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EXTENDED IMPLEMENTATION SHORTFALL FRAMEWORK: INCORPORATING AGGRESSIVENESS AND LIQUIDITY COSTS

Rahul Rangotra

Assistant Professor Department of Management Studies

Central University of Kashmir, Ganderbal, Jammu and Kashmir.

Abstract

Traditional Implementation Shortfall (IS) proposed by Perold (Perold, 1988) helps to measure trading cost arise during the implementation of trade by portfolio manager or trader. It has three components delay cost, trading cost and, and opportunity cost. This traditional IS does not consider the cost of aggressive execution at the time of trade urgency and liquidity cost in case of less liquid securities. The extended IS model in this paper tries to fill this gap and propose the framework which consider aggressiveness cost and liquidity cost. The proposed framework is more practical and intuitive to measure the trading cost and does not penalize the trader in case of trade urgency and less liquid securities. This will help the traders, brokers, investors and other stakeholders.

Keywords: Implementation shortfall, transaction costs, market microstructure, execution aggressiveness, liquidity cost.

JEL Classification: G11, G12, G14, D53

1. Introduction

Measuring trade execution costs is important to measure the portfolio performance both in the case of retail and institutional investors. Implementation Shortfall (IS), first defined by Perold (1988), is used to measure the transaction cost and is the difference between the return of a hypothetical "paper portfolio" and the actual returns of the portfolio. Implementation shortfall also help to evaluate the performance and quality of trade execution.

The, researchers (Collins & Fabozzi, 1991; Keim & Madhavan, 1997, 1998; Almgren & Chriss, 2000; Kissell, 2013) have refined the IS framework. These researchers extended the model beyond its main components such as explicit costs (commissions, fees), implicit costs (market impact, spread), delay costs, and opportunity costs from unfilled shares. But these extended IS framework developed by the researchers fails to address the two execution realities: (i) aggressiveness in execution, when traders pay higher price by chasing prices by placing the market orders instead of limit orders to ensure timely completion, and (ii) liquidity cost, when stocks are less liquid and the bid price (for long positions) or ask price (for shorts) are more important for calculation of IS, rather than the last traded price.

This paper extends the IS framework by introducing Liquidity Cost and Aggressiveness Cost in the traditional IS model proposed by Perold (1988). This will help to evaluate the trade cost not only using IS but also how much actual profit the portfolio is making after the trade. If

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trader is earning profit even if the implementation cost is high the trader should be appreciated and not penalized because of the high IS. In the trending market, beside cost returns are also important. The paper is divided into the following sections: literature review, research gap identification, formal model development, and empirical illustration.

2. Literature Review

Perold (1988) defined IS as the return difference between the paper portfolio (valued at arrival price) and the executed portfolio. This framework gained popularity for its comprehensiveness, as it aggregated explicit commissions, implicit spread/impact costs, and opportunity costs into a single measure. Collins and Fabozzi (1991) clarified the attribution categories and distinguish fixed from variable costs. Keim and Madhavan (1997, 1998) showed that IS increases with the size of the order relative to the average daily volume, urgency and volatility. Almgren and Chriss (2000) showed that execution of trade is a mean–variance optimization trade-off between market impact and execution risk. Further (e.g., Kissell, 2013) integrated stochastic dynamics, adaptive schedules, and non-linear impacts on the IS.

In practice, IS serves as the benchmark in trading cost analysis systems used by asset managers and regulators to evaluate broker performance and compliance with best-execution obligations. But the model ignores the like important components like the trade urgency and liquidity cost.

3. Research Gap

Even if IS is a robust measure, two critical gaps remain:

Aggressiveness Cost: Execution urgency often forces traders to accept unfavorable prices by paying higher price in case of buy orders and accepting the low price in case of sell orders. The existing IS framework does not consider this aggressiveness cost which is equally important for the traders and other stakeholders. Particularly in case of trending market, where the market is going up or down.

Liquidity Cost: Traditional IS measures the returns of the portfolio using last traded price. In illiquid securities or during highly volatile and stress periods, the securities are only worth the bid (for long positions) or ask (for shorts). Ignoring the liquidity cost for the calculation of IS penalizes traders inaccurately and obscures the true cost of the trade.

Addressing these gaps requires extending the IS formula to explicitly capture aggressiveness and liquidity cost.

4. Extended Implementation Shortfall Model

4.1 Traditional Implementation Shortfall Model

After implementation of trade, trader or portfolio manager has to access the trading cost as the trade implementation is not frictionless. It is important to identify where cost arise during the implementation of trade. Trade cost can be implicit as well as explicit. IS is important measure to measure the ex post trade cost and where the cost arise during the trade. Mathematically IS is equal to Returns of Paper portfolio less actual returns on the portfolio. Paper returns on the portfolio shows the hypothetical returns which the trader or portfolio manager would have

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received if all the shares were traded at desired decision price, without any friction and trading cost.

Paper returns on portfolio $= (CP - DP) * Ns$

CP: $Closing\ Price$

DP: $Decision\ Price$

Ns: $Number\ of\ total\ shares\ to\ be\ traded$

N_s is positive for buy order and negative for sell order.

Actual returns on the portfolio $= (CP - EP) * Ne - \ Fee$

EP: $Average\ Execution\ Price$

Ne: $Number\ of\ shares\ for\ which\ order\ is\ executed.$

IS can be decomposed into three components (assuming no fee): Delay Cost, Trading Cost, and Opportunity cost.

$IS = (AP - DP) * Ne + (EP - AP) * Ne + (Ns - Ne) * (CP - DP)$

Delay Cost: $(AP - DP) * Ne$

Trading Cost: $(EP - AP) * Ne$

Opportunity cost: $(Ns - Ne) * (CP - DP)$

AP is arrival price is the price at the time of release of order in the market. Delay cost arise because of adverse price movement due to non submission of order in timely manner. Trading cost arise because of difference between arrival price and execution price. It arise because of adverse movement of price at the time of execution. Opportunity cost arise because of nom fulfilment of order due to adverse price movement. Traditional IS (Perold, 1988) helps to measure the three costs during the implementation of trade. The paper extends the model to include aggressiveness cost and liquidity cost.

4.2 Extended Implementation Shortfall Model

In the extended implementation shortfall model two components are added, cost of liquidity and cost of aggressiveness. The extended implementation shortfall model is shown below:

$$EIS = (AP - DP)Ne + (EP - AP)Ne + (AGP - EP) * Ne) + (Ns - Ne)(CP - DP) - (CP - CBP)Ne$$

EIS: Extended Implementation Shortfall

Delay Cost: $(AP - DP) * Ne$

Trading Cost: $(EP - AP) * Ne$

Opportunity cost: $(Ns - Ne) * (CP - DP)$

Aggressiveness: $(AGP - EP) * Ne)$

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Cost of liquidity: $(CP - CBP)Ne$

AGP: Aggressive Price

CBP: Closing Bid Price

EP is the execution price if aggressiveness is not shown by the trader and only placed the limit order. If the trader expect the price to go up trader can place the market order and buy the additional shares at the higher price. This will increase the returns on the actual portfolio. But after a particular point, with the increase in the aggressiveness the implementation shortfall will increase. The point after which the implementation shortfall will increase depends upon the expected price increase by the trader. If the expected price increase is high the aggression can be high and vice versa. It is shown in the table 1 & 2. In table expected increase in price is \$101.7, so as shown the IS is minimum at 80% of execution with the aggressive price equal to \$101.1. In the table 2 the expected price is \$102.7, in this case trader can fulfil the complete order even can buy additional shares to increase profit. In case of sell order it's otherwise. Further in case of table 2 the trader expect the higher increase in price so he can chase the price and fulfil the orders at the market price instead of limit order to increase the returns on the actual portfolio and decrease the implementation shortfall.

$(CP - CBP)Ne$ is the cost of liquidity. It is the difference between the closing price and the closing bid price in case of buy order (as the trader has to sell to calculate the returns on the buy orders) and closing ask price in case of sell order (as the trader has to buy back the shares to calculate the returns in case of sell order). The liquidity cost is shown in the table 1 and 2. Liquidity cost increase with the decrease in the liquidity in the market as with the decrease in the liquidity the bid ask spread increases in the market. So the trader should not be penalized because of decrease or increase in liquidity in the market. For transactional cost analysis liquidity cost is important component of implementation shortfall. The liquidity cost is shown separately in the model so that the trader can assess how much of the implementation shortfall is because of the liquidity. In the table 1 &2 the returns are calculated with and without considering the liquidity.

5. Conclusion:

By introducing Aggressiveness and Liquidity Cost the paper extends the implementation shortfall model. The proposed framework helps to isolate the urgency driven execution and liquidity cost. The traditional model does not consider these costs and trader/portfolio manager gets penalized because of aggressive execution and in case of illiquid securities. This extended framework filled this research gap and enhance the understanding for both academicians and practitioners. It is a tool for transactional cost analysis which align measurement of transaction cost closely with the realities of modern electronic trading of securities.

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Table 1: Expected Price $101.7

Decision Price	Arrival Price	Execution Price	Aggressive Price	Last Traded Price	Bid Price (Closing)	Fill %	Filled Qty	Missed Qty	Delay Cost	Trading Cost	Aggr. Cost	Opp. Cost	Liq. Cost	Gross Profit	Profit after Liquidity	Total Cost	Paper Returns	Actual Returns	IS
100	10 0.2	100. 4	100. 4	10 2	101. 7	10 %	10 0	90 0	20	20	0	1,8 00. 00	3 0	16 0	130	1,8 70. 00	200 0	130	1 8 7 0
100	10 0.2	100. 4	100. 5	10 2	101. 7	20 %	20 0	80 0	40	40	20	1,6 00. 00	6 0	30 0	240	1,7 60. 00	200 0	240	1 7 6 0
100	10 0.2	100. 4	100. 6	10 2	101. 7	30 %	30 0	70 0	60	60	60	1,4 00. 00	9 0	42 0	330	1,6 70. 00	200 0	330	1 6 7 0
100	10 0.2	100. 4	100. 7	10 2	101. 7	40 %	40 0	60 0	80	80	12 0	1,2 00. 00	1 2 0	52 0	400	1,6 00. 00	200 0	400	1 6 0 0
100	10 0.2	100. 4	100. 8	10 2	101. 7	50 %	50 0	50 0	10 0	100	20 0	1,0 00. 00	1 5 0	60 0	450	1,5 50. 00	200 0	450	1 5 5 0
100	10 0.2	100. 4	100. 9	10 2	101. 7	60 %	60 0	40 0	12 0	120	30 0	800	1 8 0	66 0	480	1,5 20. 00	200 0	480	1 5 2 0
100	10 0.2	100. 4	101	10 2	101. 7	70 %	70 0	30 0	14 0	140	42 0	600	2 1 0	70 0	490	1,5 10. 00	200 0	490	1 5 1 0
100	10 0.2	100. 4	101. 1	10 2	101. 7	80 %	80 0	20 0	16 0	160	56 0	400	2 4 0	72 0	480	1,5 20. 00	200 0	480	1 5 2 0
100	10 0.2	100. 4	101. 2	10 2	101. 7	90 %	90 0	10 0	18 0	180	72 0	200	2 7 0	72 0	450	1,5 50. 00	200 0	450	1 5 5 0

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100	100.2	100.4	101.3	102	101.7	100%	1000	0	200	200	900	0	300	700	400	1,600.00	2000	400	1600

Table 2 Expected Price $102.7

Decision Price	Arrival Price	Execution Price	Aggressive Price (average)	Last Traded Price	Bid Price (Closing)	Fill %	Filled Qty	Missed Qty	Delay Cost ($)	Exec. Cost ($)	Aggr. Cost ($)	Opp. Cost ($)	Liq. Cost ($)	Gross Profit ($)	Profit after Liquidity	Total Cost ($)	Paper Returns	Actual Returns	IS
100	100.2	100.4	100.4	103	102.7	10%	100	900	20	20	0	2,700.00	30	260	230	2,770.00	3000	230	2770
100	100.2	100.4	100.5	103	102.7	20%	200	800	40	40	20	2,400.00	60	500	440	2,560.00	3000	440	2560
100	100.2	100.4	100.6	103	102.7	30%	300	700	60	60	60	2,100.00	90	720	630	2,370.00	3000	630	2370
100	100.2	100.4	100.7	103	102.7	40%	400	600	80	80	120	1,800.00	120	920	800	2,200.00	3000	800	2200
100	100.2	100.4	100.8	103	102.7	50%	500	500	100	100	200	1,500.00	150	1,100.00	950	2,050.00	3000	950	2050
100	100.2	100.4	100.9	103	102.7	60%	600	400	120	120	300	1,200.00	180	1,260.00	1,080.00	1,920.00	3000	1080	1920
100	100.2	100.4	101	103	102.7	70%	700	300	140	140	420	900	210	1,400.00	1,190.00	1,810.00	3000	1190	1810
100	100.2	100.4	101.1	103	102.7	80%	800	200	160	160	560	600	240	1,520.00	1,280.00	1,720.00	3000	1280	1720
100	100.2	100.4	101.2	103	102.7	90%	900	100	180	180	720	300	270	1,620.00	1,350.00	1,650.00	3000	1350	1650
100	100.2	100.4	101.3	103	102.7	10	10	0	200	200	900	0	300	1,700.00	1,400.00	1,600.00	3000	1400	16

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						0%	00												00
100	100.2	100.4	101.4	103	102.7	110%	1100	-100	220	220	1100	-300	330	1,760.00	1,430.00	1,570.00	3000	1430	1570
100	100.2	100.4	101.5	103	102.7	120%	1200	-200	240	240	1320	-600	360	1,800.00	1,440.00	1,560.00	3000	1440	1560
100	100.2	100.4	101.6	103	102.7	130%	1300	-300	260	260	1560	-900	390	1,820.00	1,430.00	1,570.00	3000	1430	1570
100	100.2	100.4	101.7	103	102.7	140%	1400	-400	280	280	1820	######	420	1,820.00	1,400.00	1,600.00	3000	1400	1600
100	100.2	100.4	101.8	103	102.7	150%	1500	-500	300	300	2100	######	450	1,800.00	1,350.00	1,650.00	3000	1350	1650

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