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SIMPLE AGENT-BASED FINANCIAL MARKET MODEL WITH PLAYERS HAVING THE SAME MARKET EXPECTATIONS

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Abstract: The paper presents a simple agent-based model of the financial market, in which one type of security is traded. There are two assets in the model - the speculative security and money. All agents in the model are players of the same type, their behavior obeys a simple algorithm. An important feature of the model is that the expectations of the future prices are the same for all agents. Agents differ in their investment horizon. Experimentally obtained time series were tested for the presence of well known stylized facts. In particular, the experimental time series show presence of the fat tails effect. On the other hand, the effect of volatility clustering for these time series is not revealed.

AMS Subject Classification: 91-10
Key Words: agent-based model of financial market; agent-based modelling; market expectations

1. Introduction

Financial markets have a number of properties that show themselves regardless of the specific trading place, whether it is a New York, London or any other stock exchange. Such properties are called stylized facts. Among the stylized facts, the most famous are the effect of fat tails and volatility clustering. Another well-known stylized fact associated with the effect of fat tails is that the probability of a change in the modulus of the price by x decreases approximately according to the law $1/x^\alpha$, where the value of the parameter α is usually estimated at 3,

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[1]. One of the main goals of creating simulation models of the financial market is to reproduce stylized facts within these models. In addition, one can learn about stylized facts and agent-based modeling of financial markets in following articles [2], [3], [4], [5], [6].

One of the first relatively modern agent-based models of the financial market is the model described in [7] back in 1989. It was written largely under the impression of the financial crisis that occurred in October 1987. Another significant agent-based model is the model presented in [8]. These models shed light on the possible causes of financial bubbles and the process of their collapse, causing financial crises. However, there are no stylized facts in these models, which were mentioned above. Nevertheless, the [8] model turned out to be a landmark in this field, it inspired the creation of a whole series of models both by its authors themselves and other researchers. In particular, in the works [9], [10], the same authors tried to achieve not only the effect of crisis phenomena, but also to demonstrate stylized facts within the framework of models. Nevertheless, these models have a number of disadvantages, in particular, stylized facts appear in them only under certain conditions, which in reality are not always fulfilled.

At the moment, there are many different approaches and tools for agent-based modeling of financial markets. Models can be both with discrete and continuous time [11]; it can be considered as trading one risky asset, or several [12], [13], [14]; players can be either one type or several types, for example, players trading on the basis of technical analysis, and players relying on the assessment of the fundamental value of assets [15]; agents can apply both fixed strategies throughout each simulation and adapt to a changing environment using, for example, the technology of genetic algorithms [16], [17]. Also, an approach called Zero Intelligence finds its practical application, in which agents randomly make a decision to buy or sell assets. In other words, the name of this approach is due to the fact that agents do not adhere to any strategy and all their actions are carried out randomly [18]. This approach allows us to investigate the impact of the trading mechanism on stock exchanges (double auction) without taking into account the various features of strategies of players.

The model presented in this paper implements the idea of the same expectations for the asset future value for all agents. We will call these expectations market expectations. In other words, the model implies that all agents are characterized by the same expectations. Usually, in such models, players are characterized by different expectations, due to which those who expect a decrease in value sell assets to those who expect it to increase. An example of such model is described in the work [19]. In this paper, trading is carried out due to the fact that all agents have different rates of return and different investment terms. Agents can also sell their assets if their value falls below a certain pre-determined level, which is individual for each agent. For simplicity, the agents in the presented model do not try to choose the optimal balance between risk-free asset and risky one, as for example is done in the classical model [8], they either purchase risky asset spending all available money, or sell these assets completely.

The algorithm for determining the price in the model is based on the search for an equilibrium state in which the total supply of assets for sale is equal to or differs by some minimum value from the total demand. Similar approach of finding an equilibrium price was used in the works [19] and [20]. An important feature of the proposed method of determining the price here is that, unlike these works, in the presented model we are talking about the search for a discrete number of assets, not a continuous one.

2. Model description

The model presented in this paper is an agent-based model made in AnyLogic software. The market in the model consists of one risky asset that agents can purchase for money. Thus, there are two types of assets in the model shares and money. Shares are speculative assets, since they do not bring any income by themselves (i.e. no dividends or any similar payments are made on them), the desire to purchase them for agents is based on expectations to see higher price levels in the future what allows to sell them with profit. The total amount of money in the model and the total number of assets are determined at the beginning of each simulation and during the simulation the values of these indicators do not change. The model consists of agents of the same type who make transactions on the market of one risky asset. The model is characterized by discrete time, it will be assumed that one period of model time corresponds to one day of real time. At each time period, one aggregated transaction is made, as a result of it all agents selling their assets receive money from those who purchase these assets at a single specially determined price. The price and volume of such transaction correspond to the intersection of the supply and demand curves for the asset, which are determined based on the requests of agents for purchase (bids) and requests for the sale (asks). The algorithm for price and volume search in more details is discussed below. In the model, investors can open only long positions, short positions cannot be opened.

All agents are investors of the same type. At each moment of time, all investors form a forecast price of the asset for each of the future periods. The value of this indicator is the same for all agents, so is assumed that this indicator is general knowledge and corresponds to the expectations of the entire market for the future value of the asset. Denote the parameter responsible for estimation of daily percentage growth of the asset price as $daily\_return\_estimation$. For simplicity, regardless of previous dynamics of the price,it is assumed that agents always estimate percentage daily growth of the price at constant positive level, i.e. value of parameter $daily\_return\_estimation$ is positive and does not change during each simulation.

Each investor is characterized by several individual parameters, which do not change during simulation. Firstly, all agents differ in the investment horizon for which they can make their investments. Agents cannot hold assets longer than this period, the purchased securities are either sold before this period or just after it expires. For each agent, this indicator is determined randomly at the beginning of each simulation. Secondly, each agent has individual characteristic corresponding to the daily rate of return that this agent would like to receive. Thus, making decision to purchase an asset, the agent calculates the yield for the period corresponding to its investment horizon according to his daily rate of return and compares this yield with the return that the market expects.

Each agent can be in one of two states. In the "buyer" state, an agent does not have assets on his balance sheet, there is a certain amount of money, he chooses the moment to buy assets spending all available money. From the moment of purchase of assets, the countdown of periods begins until the end of the investment period. In the "seller" state, an agent has a certain number of assets on the balance sheet, which he seeks to sell with a profit for himself. He sells them in one of two cases either the price has reached the target values, or the investment period has come to end.

Below parameters and variables of the model are described.

Each agent is characterized by the following parameters:

$period\_of\_investment$ investment period. Parameter, which characterizes duration of investment by the player, the player sells shares either before this period, or just after it expires. For any player, the value of this parameter within each simulation is determined at the beginning and does not change during the simulation.

$daily\_profit\_requirement$ the daily rate of return that the agent would like to receive. In order to get the rate of return for any other number of days, it is necessary to raise the value of this indicator to the degree corresponding to this number of days. In order to get the rate of return for any other number of days, it is necessary to raise the value of this indicator to the degree corresponding to this number of days. This parameter is also determined at the beginning of the simulation and its value does not change during the simulation.

The state of each agent is characterized by the following variables:

assets current number of assets on the agent's balance sheet.

cash amount of the agent's cash at the current time.

buy_price price at which agent, who is in the "seller" state, last purchased securities.

Based on the presented parameters *period_of_investment* and *daily_profit_requirement*, it is possible to determine a variable that will characterize the target value of asset for this agent at the end of the investment period, provided that the last fixed price of the security p_t at time t is known. Then this variable will be defined as follows:

$$target\_price(t) = daily\_profit\_requirement^{period\_of\_investment} p_t.$$

The key variable in the model is the *price* variable, which characterizes the price at which an aggregated transaction was made in the current period.

Each agent in the "buyer" state evaluates the value of the price that is expected by the end of its investment period. In order to get procentege price growth forecast for an arbitrary period of time, it is necessary to raise parameter *daily_return_estimation* to a power equal to the number of periods of selected forecast period. Thus, it is possible to determine a discrete function $price\_forecast(t, T)$ that characterizes the forecasted value of the asset at time $t + T$, provided that p_t is known the value of asset at time t. This discrete function is determined by the rule:

$$forecast\_price(t, T) = daily\_return\_estimation^T p_t.$$

Agents calculate the forecast price for the period T equal to $period\_of\_investment$, i.e. they calculate the following value

$$forecast\_price(t, period\_of\_investment).$$

Everywhere below, when considering a single agent, its forecast price for $T = period\_of\_investment$ periods at time t will be written as $forecast\_price(t)$, i.e. with one argument instead of two.

An agent in the "buyer" state, on the one hand, calculates such a target value of the stock, which, relative to the current price, will give him desired level of profitability for the corresponding investment period, i.e. he calculates $target\_price(t)$. On the other hand, he takes the forecasted value of the market price $forecast\_price(t)$ for his investment period and compares it with the target value of the stock. If the forecast price is higher than the target price, i.e. $forecast\_price(t) \geq target\_price(t)$, then the agent decides to buy assets with all the available money to him. When transaction is made the state of the agent switches from "buyer" to the "seller".

An agent who is in the "seller" state at the time of determining new price submits request for the sale of his assets in two cases: a) at the expiration of the investment period, regardless of the current price, and b) before the investment period, if the proposed price reaches or exceeds his target value. If one of these conditions is satisfied, the agent sells the entire volume of assets available to him and goes into the "buyer" state.

Algorithm description of price and volume determination

Let us describe the algorithm of the market mechanism of the model, which determines the volumes and prices of transactions. The algorithm consists in varying of price per asset, while, at each of its values, on the one hand, orders are collected from agents who are in the "seller" state, ready to sell all their assets at this price, on the other hand, orders are collected from agents who are in the "buyer" state, ready to buy assets spending all their available money. If the total number of assets in the purchase orders coincides with the total number of assets in the sale orders, or differs by a minimum value, then in this case the algorithm fixes this price and an aggregated transaction is made. At the same time, if the total number of assets in the sale orders does not coincide with the total number of assets in the purchase orders, then the value of the first indicator should exceed the value of the second, in other words, all purchase orders are going to be fully satisfied, while the sale orders are going to be satisfied only partially, herewith first of all, orders with highest prices are going to stay unrealized or realized partially. Those agents, who sell their assets partly also change their state from "seller" to "buyer", despite the fact that they have not sold all of their assets.

Initialization of parameters and variables

The behavior of agents is determined by the values of their main parameters. At the initial time of each experiment, agents are initialized, which consists in assigning values to the parameters and variables of the agents. Most parameters are assigned randomly based on a given random distribution. Further, distribution laws, which are used to select random values for parameters and variables of agents are presented.

Amount of money for each agent, the *cash* variable, at the initial moment of time is randomly selected in accordance with discrete uniform distribution law in the range from 0 to 7000.

Number of assets that are initially on the balance sheet of each agent, the *assets* variable, is randomly selected in accordance with a discrete uniform distribution law in the range from 0 to 10.

Price at which assets were last purchased, the variable *buy_price* is selected randomly in accordance with a continuous uniform distribution law with boundaries from 90 to 110.

Parameter *daily_profit_requirement* is selected randomly according to a continuous uniform distribution law with boundaries from 1 to 1.015.

Parameter *period_of_investment* is selected randomly according to a discrete uniform distribution law in the range from 5 to 55.

Initial value of the *price* variable is selected at the level of 100.

Initial value of the *daily_return_estimation* is taken at level 1.001, i.e. it is assumed that agents estimate growth of the price by 0.1% every period.

3. Results

In this section the results of one of the representative experiments are analyzed. Duration of this experiment is 2000 periods. Number of agents is taken at level of 1000.

Below graphs of price change of the model asset and graph of changes in the volume of transactions are presented in Fig. 1 and Fig. 2.

Let us evaluate how the results of the presented model are consistent with the stylized facts. Let us also compare the statistical characteristics of the time series obtained in the model with the time series that correspond to the dynamics of the S&P 500 index and the dynamics of the value of IBM shares. Consider two main stylized facts. The first stylized fact is well known fat tails effect, the essence of which is that the distribution functions for assets returns are characterized by fat tails compared to the normal distribution [1]. The second stylized fact is known as volatility clustering. The main idea of this stylized fact is that high volatility is clustered over time, i.e. more significant changes in price are observed at relatively short time intervals, after which the price change is getting smaller.

Fat tails effect

Graph of histogram and quantile-quantile graph for model time series are presented bellow on Fig. 3 and 4.

Both figures show that the effect of fat tails is present, more clearly this effect is seen in the field of positive price changes. Let us compare these charts with similar charts made for S&P 500 index shown in Fig. 4 and Fig. 5 (daily

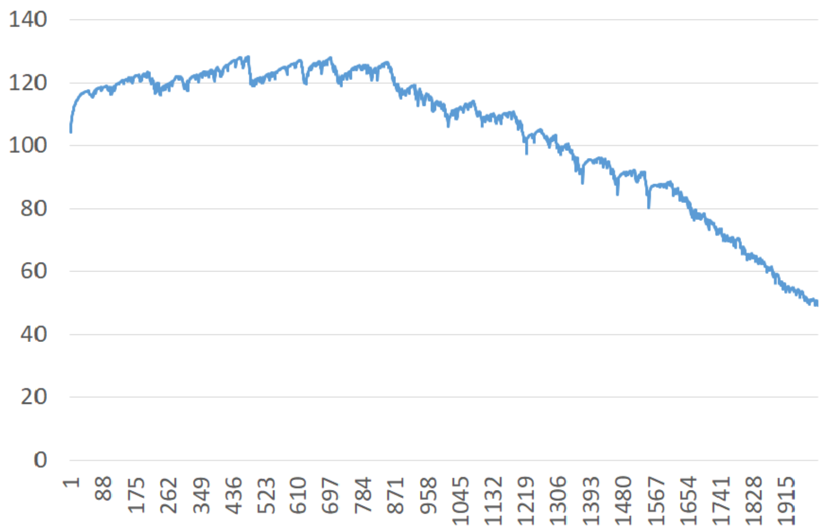

Figure 1: Price dynamics of the model asset

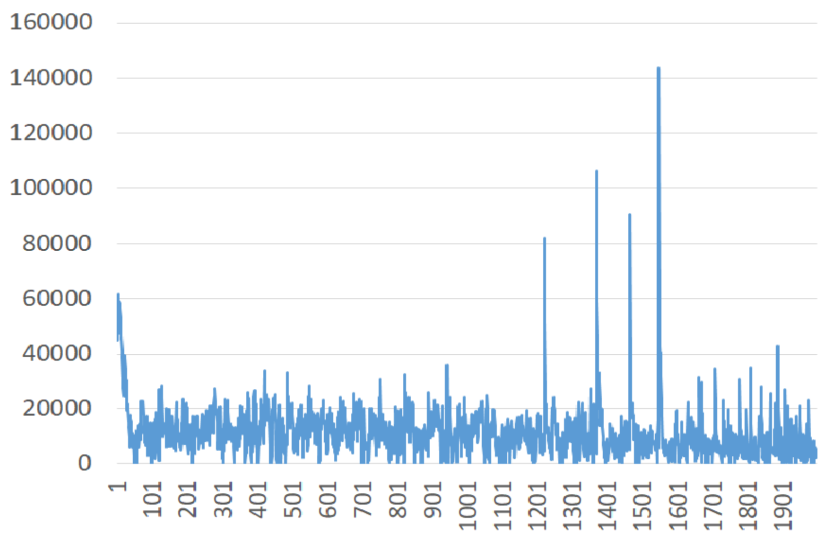

Figure 2: Volume dynamics of the model asset

time series taken in the period from 04.01.2006 to 31.12.2020, i.e. data consist of 3774 periods, source: URL: https://www.investing.com/).

It is easy to see, that charts for both time series are qualitatively similar,

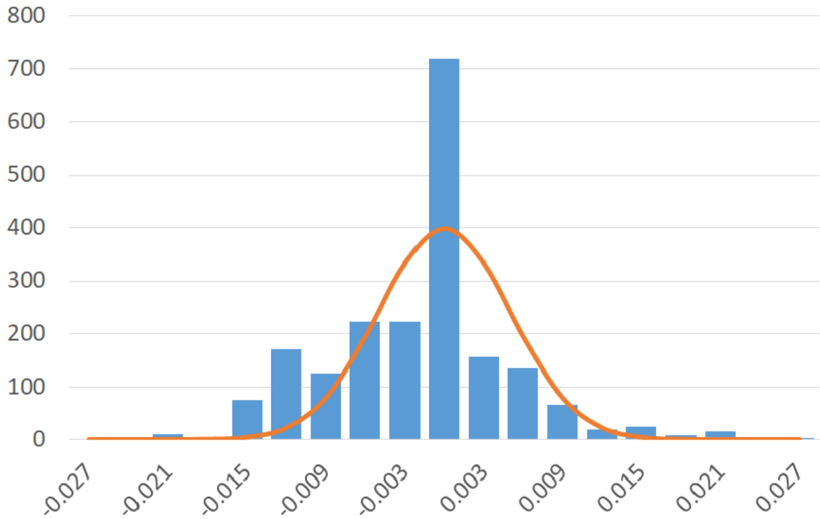

Figure 3: Histogram of returns for the model time series compared to scaled density function of normal distribution (red line)

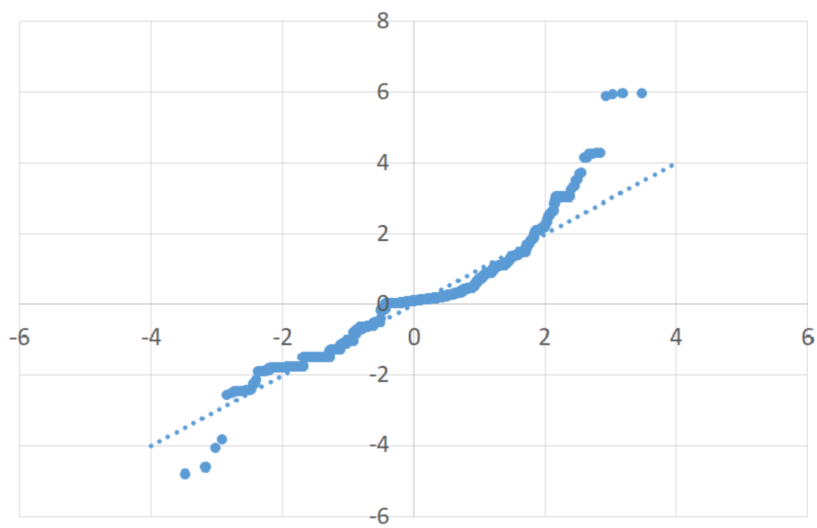

Figure 4: Quantile-quantile plot of the model time series compared to normal distribution (dot line)

they have fat tails. For the model time series, the effect of fat tails is more

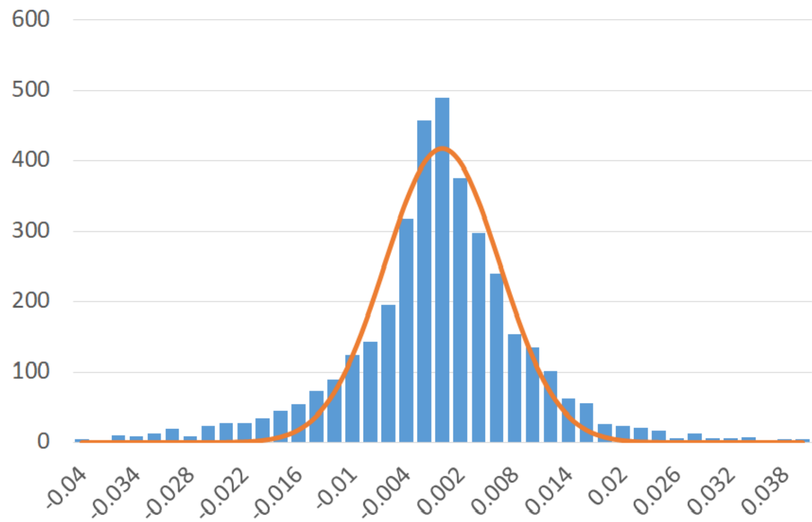

Figure 5: Histogram of returns for S&P 500 index compared to scaled density function of normal distribution (red line)

obviously seen in the area of positive returns, for negative returns this effect is also observed, but not so clearly, only few dots lay bellow dot line in Fig. 4.

Another way to make sure that the resulting distribution is characterized by weighted tails is to calculate the kurtosis coefficient $\kappa = \frac{E(r-Er)^4}{Var(r)^2} - 3$. If this indicator is close to 0, then the distribution by the magnitude of its peak and by the mass of its tails is close to the normal distribution. If $\kappa > 0$, then the distribution is characterized by higher peak and heavier tails compared to the normal distribution.

For the distribution of returns obtained in the model, the kurtosis coefficient turns out to be equal to $\kappa = 4.21$, for the S&P 500 index this indicator is equal to $\kappa = 13.27$. Thus, it can be seen that both distributions are characterized by fat tails, but for the S&P500 index the value of this indicator is noticeably higher.

On the other hand, as it was shown in [1], the distribution function of the absolute value of return in a certain field by its nature behaves like a power function, i.e. $Prob(|return| \geq x) \sim x^{-\alpha}$, with x taking values not close to 0. The parameter α is estimated at level 3. Bellow the distribution functions for the S&P 500, IBM shares and for the model time series are displayed in logarithmic scales (Fig. 7). The dotted line indicates the cubic function. It can be seen that starting from some point, the behavior of the distribution

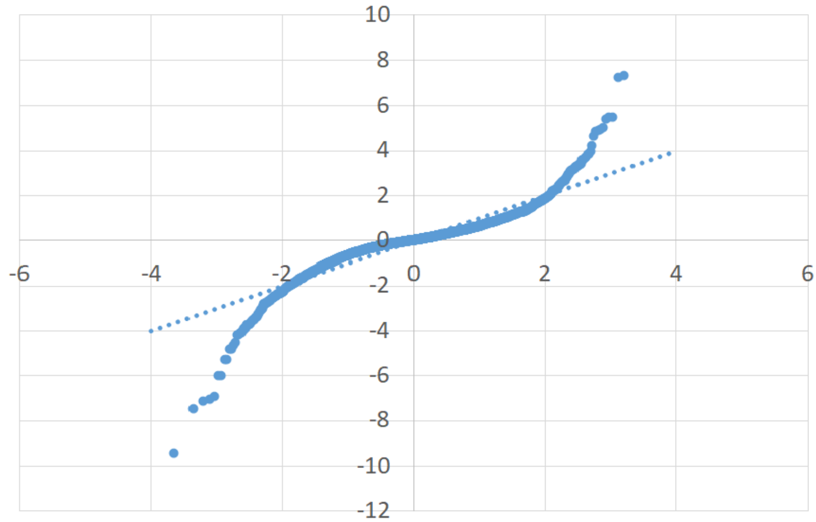

Figure 6: Quantile-quantile plot of S&P 500 index compared to normal distribution (dot line)

functions is similar to the cubic function for model time series, S&P 500 and IBM.

Volatility clustering

At the first stage, autocorrelation charts of returns for both model time series and S&P 500 index time series are considered. It can be seen that the graphs are qualitatively similar, namely, autocorrelation of the first order is significant, autocorrelations of other orders are much less significant.

However, in order to identify the effect of volatility clustering, it is best to consider the autocorrelation graph of either module of returns or squared returns. Consider the autocorrelation graph of the squared returns for the model time series and for the S&P 500 index time series (Fig. 10 and 11). It can be seen that the graphs are very different. If only the first-order autocorrelation is significant for the model time series, then autocorrelations of at least all twenty orders of the S&P500 index time series marked on the chart are significant. The significance of high-order autocorrelation for the squared returns indicates the presence of clustering volatility effect, respectively, the absence of high-order correlation indicates the opposite. Thus, it can be seen that the dynamics of the model asset does not demonstrate the effect of clustering volatility, whereas the dynamics of the S&P 500 index has such an effect (this effect is also seen for IBM and Microsoft stocks). Summing up some of

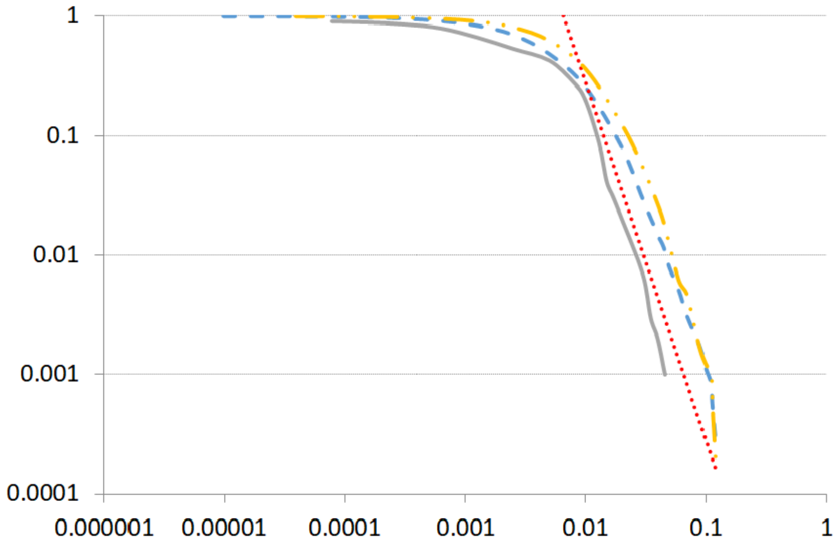

Figure 7: Distribution functions of the form $F(x) = Prob(|return| \geq x)$ for the model time series (solid green line), S&P 500 index (blue discontinuous line), IBM (yellow discontinuous with dots line) and cubic function (dot red line). The graph is displayed in logarithmic scales

the results of this section, it can be noted that the obtained dynamics for the model asset on the one hand is characterized by the presence of the effect of fat tails, also $Prob(|return| \geq x) \sim x^{-\alpha}$, $\alpha = 3$ is satisfied, on the other hand, the price dynamics of model asset does not show the effect of clustering volatility, which is stylized fact for financial indicators, what was demonstrated by the example of the S&P 500. Thus, not all stylized facts can be obtained using the presented model.

4. Conclusion

This article presents financial market model with one risky asset, the role of which is performed by securities, and one riskfree asset - money. The peculiarity of this model is that all agents are characterized by the same expectations of the future value of the risky asset. Also, within the framework of the model, all agents are agents of the same type and their behavior obey to simple algorithm. The experiments have shown that not all stylized facts are fulfilled for the time

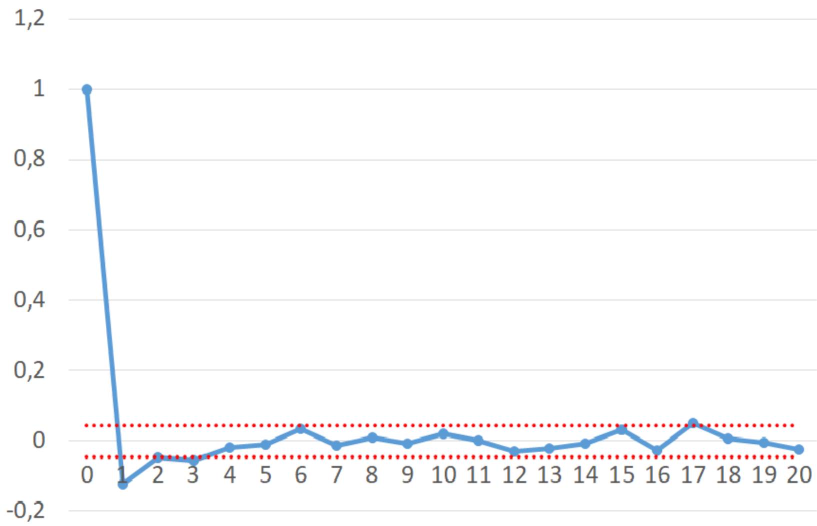

Figure 8: Autocorrelation of returns for the model time series (red dot lines are 5% confidence level)

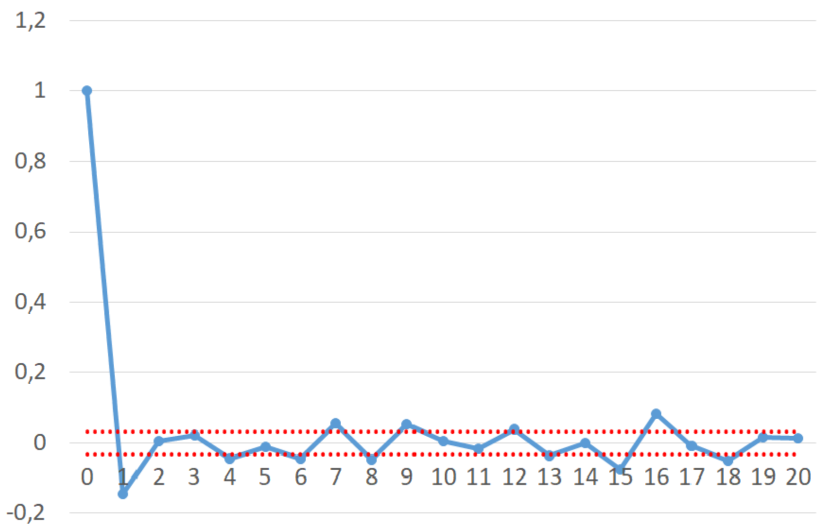

Figure 9: Autocorrelation of returns for S&P 500 index (red dot lines are 5% confidence level)

series obtained in the model. The effect of fat tails for experimental time se-

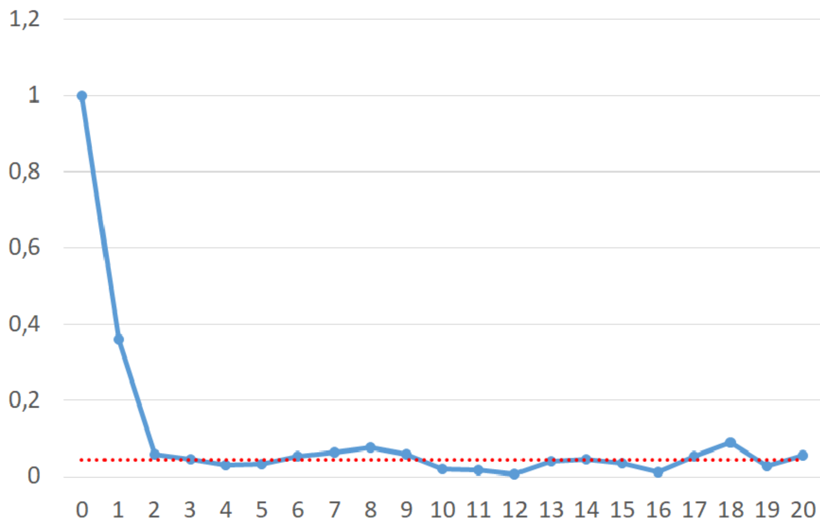

Figure 10: Autocorrelation of the squared returns for the model time series (red dot line is 5% confidence level)

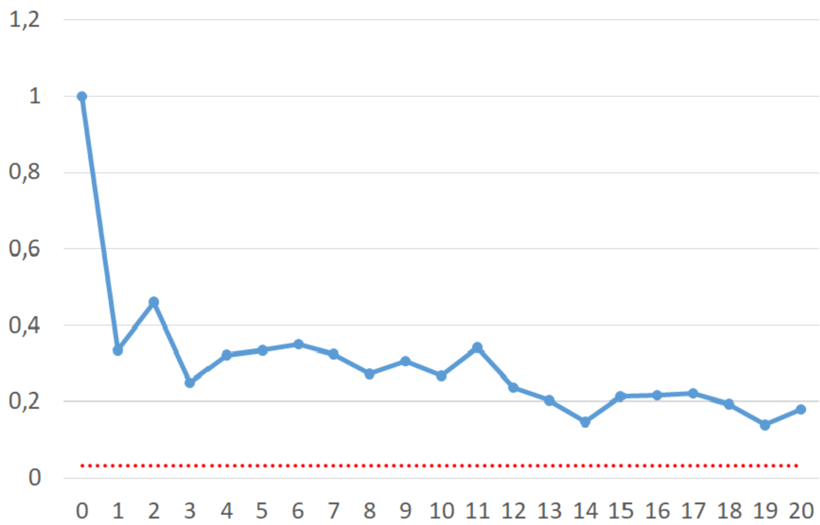

Figure 11: Autocorrelation of the squared returns for S&P 500 index (red dot line is 5% confidence level)

ries is revealed. It is graphically demonstrated that the probability density of price returns modulo on certain area looks like a cubic function, which is also consistent with empirical observations. It is shown that the autocorrelation graph is qualitatively similar to analogous graphs observed for time series of real securities. However, the effect called volatility clustering was not found for experimentally obtained time series. The latter may indicate that the assumption of the same market expectations for all agents may be too strong and it may be necessary to weaken it.

Note, that another feature of the model is the constant total amount of money and total number of assets, which do not change from initial period till the end of each experiment. This feature makes it possible to conduct various other studies, in particular, it is possible to study how the distribution of welfare between all agents changes over time. So, if at the initial moment of time, during the initial distribution of cash and assets among agents, their welfare does not differ from each other too much, then as the trade proceeds, the gap between rich and poor agents is getting more and more noticeable. In other words, towards the end of each experiment only a small percentage of agents may be characterized by significant wealth, while most of the agents could be classified as poor. As part of the current work, detailed research in this direction has not been carried out, similar study can be done in the future.

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