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DESIGNING AN OUTPATIENT-APPOINTMENT SCHEDULING USING AHP AND SIMULATED ANNEALING

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Abstract: In this paper, we consider the problem of designing an efficient appointment system of an outpatient department of a healthcare system in order to optimize the performance of the clinic. This problem includes optimizing three objectives, therefore it is considered as a multi-objective optimization problem (MOOP). One way for solving the MOOP is to use the weighted sum method at which all objectives are aggregated into a single objective using relative weights for each objective based on their importance, then one can use any optimization method to solve the aggregated problem. The analytic hierarchy process (AHP) is used to select these relative weights, then the simulated annealing (SA) method is implemented to solve the aggregated optimization problem. The proposed AHP-SA algorithm is used to solve a real case appointment system. The obtained numerical results indicate that the proposed method indeed gives relatively good solutions based on the importance level of each objective.

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Key Words: multi-objective optimization problem; simulated annealing; analytic hierarchy process; appointment system

1. Introduction

We consider designing an appointment scheduling in outpatient health care

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system. The aim of this scheduling is to increase the efficiency of the clinic by minimizing the waiting time per patient and the number of patients in the clinic and at the same time increase the utilization of the physician. This problem is modeled as a multi-objective optimization problem (MOOP).

Outpatient clinics are a main healthcare service which are designed for the treatment of outpatients and these patients visit it for consultation, without need for a bed or to be admitted for overnight care. Patients often follow different paths in these clinics according to many factors. For example, the doctor whom they are scheduled to see, their medical conditions and the results of laboratory's tests [8]. So, there is a common problem that the patients who are getting consultation must wait for long time. Hence, the idea came for designing an appointment system (AS). The AS problem becomes one of the most widely studied problems in MOOPs.

A general MOOP is defined as follows:

$$\min_{x\in S}(f_1(x), f_2(x), ..., f_k(x)), \tag{1}$$

where S denotes the set of all feasible solutions, and $f_i(x) : S \to R$ is the ith objective function for $i = 1, 2, ..., k$. Each function $f_i(x)$ is an expected performance of stochastic system.

One way for solving the MOOP is to aggregate the objectives in a single objective through a weighted sum, where all objectives are given individual weights based on their importance. In more details, the weighted sum method, solves a positively weighted sum of the objectives, that is,

$$F(x) = \sum_{i=1}^{k} w_i.f_i(x), \tag{2}$$

where $\sum_{i=1}^{k} w_i = 1$, $w_i \geq 0$, $x \in S$, and w_i is the weight or relative importance of the ith objective. Many methods have been carried out to choose the weights w_i's, [15]. One way of selecting the weights is the analytical hierarchy process (AHP) designed by Saaty [12, 14]. The decision maker need to know these weights before the optimization is done based on their knowledge and experience. This means that the optimal solution will be selected depending on the preference of the decision makers. Once the decision maker changes these weights, the optimal solution is changed.

For solving this problem, we adopt the weighted sum multi-objective simulated annealing presented by Alrefaei and Diabat [5]. AHP is used to determine the wights for the aggregated function. Simulated annealing is a hill-climbing random search method that allows the search to escape local minimality and jumps over hills in order to locate a global minimum solution, and this global minimum solution, obtained by SA, does not depend on the initial solution, [1]. SA is an optimization method that mimics the physical annealing problem. The annealing is a physical process that heated a solid to high temperature with subsequent cooling, slowly cooling with specific rate, so as to obtain a high quality crystal, Metropolis [10].

Kirkpatrick et al. [9] developed the simulated annealing to be used for solving deterministic discrete optimization problems. Alrefaei and Andradottoir [3] have used the SA to solve stochastic optimization problem and Serafini [17] proposed the first version of SA to solve MOOPs.

In general, the algorithm of SA for MOOPs is almost the same algorithm for a single optimization problem that starts by an initial solution, say x. Then a candidate solution y is selected from the neighborhood $N(x)$ of the current solution x where $N(x)$ is the set of all neighboring solutions of x. If the objective function $f(y) < f(x)$ then y is better than x so it is accepted as a new solution. On the other hand, if y is not better than x, i.e $f(y) \geq f(x)$, we should not rush to reject it because it might hide a good solution behind, so there is a chance to accept y as a new solution based on a selection probability P that depends on the difference $f(y) - f(x)$; if it is large then the acceptance probability is small but if the difference is small, then the acceptance probability is high. Serafini [17] examined several alternatives criteria to determine the acceptance probability of the new solution versus the current solution. One of them is given by $P = exp[-(f(y) - f(x))/T_m]$, where T_m is a control parameter in iteration m called a temperature that decreases with some rate called the cooling rate.

2. AHP and SA for MOOP

The Analytic Hierarchy Process (AHP) procedure was originally developed by Saaty [12], since then it becomes one of the most popular tools and effective techniques for dealing with complex decision problems with multi objectives.

Several studies [18, 16] demonstrate the relevance of this method in many different sectors such as agriculture, education, and health, etc. The strength of the AHP technique is that it organizes tangible and intangible factors in a systematic way. A review of applications of analytic hierarchy process in operations management can be found in [18]. In addition, the AHP technique provides a structured and relatively simple solution to the decision makers, [11]. The AHP is a theory of measurement through pairwise comparisons and relies on the judgements of experts based on their needs, knowledge and experience of each problem.

The AHP is flexible and intuitive method for decision makers, which also identifies and elicits a corresponding vector of priorities or weights $\mathbf{w} = (w_1, w_2, ..., w_n)$ based on the pairwise comparison values of a set of objects. Pairwise comparison values are the judgments obtained from an appropriate semantic scale as presented in Table 1. It is suggested to use scale 1 to 9, but not necessary.

Table 1: Comparison Scale (adapted by [12])

Level of importance	Definition
1	Equal importance
3	Essential or strong importance
5	Essential or strong importance
7	Very strong or demonstrated importance
9	Absolute importance
2,4,6,8	Intermediate values

The mathematical technique behind computing the corresponding priority vector is based on linear algebra. A matrix of pairwise comparisons $\mathbf{A} = \left(a_{ij}\right)$ is a positive and reciprocal square matrix where the terms a_{ij} are the result of the comparison between the elements i and j. The values on the diagonal are equal to 1 and the opposite values of the comparisons are placed in the a_{ji} position of $\mathbf{A}$. Thus, the matrix $\mathbf{A}$ is presented as

$$\mathbf{A} = \begin{bmatrix} 1 & a_{12} & ...... & a_{1n} \\ a_{21} & 1 & ...... & a_{2n} \\ . & . & ...... & . \\ a_{1n} & a_{1n} & ...... & 1 \end{bmatrix},$$

where $a_{ij} = 1/a_{ji}$ for all $i, j = 1, 2, ..., n$.

Recently, several techniques are presented for computing the relative priorities, including Saaty's eigenvector method [12]. Other researchers apply other techniques, for example, the least squares method, [7] and the logarithmic least squares method or geometric mean vector [6]. Saaty and Ho [13] have done comparative work between these techniques and they prove that when the pairwise comparison matrix $\mathbf{A}$ is consistent (i.e., $a_{ik}a_{kj} = a_{ij}$), all these techniques lead to the same priority vector $\mathbf{w}$. But in real life, judgments are frequently inconsistent, and these different methods give rise to different priority vectors. We are interested in Saaty's eigenvector method which computes the vector $\mathbf{w}$

as a principal right eigenvector of the matrix $\mathbf{A}$ as $\mathbf{Aw} = \lambda_{\max}\mathbf{w}$, where $\lambda_{\max}$ is the maximum eigenvalue of the matrix $\mathbf{A}$. The consistency index (C.I) which is the first indicator of result accuracy of the pairwise comparisons is defined as

$$C.I = \frac{\lambda_{\max} - n}{n - 1}. \tag{3}$$

It has been proven that if a decision maker is perfectly consistent in specifying the entries of the matrix $\mathbf{A}$, then $\lambda_{\max} = n$ and $C.I = 0$, [12]. The $C.I$ value is compared with the random consistency index ($R.I$). This parameter is defined as a mean $C.I$ values from 500 $n \times n$ positive reciprocal pairwise comparison matrices whose entries were randomly generated using the proposed scale by Saaty. ($R.I$) was computed for several values of n and shown in Table 2.

Table 2: Some values of the random consistency index ($R.I$)

n	2	3	4	5	6	7	8	9	10
$R.I$	0	0.58	0.90	1.12	1.24	1.32	1.41	1.45	1.51

The consistency ratio ($C.R$) measures the degree of inconsistency (i.e the accuracy of pairwise comparisons) which is the ratio of the $C.I$ to the $R.I$, thus

$$C.R = \frac{C.I}{R.I}. \tag{4}$$

A value of the $C.R \leq 0.1$ is typically considered acceptable and indicates that the decision maker has been sufficiently consistent in specifying entries for the matrix $\mathbf{A}$, [14]. Larger values indicate that the data are inconsistent and require the decision maker's judgments must be reviewed to reduce the inconsistencies. For a matrix of order n, $n(n-1)/2$ comparisons are required.

3. Designing an Appointment System using AHP-SA Algorithm

Consider an appointment system of a healthcare department at which one needs to select an optimal scenario that optimize three types of objectives, namely, minimizing the average waiting time per patient, minimizing the average number of patients in the clinic and maximizing the utilization of the physician.

We assume that there is one physician and one lab test, this problem was considered in [18, 4]. Patients arrive at the clinic and go directly to the reception unit to register, then wait until the physician becomes available to receive consultation. After receiving consultation, the doctor decides whether the patient leave or needs to take some laboratory tests, in this case, the patient goes to the laboratory process and then comes back and waits in the doctor queue to receive consultation again but in this case the patient is served with priority, so he or she sees the doctor at the next available time immediately.

It is assumed that there are three types of patients arrive at the clinic, namely new patients, follow-ups and return patients. The two first types require a number from the reception unit, while return patients are those who run lab tests and return again to visit the doctor's office. After the patient enters the doctor's diagnoses room, the patient remains there for a service time depends on the patient type. New patients service time is usually the longest due to the fact that the doctor needs to fill and complete the patient's file and identify problems and conditions. Follow-up patients service time is not as long, because the doctor already has a record for these patients and they usually come for check-up. Finally, return patients require the least time, as the doctor only examines results of the tests and provides the appropriate prescription based on these results. The patient who goes to do lab tests based on the doctor's opinion normally takes an average of ten minutes to finish the lab test. After finishing, he comes back to the doctor based on priority rule to see the results of the tests.

The following data types are needed in the system: arrival time for each patient, type of patient; new patient, follow up patient or return patient, waiting time in the doctor queue, service time in the doctor room, number of patients sent to the lab and the time taken until the patient comes back from the lab to the doctor queue. The processing times are estimated as follows: for the new patients it is 8.86 minutes. For follow up patients it is 6.3 minutes and for return patients from the lab it is 2.89 minutes. The lab time for return patients is 10.77 minutes. It is also noticed that 87.5% of the arrival patients are follow up patients and the others are new patients and only 9.5% of the patients need lab tests, [2].

Arena simulation package is used to simulate each scenario and get an estimate of three objectives: waiting time per patient, number of patients in the clinic and doctor's utilization. We then normalize the average waiting time per patient and the number of patients in the clinic using the formula

$$x_{new} = \frac{x - x_{\min}}{x_{\max} - x_{\min}}, \tag{5}$$

and keep the average doctor's utilization without any change because it is already in the interval $[0, 1]$. Secondly, we use the AHP method, to determine the relative priority for each performance measure at certain conditions based on the vision and need of the decision maker. After choosing the relative priorities, we applied The AHP-SA algorithm proposed in the previous section.

We model each solution as a vector of the form (I, T, N), where I is the number of initial arrivals which is assumed between 1 and 4. T is the period between any two consecutive arrivals which is assumed to be between 7 and 12 minutes and N is the number of patients scheduled to arrive in each block which is between 1 and 4 patients. The neighborhood is defined by adding or subtracting 1 for one entry, this means each solution has at most six neighbors. Now we present some different numerical results for different relative priorities.

3.1. High priority assigned to the number of patients in the system

Many clinics are competing among each others to deliver the best services to the patients. Here, we give the average number of patients in the clinic the highest priority, especially with the spread of the Covid-19 virus, all clinics intend to reduce the number of patients in order to prevent the transmission of infection between patients. To tackle this problem, we give the average number of patients in the clinic high relative priority (i.e absolute importance according to the comparison scale Table 1). The pairwise comparison matrix of the three objectives is shown in Table 3. We found $\lambda_{\max} = 3$, therefore $C.R = 3$. The priority vector $\mathbf{w}$ is given in Table 4.

Table 3: Pairwise comparison matrix when high priority is assigned to the number of patients objective

Objective	No. of Patients	Waiting Time	Utilization
No. of Patients	1	9	9
Waiting Time	1/9	1	1
Utilization	1/9	1	1

After computing the priority vector, we implement AHP-SA algorithm and found that the optimal solution is $(2, 9, 1)$ which is to assign 2 patients at the beginning of the clinic hours and 1 patient every 9 minutes, note that the average consultation time for new patients is about 8.4 minutes.

The average performance of the algorithm over 20 replications is depicted in Figure 1. It is clear that the algorithm finds the solution in about 130 iterations and that the number of patients' objective decreases more than the

Table 4: The priority vector when high priority is assigned to the number of patients

The objective	The priority vector **w**
Number of patients	0.818181818
Waiting time	0.090909091
Utilization	0.090909091

Table 5: The optimal solution with respect to high priority is assigned to the number of patients

Obtained solution	Average waiting time	Average no. of patients	Average utilization
(2, 9, 1)	4.469	0.5177	69.36 %

two objectives. Note that if the clinic session is 5 hours then the average number of scheduled patients in this case is $(2 + (300 \div 9) = 35$ patients.

3.2. High priority assigned to the waiting time per patient in the system

Some clinics intend to reduce the patient's waiting time. This depends on many factors. Especially if it is treating the elderly patients or those with chronic diseases, even these days, with the spread of the Covid-19 virus. So our goal in this section is minimizing the average waiting time per patient in the clinic. Therefore, the decision maker assigns the average waiting time a high relative priority (i.e absolute importance according to the comparison scale Table 1). The pairwise comparison matrix is shown in Table 6.

The results are given in Table 7. The optimal solution now is $(1, 11, 1)$.

The average performance of the algorithm over 20 replications is depicted in Figure 2. It is clear that the algorithm finds the solution around 80 iterations, and the three objectives are decreasing. If the clinic session is 5 hours then the average number of scheduled patients in this case is $(1 + (300 \div 11)) = 28$ patients.

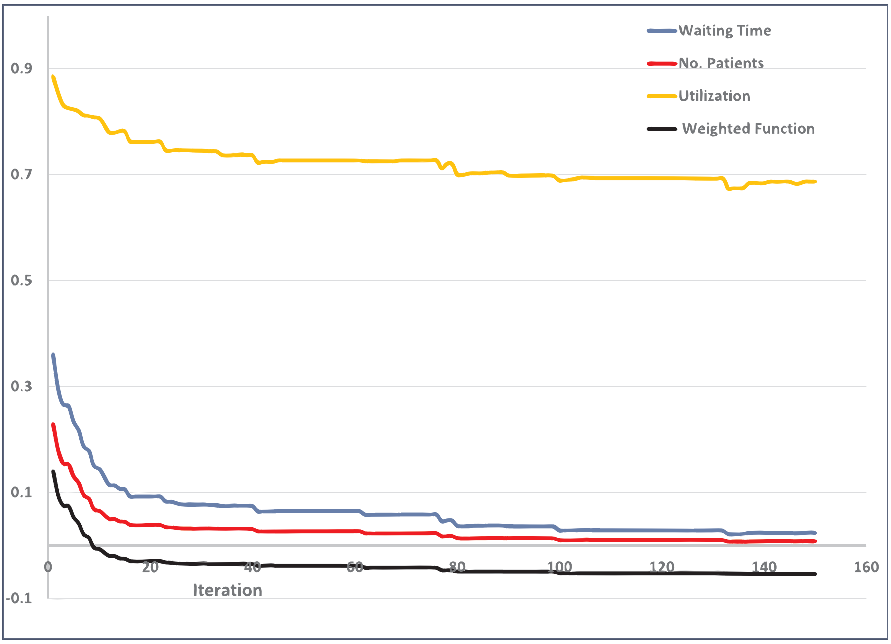

Figure 1: Average performance of normalized 3-objectives and weight function where high priority assigned to the number of patients.

Table 6: Pairwise comparison matrix when high priority is assigned to the waiting time per patient

Objective	Waiting Time	No. of Patients	Utilization
Waiting Time	1	9	9
No. of Patients	1/9	1	1
Utilization	1/9	1	1

3.3. High priority assigned to the doctor's utilization

The decision maker expects that the number of patients will increase at the beginning of weeks and after the holidays. Therefore, the decision maker is seeking to maximize the doctor's utilization. The doctor's utilization is given a high relative priority (i.e absolute importance according to the comparison

Table 7: The optimal solution with respect to high priority assigned to the waiting time per patient

Obtained solution	Average waiting time	Average number of patients	Average utilization
(1, 11, 1)	2.1692	0.195	55.47 %

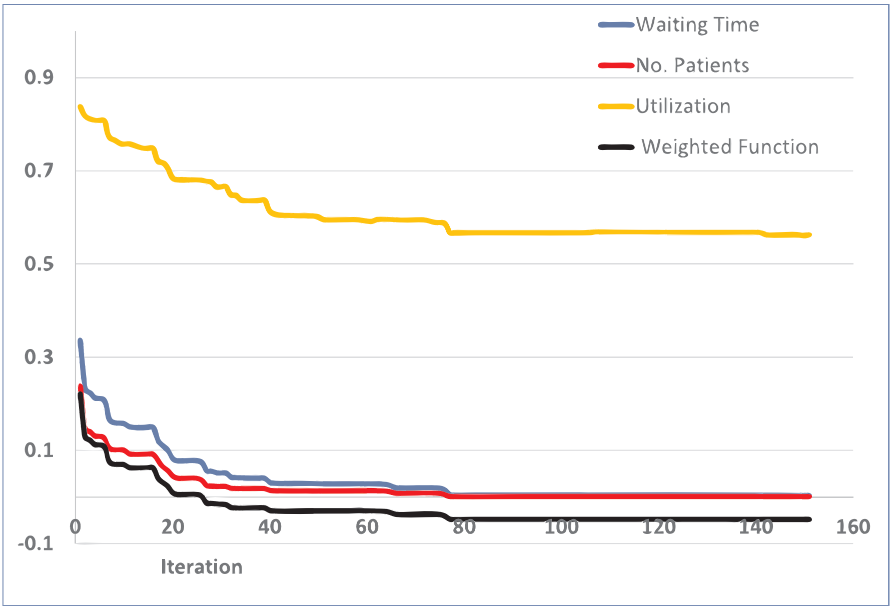

Figure 2: Average performance of AHP-SA when the waiting time per patient is given high priority

scale Table 1). The pairwise comparison matrix is shown in Table 8.

The results are given in Table 9. Note that the solution is $(4, 10, 2)$.

The average performance of the algorithm over 20 replications is depicted in Figure 3. It is clear that the algorithm finds the solution around 15 iterations and that the utilization's objective increases while the other objectives decrease. If the clinic session is 5 hours then the average number of scheduled patients in this case is $(4 + (300 \div 10 \times 2)) = 64$ patients. This increase of the number of patients that get consultation is due to the increase of doctor utilization.

Table 8: Pairwise comparison matrix when a higher priority is assigned to the doctor's utilization

Objective	Waiting Time	No. of Patients	Utilization
Waiting Time	1	1	1/9
No. of Patients	1	1	1/9
Utilization	9	9	1

Table 9: The optimal solution doctor's utilization is assigned high priority

Obtained solution	Average waiting time	Average number of patients	Average utilization
(4, 10, 2)	44.4449	8.8292	98.67 %

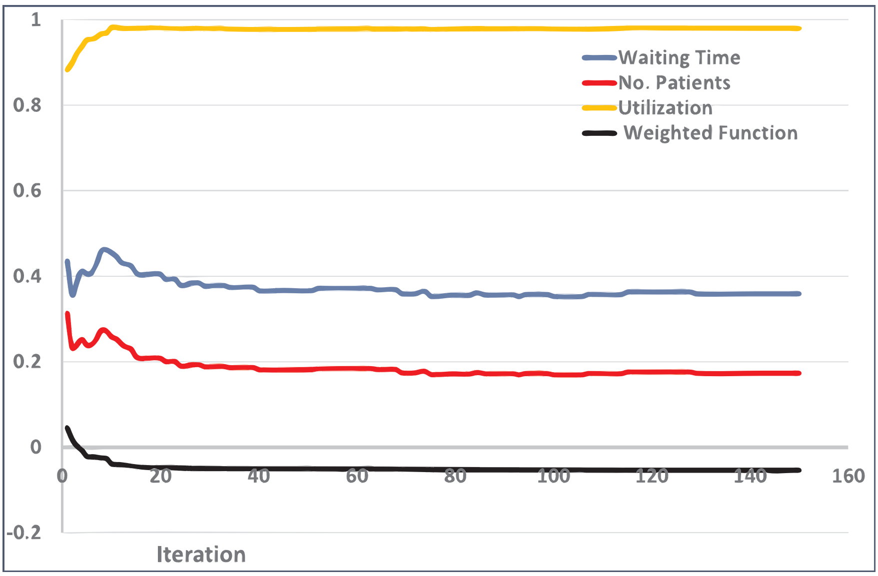

Figure 3: Average performance of AHP-SA when the doctor's utilization is given higher priority

3.4. Equal priority assigned to all objectives

If the decision maker decided to give equal importance for all studied objectives, according to the comparison scale in Table 1. This case is the same as in [5]. The pairwise comparison matrix is shown in Table 10. We first the values of $\lambda_{\max} = 3$ and $C.R = 0$. The priority vector is shown in Table 11. The results of the algorithm is given in Table 12, the optimal solution is $(2, 7, 1)$. Which means that 45 patients can be served in 5 hour clinic session.

Table 10: Pairwise comparison matrix when equal priority assigning to all objectives

Objective	Waiting Time	No. of Patients	Utilization
Waiting time	1	1	1
No. of Patients	1	1	1
Utilization	1	1	1

Table 11: The priority vector when equal priorities are assigned

The objective	The priority vector **w**
Number of patients	0.333333333
Waiting time	0.333333333
Utilization	0.333333333

Table 12: The optimal solution when equal priorities are given to all objectives

Obtained solution	Average waiting time	Average number of patients	Average utilization
(2, 7, 1)	11.7967	1.7328	85.73%

3.5. High priority is given to the utilization then the waiting time then the number of patients

Now we assume that there are more patients want to see the consultant and at the same time we are interested in minimizing the waiting time per patient. The pairwise comparison matrix of the objectives is shown in Table 13. The value of $\lambda_{\max} = 3.072444619$ and $C.R = 0.062452257$. The priority vector is shown in Table 14.

Table 13: Pairwise comparison matrix with different priority

Objective	Waiting Time	No. of Patients	Utilization
Waiting time	1	4	1/5
No. of Patients	1/4	1	1/9
Utilization	5	9	1

Table 14: The priority vector when high priority is assigned to the utilization of then the waiting time the number of patients

The objective	The priority vector **w**
Waiting time	0.199418886
Number of patients	0.065391445
Utilization	0.735189669

The results of implementing the algorithm is given in Table 15. The solution is the vector $(4, 12, 2)$. For a 5 hours clinic session, the average number of scheduled patients in this case is 54 patients.

Table 15: The optimal solution based on the decision maker

Obtained solution	Average waiting time	Average number of patients	Average utilization
(4, 12, 2)	26.5007	4.4737	94.62 %

4. Conclusion

We have proposed an AHP-SA algorithm that combined AHP method and simulated annealing to design an efficient appointment system for outpatient clinics. We modelled the problem as DSMOOP of three objectives; minimizing average waiting time per patients in clinic, minimizing average number of patients in clinic and maximizing doctor's utilization. The three objective functions are aggregated in one objective function which is called the weighted sum function. Then we used the AHP technique to determine the weights in the aggregation function, and developed a simulated annealing algorithm for solving the resulting aggregated function. Further, we observed that the AHP-SA algorithm is robust and is easy to incorporate into most existing scheduling operations. The good thing is that the solution is given based on the decision maker priority.

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