov [9] initiated the concept of soft sets. "Let U be an initial universe set and let K be a set of parameters. A pair (F, K) is called a soft set (over U) if and only if F is a mapping of K into the set of all subsets of the set U. That is, $F : K \to \mathcal{P}(U)$".

2.3. **Soft Semigraph.** George, Thumbakara and Jose [2], [3] introduced soft semigraph by applying the concept of soft set in semigraph as follows: "Let $S^* = (T, D)$ be a semigraph having vertex set T and edge set D. Consider a subset T_1 of T. Then a partial edge formed by some or all vertices of T_1 is said to be a *maximum partial edge* or *mp edge* if it is not a partial edge of any other partial edge formed by some or all vertices of T_1. Let D_p be the collection of all partial edges of the semigraph S and K be a nonempty set. Let a subset R of $K \times T$ be an arbitrary relation from K to T. We define a mapping I from K to $\mathcal{P}(T)$ by $I(k) = \{y \in T | kRy\}, \forall k \in K$, where $\mathcal{P}(T)$ denotes the power set of T. Then the pair (I, K) is a soft set over T. Also define a mapping J from K to $\mathcal{P}(D_p)$ by $J(k) = \{\text{mp edges} < I(k) >\}$, where $\{\text{mp edges} < I(k) >\}$ denotes the set of all mp edges that can be formed by some or all vertices of $I(k)$ and $\mathcal{P}(D_p)$ denotes the power set of D_p. The pair (J, K) is a soft set over D_p. Then we can define a soft semigraph as follows:

The 4-tuple $S = (S^*, I, J, K)$ is called a *soft semigraph* of S^* if the following conditions are satisfied:

(1) $S^* = (T, D)$ is a semigraph having vertex set T and edge set D,
(2) K is the nonempty set of parameters,
(3) (I, K) is a soft set over T,
(4) (J, K) is a soft set over D_p,
(5) $L(a) = (I(a), J(a))$ is a partial semigraph of $S^*, \forall k \in K$.

Let $S^* = (T, D)$ be a semigraph and $S = (S^*, I, J, K)$ be a soft semigraph of S^* which is also given by $\{L(k) : k \in K\}$. Then the partial semigraph $L(k)$ corresponding to any parameter k in K is called a *p-part* of the soft semigraph S. An edge present in a soft semigraph S of S^* is called an *f-edge*. It may be a partial edge of some edge in S^* or an edge in S^*. A partial edge of any f-edge of a soft semigraph S is called a *p-edge* of S. An f-edge is a p-edge of itself. An f-edge or a p-edge of a soft semigraph S is called an *fp-edge* of S."

Example 2.1. [12, 13]: "An example of a soft semigraph is given below. Let $S^* = (T, D)$ be a semigraph given in Figure 1 having vertex set $T = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7\}$ and the edge set $D = \{(v_1, v_2, v_3, v_4), (v_4, v_5, v_6, v_7), (v_3, v_6)\}$. Let the parameter set be $K = \{v_2, v_6\} \subseteq T$. Define $I : K \to$

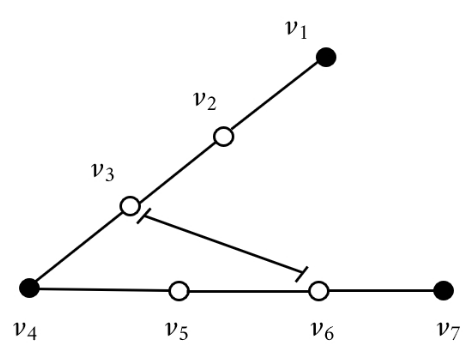

Figure 1. Semigraph $S^* = (T, D)$

$\mathcal{P}(T)$ by $I(k) = \{y \in T | kRy \Leftrightarrow k = y$ or k and y are adjacent$\}$, for all k in K and $J : K \to \mathcal{P}(D_p)$ by $J(k) = \{mp\ edges < I(k) >\}$, for all k in K. That is, $I(v_2) = \{v_1, v_2, v_3, v_4\}$ and $I(v_6) = \{v_3, v_4, v_5, v_6, v_7\}$. Also $J(v_2) = \{(v_1, v_2, v_3, v_4)\}$ and $J(v_6) = \{(v_3, v_6), (v_4, v_3), (v_4, v_5, v_6, v_7)\}$. Then $L(v_2) = (I(v_2), J(v_2))$ and $L(v_6) = (I(v_6), J(v_6))$ are partial semigraphs of S^* as shown below in Figure 2. Hence $S = \{L(v_2), L(v_6)\}$ is a soft semigraph of S^*. "

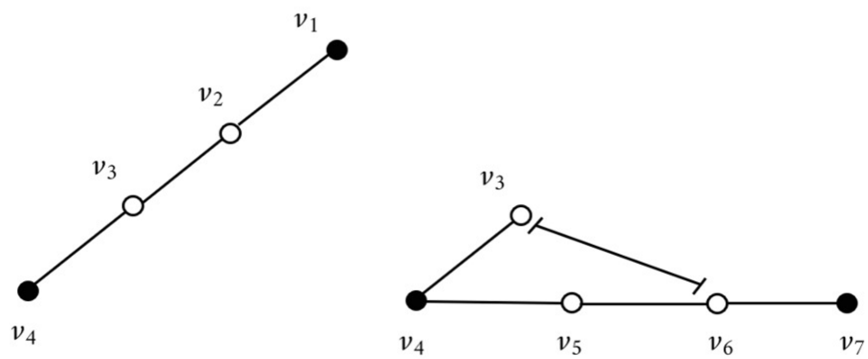

FIGURE 2. Soft Semigraph $S = \{L(v_2), L(v_6)\}$

3. **Combined Graphs Associated with Soft Semigraphs**

DEFINITION 3.1. Let $S^* = (T, D)$ be a semigraph and $S = (S^*, I, J, K)$ be a soft semigraph of S^* given by $S = \{L(k) : k \in K\}$. Then the combined adjacency graph of the soft semigraph S is the graph S'_{ca} having vertex set $T'_a = \cup_{k\in K} I(k)$ and two vertices in S'_{ca} are adjacent if they are adjacent in any one of the partial semigraph $L(k), k \in K$

EXAMPLE 3.1. Consider the semigraph $S^* = (T, D)$ given in Figure 1 and its soft semigraph S given in Figure 2. Then the combined adjacency graph S'_{ca} of S is given in Figure 3.

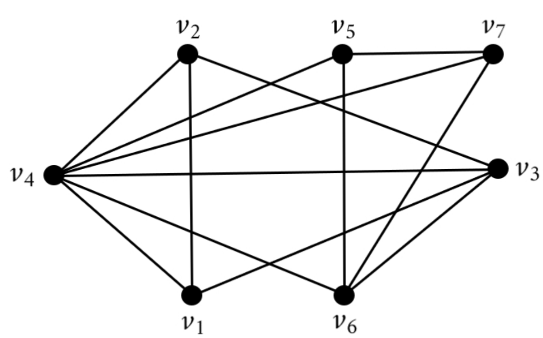

FIGURE 3. Combined Adjacency Graph S'_{ca}

THEOREM 3.1. *Let $S^* = (T, D)$ be a semigraph and $S = (S^*, I, J, K)$ be a soft semigraph of S^* given by $S = \{L(k) : k \in K\}$. Then the combined* *adjacency graph of S is a subgraph of the adjacency graph of S^*, i.e., S'_{ca} is a subgraph of S^*_a*

P r o o f. If $S^* = (T, D)$, then the vertex set of S^*_a is T. The vertex set of S'_{ca} is $\cup_{k \in K} I(k) \subseteq T$. Two vertices u and v are adjacent in S'_{ca} if they are adjacent in any of the p-parts $L(k)$ of S. i.e, u and v belong to the same f-edge in any of the p-parts $L(k)$. But, that f-edge is an edge in S^* or a partial edge of an edge in S^*. Hence, u and v are definitely adjacent in S^*. i.e, every edge in S'_{ca} is an edge in S^*_a. i.e., The vertex set and edge set of S'_{ca} are subsets of the vertex set and edge set of S^*_a respectively. Hence S'_{ca} is a subgroup of S^*_a. □

Definition 3.2. Let S^* be a semigraph and S be a soft semigraph of S^* which is given by $S = \{L(k) : k \in K\}$. Then the combined consecutive adjacency graph of the soft semigraph S is the graph S'_{cca} having vertex set $T'_{ca} = \cup_{k \in K} I(k)$ and two vertices S'_{cca} are adjacent if they are consecutively adjacent in any one of the partial semigraphs $L(k), k \in K$.

Example 3.2. Consider the semigraph $S^* = (T, D)$ and its soft semigraph S given in Figures 1 and 2, respectively. The consecutive adjacency graph S'_{cca} of S is given in Figure 4.

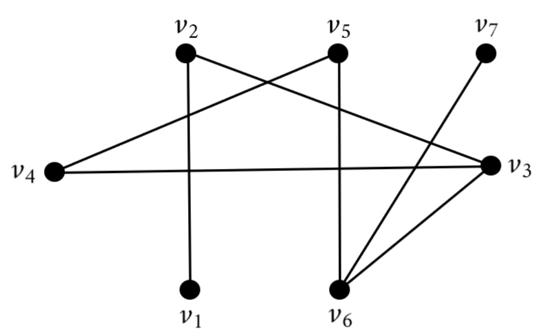

Figure 4. Combined Consecutive Adjacency Graph S'_{cca}

Theorem 3.2. *Let $S^* = (T, D)$ be a semigraph and $S = (S^*, I, J, K)$ be a soft semigraph of S^* given by $S = \{L(k) : k \in K\}$. Then,*

(i) *S'_{cca} is always a subgraph of S^*_{ca},*
(ii) *S'_{cca} is always a spanning subgraph of S'_{ca}.*

Proof. (i) The vertex set of S^*_{ca} is T if $S^* = (T, D)$. The vertex set of S'_{cca} is $\cup_{k\in K} I(k)$ and we have $\cup_{k\in K} I(k) \subseteq T$. Two vertices u and v are adjacent in S'_{cca} if they are consecutively adjacent in any of the p-parts $L(k)$ of S. i.e, u and v belong to an f-edge of $L(k)$ for some $k \in K$ and they are consecutively adjacent in that f-edge. This f-edge is an edge in S^* or a partial edge of an edge in S^*. Hence, u and v are also consecutively adjacent in S^*. i.e, Every edge present in S'_{cca} is also an edge in S^*_{ca}. Thus vertex set and edge set of S'_{cca} are subsets of the vertex set and edge set of S^*_{ca} respectively. Hence, S'_{cca} is always a subgraph of S^*_{ca}.

(ii) The vertex set of both S'_{cca} and S'_{ca} is $\cup_{k\in K} I(k)$. Two vertices u and v are adjacent in S'_{cca} if they are consecutively adjacent in any of the p-parts $L(k)$ of S. Then definitely u and v will be adjacent in that p-part. i.e, u and v are adjacent in S'_{ca} also. i.e, every edge present in S'_{cca} is also an edge in S'_{ca}. Thus the vertex sets of S'_{cca} and S'_{ca} are the same and the edge set of S'_{cca} is a subset of the edge set of S'_{ca}. Hence, S'_{cca} is always a spanning subgraph of S'_{ca}.

□

Definition 3.3. Let $S^* = (T, D)$ be a semigraph and $S = (S^*, I, J, K)$ be a soft semigraph of S^* given by $S = \{L(k) : k \in K\}$. Then the combined one end vertex graph of the soft semigraph S is the graph S'_{c1e} having vertex set $T'_{1e} = \cup_{k\in K} I(k)$ and two vertices in S'_{c1e} are adjacent if one of them is an end vertex or a partial end vertex of an f-edge containing the two vertices in any one of the partial semigraph $L(k)$, $k \in K$.

Example 3.3. Consider the semigraph S^* and its soft semigraph S in Figures 1 and 2 respectively. Then the combined one end vertex graph S'_{c1e} of S is given in Figure 5.

Theorem 3.3. *Let $S^* = (T, D)$ be a semigraph and $S = (S^*, I, J, K)$ be a soft semigraph of S^* given by $\{L(k) : k \in K\}$. Then*

(i) *S'_{c1e} is always a subgraph of S^*_a,*
(ii) *S'_{c1e} is always a spanning subgraph of S'_{ca}.*

Proof. (i) The vertex set of S'_{c1e} is $\cup_{k\in K} I(k)$ which is a subset of the vertex set of T of S^*_a. Also, two vertices u and v in S'_{c1e} are adjacent if one of them is an end vertex or a partial end vertex of an f-edge containing the two vertices in any one of the p-parts $L(k)$. That f-edge is an edge in S^* or a partial edge of an edge in S^*. Hence, u and v

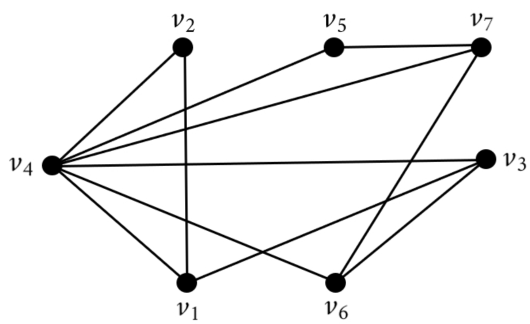

FIGURE 5. Combined One End Vertex Graph S'_{c1e}

are adjacent in S^*. i.e, An edge present in S'_{c1e} is also an edge in S^*_a. Hence, The edge set of S'_{c1e} is a subset of the edge set of S^*_a. Hence, S'_{c1e} is always a subgraph of S^*_a.

(ii) The vertex set of both S'_{c1e} and S^*_a is $\cup_{k\in K} I(k)$. Also, two vertices u and v are adjacent in S'_{c1e} if one of them is an end vertex or a partial end vertex of an f-edge containing the two vertices in any one of the partial semigraphs $L(k)$. Definitely, u and v are adjacent in that $L(k)$. Hence, u and v are adjacent in S'_{ca}. That is, the edge set of S'_{c1e} is a subset of the edge set of S'_{ca}. Hence,S'_{c1e} is always a spanning subgraph of S'_{ca}.

□

DEFINITION 3.4. Let $S^* = (T, D)$ be a semigraph and $S = (S^*, I, J, K)$ be a soft semigraph of S^* given by $S = \{L(k) : k \in K\}$. Then the combined end vertex graph of the soft semigraph S is the graph S'_e having vertex set $T'_e = \cup_{k\in K} I(k)$ and two vertices in S'_{ce} are adjacent if they are the end vertices or the partial end vertices of an f-edge containing them in any one of the p-parts, $L(k), k \in K$.

EXAMPLE 3.4. Consider the semigraph S^* and its soft semigraph S given in Figures 1 and 2, respectively. Then the combined end vertex graph S'_{ce} is given in Figure 6.

THEOREM 3.4. *Let $S^* = (T, D)$ be a semigraph and $S = (S^*, I, J, K)$ be a soft semigraph of S^* given by $S = \{L(k) : k \in K\}$. Then*

(i) *S'_{ce} is a subgraph of S^*_a,*
(ii) *S'_{ce} is a spanning subgraph of S'_{ca},*
(iii) *S'_{ce} is a spanning subgraph of S'_{c1e}.*

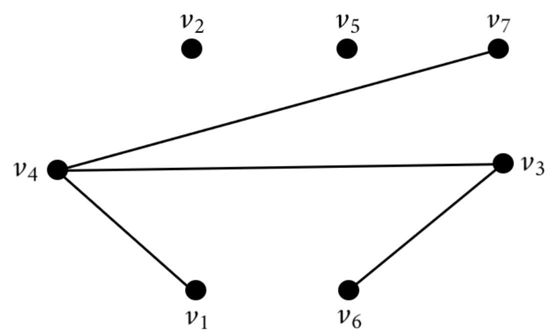

FIGURE 6. Combined End Vertex Graph S'_{ce}

Proof. (i) The vertex set of S'_{ce} is $\cup_{k\in K} I(k)$ which is a subset of T, the vertex set of S^*_a. Also, two vertices u and v are adjacent in S'_{ce} if they are the end vertices or partial end vertices of an f-edge containing them in any one of the p-parts $L(k), k \in K$. That f-edge is an edge in S^* or a partial edge of an edge in S^*. Hence, u and v belong to an edge in S^* and hence adjacent in S^*. Therefore, u and v are adjacent in S^*_a. That is, every edge present in S'_{ce} is also an edge S^*_a. That is the edge set of S'_{ce} is a subset of the edge set of S^*_a. Hence, S'_{ce} is always a subgraph of S^*_a.

(ii) The vertex set of both S'_{ce} and S'_{ca} is $\cup_{k\in K} I(k)$. Also, 2 vertices u and v are adjacent in S'_{ce} if they are the end vertices or partial end vertices of an f-edge containing in any one of $L(k); k \in K$. Then definitely u and v are adjacent in that $L(k)$. Hence, u and v are also adjacent in S'_{ca}. i.e, Any edge present in S'_{ce} is also an edge in S'_{ca}. i.e, The edge set of S'_{ce} is a subset of the edge set of S'_{ca}. Thus S'_{ce} is a spanning subgraph of S'_{ca}.

(ii) The vertex set of both S'_{ce} and S'_{c1e} is $\cup_{k\in K} I(k)$. Also, u and v are adjacent in S'_{ce} if they are the end vertices or partial end vertices of an f-edge containing them in any one of the p-parts $L(k), k \in K$. Then definitely one vertex is an end vertex or partial end vertex of an edge containing them in any one of $L(k); k \in K$. i.e, u and v are adjacent in S'_{c1e}. i.e, the edge present in S'_{ce} is also an edge in S'_{c1e}. Thus the vertex sets of S'_{ce} and S'_{c1e} are the same and the edge set of S'_{ce} is a subset of S'_{c1e}. Hence, S'_{ce} is a spanning subgraph of S'_{c1e}.

□

4. **Combined Matrices Associated with Soft Semigraphs**

DEFINITION 4.1. Let $S^* = (T, D)$ be a semigraph and $S = (S^*, I, J, K)$ be a soft semigraph of S^* which is given by $S = \{L(k) : k \in K\}$. Suppose that

$\cup_{k\in K}I(k)$ contains r elements. Then the combined adjacency matrix $X_{ca}(S)$ of the soft semigraph S is the $r\times r$ matrix $[d_{ij}]$, where

$$d_{ij}=\begin{cases}1, \text{ if the vertex } v_i \text{ and } v_j \text{ are adjacent } \textit{in any of the } \mathrm{L(k)}\\ 0, \text{ if not.}\end{cases}$$

Example 4.1. Consider the semigraph S^* and its soft semigraph S given in Figures 1 and 2 respectively. Then the combined adjacency matrix of S is given by

$$X_{ca}(S)=\begin{array}{c} \begin{matrix} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 \end{matrix} \\ \begin{bmatrix} 0 & 1 & 1 & 1 & 0 & 0 & 0\\ 1 & 0 & 1 & 1 & 0 & 0 & 0\\ 1 & 1 & 0 & 1 & 0 & 1 & 0\\ 1 & 1 & 1 & 0 & 1 & 1 & 1\\ 0 & 0 & 0 & 1 & 0 & 1 & 1\\ 0 & 0 & 1 & 1 & 1 & 0 & 1\\ 0 & 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix} \end{array} \begin{matrix} v_1\\ v_2\\ v_3\\ v_4\\ v_5\\ v_6\\ v_7 \end{matrix}.$$

Remark 4.1. $X_{ca}(S)$ is a symmetric matrix of order r if $\cup_{k\in K}I(k)$ contains r elements.

Theorem 4.1. *The combined adjacency matrix $X_{ca}(S)$ of the soft semigraph S is the same as the adjacency matrix of the combined adjacency graph S'_{ca} of S.*

P r o o f. We know that $X_{ca}(S)$ may contain only two elements 1 and 0 and is an $r\times r$ matrix if $\cup_{k\in K}I(k)$ contains r elements. Also, the adjacency matrix of the combined adjacency graph S'_{ca} of S will be an $r\times r$ matrix. The general element d_{ij} is 1 if the corresponding vertices v_i and v_j are adjacent in any one of the p-parts $L(k)$ of S and 0 otherwise. Also, if v_i and v_j are adjacent in any one of the p-parts there will be an edge connecting these vertices in the combined adjacency graph S'_{ca} of S. Hence the corresponding entry in the adjacency matrix of the combined adjacency graph S'_{ca} of S is also 1. The situation is the same when the entry is 0. Hence the combined adjacency matrix $X_{ca}(S)$ of the soft semigraph S is the same as the adjacency matrix of the combined adjacency graph S'_{ca} of S. □

Definition 4.2. Let $S^*=(T,D)$ be a semigraph and $S=(S^*,I,J,K)$ be a soft semigraph of S^* which is given by $S=\{L(k):k\in K\}$. Suppose that $\cup_{k\in K}I(k)$ contains r elements. Then the combined consecutive adjacency matrix $X_{cca}(S)$ of the soft semigraph S is the $r \times r$ matrix $[e_{ij}]$, where

$$e_{ij} = \begin{cases} 1, \text{ if the vertex } v_i \text{ and } v_j \text{ are consecutively adjacent } \textit{in any} \\ \textit{ of the } \mathrm{L(k)} \\ 0, \text{ if not.} \end{cases}$$

EXAMPLE 4.2. Consider the semigraph S^* and its soft semigraph S given in Figures 1 and 2, respectively. Then the combined consecutive adjacency matrix of S is given by

$$X_{cca}(S) = \begin{array}{c} \begin{matrix} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 \end{matrix} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array} \begin{matrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \end{matrix}$$

REMARK 4.2. $X_{cca}(S)$ is a symmetric matrix of order r if $\cup_{k \in K} I(k)$ contains r elements.

THEOREM 4.2. *The combined consecutive adjacency matrix $X_{cca}(S)$ of the soft semigraph S is the same as the consecutive adjacency matrix of the combined consecutive adjacency graph S'_{cca} of S.*

P r o o f. We know that $X_{cca}(S)$ may contain only two elements 1 and 0 and is an $r \times r$ matrix if $\cup_{k \in K} I(k)$ contains r elements. Also, the consecutive adjacency matrix of the combined consecutive adjacency graph S'_{cca} of S will be an $r \times r$ matrix. The general element e_{ij} is 1 if the corresponding vertices v_i and v_j are consecutively adjacent in any one of the p-parts $L(k)$ of S and 0 otherwise. Also, if v_i and v_j are consecutively adjacent in any one of the p-parts there will be an edge connecting these vertices in the combined consecutive adjacency graph S'_{cca} of S. Hence the corresponding entry in the consecutive adjacency matrix of the combined consecutive adjacency graph S'_{ca} of S is also 1. The situation is exactly the same when the entry is 0. Hence, the combined consecutive adjacency matrix $X_{cca}(S)$ of the soft semigraph S is the same as the consecutive adjacency matrix of the combined consecutive adjacency graph S'_{cca} of S. □

5. Conclusion

The concept of the soft semigraph was developed by incorporating the idea of soft sets into semigraph theory. Through parameterization, a soft semigraph generates multiple descriptions of relationships depicted by a semigraph. Undoubtedly, the theory of soft semigraphs will become a significant aspect of semigraph theory due to its effective use of parameterization.

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