A SHAPLEY TRADE-OFF RANKING METHOD FOR MULTI-CRITERIA DECISION-MAKING WITH DEFUZZIFICATION CHARACTERISTIC FUNCTION

Muhammad Akram Ramadhan Ibrahim[1], Nor Izzati Jaini[2] [§]
Ku Muhammad Naim Ku Khalif[3]

[1,2,3] Centre for Mathematical Sciences
College of Computing and Applied Sciences
Universiti Malaysia Pahang, Lebuhraya Tun Razak
Gambang – 26300, MALAYSIA

Abstract: More studies tend to hybrid the game theory technique with the MCDM method to cater to real-situation problems. This paper provides a novel hybrid Shapley value solution concept in the cooperative game with the trade-off ranking method in MCDM. The fundamental methodology of the Shapley value solution concept and trade-off ranking method are explained to make the methodology clear to the readers. A Shapley trade-off ranking (S-TOR) method has been proposed to obtain the best solution to the fuzzy conflicting MCDM in the personnel selection problem. Thus, the triangular fuzzy number is used to represent the DMs evaluation. Then, the fuzzy number be transformed into crisp values using the defuzzification process. The future suggestions are the fuzzy system may be changed to real data for more practical problems, attempt to incorporate a comprehensive method to increase sharing-profit and decrease sharing-loss in the economy or financial problems, and other types of fuzzy numbers may be used to represent an evaluation of the DMs.

1. Introduction

Multi-Criteria Decision-Making (MCDM) is a sub-discipline in an operations research field that involves many criteria and several alternatives in making a decision. In most real-life situations, people are facing problems that need a solution among several choices with conflicting criteria or objectives to be solved simultaneously. Numerous MCDM methods have been explored over the last several decades with the most well-known MCDM methods such as Analytic Hierarchy Process (AHP), Analytic Network Process (ANP), Technique for Order Preference by Similarity to the Ideal Solution (TOPSIS), Vlsekriterijuska Optimizacija I Komoromisno Resenje (VIKOR), Decision Making Trial and Evaluation Laboratory (DEMATEL) and Data Envelopment Analysis (DEA) as well as the extended version of these methods have been used actively. Also, the hybrid MCDM methodology has been used progressively during the last decade to diverse decision-making methods among other MCDM methods [1, 14, 18]. It can aid the confidence level of a decision-maker (DM) on the result obtained in complex and challenging problems.

The main goal of the MCDM method is to obtain the most preferred alternative from a set of feasible alternatives with respective criteria. One of the latest MCDM methods is the trade-off ranking (TOR) method originally introduced by Jaini and Utyuzhnikov [5]. It is an effective method based on the Euclidean distance equation to manage the MCDM problems concerning the DMs' preferences. The base of this method was to capture the least compromise of the solution. Also, the TOR method can aid the DMs with their preferences in a conflicting MCDM problem. Conventionally, the weights of the criteria used are the average calculation.

The assumption that criteria have equal importance to each other is not practical in real-life cases. By using the average of additive measures of each criterion, it is difficult to assign the weight because the assumption is contravened to solve many decision-making problems [10]. A DM decision is based on their preference to choose any alternatives concerning some criteria. Thus, such a hybrid is capable to obtain a result in more realistic and stable criteria weights for the decision-making process [9]. Therefore, a new criteria weight calculation is considered based on the fairness concept in the cooperative game to cater to this issue in the TOR method, so that it will be the contribution of this paper. The Shapley value solution concept [16] from the cooperative game is used to obtain the fairness-based calculation.

Most of the studies [6, 8, 9, 10, 15, 19, 20, 21] are using fuzzy sets to determine the values of weights and criteria to cope with the uncertainty or

vague conditions. Therefore, this paper will use the triangular fuzzy number to represent the data and then use the defuzzification calculation to make it simpler. To the best of our knowledge, the contribution of this paper is the first hybrid between Shapley value solution concept in cooperative game theory with TOR method in MCDM, or acronym as S-TOR method.

This paper is focused on the improved TOR method proposed in [5] by using the Shapley value solution concept in cooperative game theory. The improvement regarding the weights of the criteria is the fairness-based weightage from the Shapley value solution concept. The remainder of this paper is structured as follows. The Shapley value and MCDM methods literature are highlighted in Section 2. In Section 3, the Shapley value solution concept in a cooperative game is briefly introduced. Section 4 is discussed about the fuzzy TOR method wherein Subsections 4.1 and 4.2 are discussed about the triangular fuzzy number and TOR method with defuzzification, respectively. The Shapley TOR method (S-TOR) is explained in Section 5. Section 6 provided the numerical example of personnel selection with some modification case to test the model applicability. Lastly, Section 7 concludes the paper overall.

2. Shapley value solution concept and MCDM methods

Many studies have been done integrating cooperative game theory and MCDM. Shyr and Kuo [18] studied the TOPSIS and Shapley value solution concept in a cooperative game where the TOPSIS is used to the importance of the factor in making a decision to code sharing and merging while cooperative game part makes a priority raking of target airlines. Optimal fare rates and daily service frequency is solved using Nash equilibrium and payoffs of market shares are marked. Then, the Shapley value solution concept is used to calculate the profit distribution. Then, the TOPSIS method is used to rank the factors that affecting the airlines merging and all its coalition choices. Opricović [13] combined the transferable utility cooperative game framework with the VIKOR method in MCDM to aid the conflict condition. The VIKOR method was calculated to get the compromise solution in the MCDM problem. The game theory part was explained that criteria are more valuable to measure rather than utilities in the game. It can give a set of efficient compromise solutions instead of one single solution.

Lv and Zhao [7] used an improved Shapley value solution concept to allocate the profit of the software outsourcing alliances. The AHP method is used to examine the influence factors. The model argued Shapley model ignored

dissimilar alliance members' motives despite its ability to avoid average allocation. Also, Wei and Zhang [22] proposed the hybrid method between VIKOR in MCDM and Shapley value in a cooperative game. The Shapley value solution concept is used to calculate the weight of the alone contribution of each criterion. Then they have done a comparative analysis by applying the TOPSIS method to solve the problem based on the Shapley value solution concept. The numerical example to plan the development of large projects by the board of directors of enterprises has been considered.

Hindia et al. [4] suggested a new proposed algorithm of integration between the cooperative game and the TOPSIS to improve resource allocation for three smart grid applications. The first step was to calculate the bankruptcy and Shapley value from cooperative game theory to distribute fairly the resources among smart grid applications. Then, the allocation process of the resources to users' applications proceeded for its criteria and preferences. using the TOPSIS method. The findings showed a significant enhancement of the scheduling scheme from other algorithms. Mousavi-Nasab [11] replied to the unfair resource allocation using DEA and the Nash bargaining solution (NBS) to solve the resource allocation problem based on the overall equipment effectiveness. The DM performance was evaluated using the DEA-NBS model whereas the agreement between DMs' weights contribution in the cooperative game can be achieved efficiently. The Spearman's rank correlation coefficient was used to compare the proposed methods and TOPSIS to determine the equity. The result found that the DEA-NBS method was the fairest and impartial.

Numerous studies have been done involving cooperative games and MCDM with the fuzzy system. Singh and Tiong [19] integrated fuzzy numbers with the Shapley value solution concept in the decision-making problem. The grand value of all coalitions is taken into account for decision-making process. The applicability tested for contractor selection problem.

Sun et al. [20] combined the merits of fuzzy set theory, game theory and modified evidence combination extended by D numbers to introduce a new decision-making model. The practicability was tested in the evaluation of the health condition of transformers. The assignment of the probability for all indices was obtained using the fuzzy set theory. Then, the subjective weight of indices was done using fuzzy AHP and the objective weight of indices was done using EW respectively. These two calculations were combined to be used in the game theory model as comprehensive weights. Lastly, the modified evidence combination extended by D numbers was suggested to get the final evaluation of the transformers.

Mishra et al. [10] proposed a new hybrid Shapley value solution concepts in

cooperative game theory and the complex proportional assessment (COPRAS) method in MCDM with hesitant fuzzy information. The new entropy and divergence measures were used to calculate weights of the criteria based on Shapley value solution concepts. The proposed method was tested to service quality decision making then be compared with Shapley TOPSIS for validation of the approach. Rani et al. [15] proposed an extended interval-valued intuitionistic fuzzy VIKOR. To measure the interval-valued intuitionistic sets, the new entropy and similarity measures were used based on the exponential function. The Shapley value solution concept was calculated to cater to incomplete information about the criteria weights. The proposed method was applied to the investment problem.

Jing et al. [6] proposed a fuzzy DEMATEL cooperative game model to perform a relative equilibrium decision approach for concept design in the selection process of the cutting device case study. First, the proposed model used fuzzy DEMATEL to get the objectives' weights. Second, to integrate the weights of the objectives with impact utility into the negotiation theory in the cooperative game model to get the relative equilibrium to fulfill objectives requirements from different strategies. The weighted product method and TOPSIS were used to make a comparative analysis. Mishra and Rani [8] proposed a new methodology that integrated the VIKOR method in MCDM and Shapley value solution concepts in cooperative game theory framework with intuitionistic fuzzy sets. The proposed methodology had been tested for pattern recognition and real cloud service selection problem. Teng et al. [21] developed a modified Shapley value solution concept technique with fuzzy and AHP methods to perform the proposed model. The fuzzy evaluation and AHP were used to evaluate the risk stages of each stakeholder. The fair profit allocation among stakeholders can reduce the risk stage of every stakeholder using the proposed modified Shapley value technique.

Mishra et al. [9] continued the study between Shapley and MCDM methods using Portuguese for Interactive Multicriteria Decision Making (TODIM) with exponential-type divergence measures that applied to fuzzy sets. This proposed methodology was applied to service quality in vehicle insurance firms. The proposed methodology has a unique procedure for the MCDM field.

The importance of all criteria in fuzzy MCDM problems must take into account. Besides, the weight allocated to each criterion is neglected the combination of criteria where there are difficulties to compare two or more alternatives with one is better while the others are not, or vise versa [19]. Therefore, the importance among all criteria is critical and sufficient for fuzzy decision-making since the combinations of criteria shows the DMs' fair preferences using the

Shapley value solution concept.

3. Shapley value solution concept and MCDM methods

Cooperative game theory is one of the branches of the game theory field. The study is about games in coalition form that has been introduced by von Neumann and Morgenstern [12] in 1944. There are two types of payoffs in cooperative game theory which are transferable and nontransferable. Transferable payoff means that there is a medium of exchange between the players and the gain of each coalition can be expressed as one number, for instance, money that can be a profit or a cost. It can be distributed in any conceivable way to all players in a coalition. However, a nontransferable payoff means that there is no such medium of exchange. Each member in a coalition receives an individual payoff that does not come from the coalition's payoff. This research is focusing on the cooperative game with a transferable payoff.

In this section, the Shapley value solution concept in cooperative game theory is introduced. The Shapley value solution concept [16] is one of the well-known single-valued solution concepts in cooperative game theory which assigns to every player its expected marginal contribution. The possible orders of the entrance of the players to the grand coalition occur with equal probability. The Shapley value of a transferable payoff of a game is the payoff allocation $\phi_i(v)$ of player i that be defined as follows:

$$\phi_i(v) = \sum_{C \subseteq N, i \in C} \frac{(|N| - |C|)! \, (|C| - 1)!}{|N|!} \left[v(C) - v(C \setminus \{i\}) \right]. \tag{1}$$

The value $v(C \setminus i)$ represents a coalition without the player i. This equation describes the expected marginal contribution of a player i to the coalition in the following arrival order of players. As an example, there are two possible orders of arrival for two players' cases. First is player 1 arrives first then player 2 and second is player 2 arrives first then player 1. A player i will be paid based on his marginal contribution when joining the coalition of earlier arrivers C. The solution concept of Shapley value encompasses fairness by following four axiomatic characterizations.

Theorem 1. ([16]) *The Shapley value is a unique value that satisfies efficiency, symmetry, dummy player and additivity.*
Axiom 1. Efficiency: $\sum_{i \in N} \phi_i(v) = v(N)$.

Linguistic scale	Fuzzy number scale
Very Low (VL)	$(0, 0, 0.1)$
Low (L)	$(0, 0.1, 0.3)$
Medium Low (ML)	$(0.1, 0.3, 0.5)$
Medium (M)	$(0.3, 0.5, 0.7)$
Medium High (MH)	$(0.5, 0.7, 0.9)$
High (H)	$(0.7, 0.9, 1.0)$
Very High (VH)	$(0.9, 1.0, 1.0)$

Table 1: The linguistic and fuzzy number scale for the criteria weight.

Axiom 2. Symmetry: If for two players i and j , $v(C \cup \{i\}) = v(C \cup \{j\})$ holds for every C, where $C \subset N$ and $i, j \notin C$, then $\phi_i(v) = \phi_j(v)$.

Axiom 3. Dummy: If $v(C \cup \{i\}) = v(C)$ holds for every C , where $C \subset N$ and $i \notin C$, then $\phi_i(v) = 0$.

Axiom 4. Additivity: For any pair of games $v, w : \phi(v + w) \geq \phi(v) + \phi(w)$, where $(v + w)(C) \geq v(C) + w(C)$ for all C.

The efficiency axiom is the distribution of the solution should be the maximum total payoff. The symmetry axiom is the payoff paid refers to the individual player's contribution. The dummy axiom is any player who does not contribute to the coalition should get nothing as his value. Additivity axiom is by adding a solution of two games will produce the solution more or equal to the sum of the game.

4. Fuzzy TOR method

In this section, triangular fuzzy numbers and the TOR method with defuzzification are briefly explained in Subsections 4.1 and 4.2 respectively.

4.1. Triangular fuzzy number

In this subsection, the positive triangular fuzzy number is used to represent the linguistic scale in the fuzzy MCDM problem. The linguistic and fuzzy number scale for the weights of the criteria is shown in Table 1 while the linguistic and fuzzy number scale for the performance of the alternative is shown in Table 2.

Assuming that there are a alternatives, c criteria and $\bar{P}_{ij} = (a_{ij}, b_{ij}, c_{ij})$ denotes the performance of criterion j in terms of alternative i and $\bar{w}_j =$

Linguistic scale	Fuzzy number scale
Very Poor (VP)	$(0, 0, 1)$
Poor (P)	$(0, 1, 3)$
Medium Poor (MP)	$(1, 3, 5)$
Fair (F)	$(3, 5, 7)$
Medium Good (MG)	$(5, 7, 9)$
Good (G)	$(7, 9, 10)$
Very Good (VG)	$(9, 10, 10)$

Table 2: The linguistic and fuzzy number scale for alternative performance.

Alternative	Criterion				
	C_1	C_2	C_3	$\ldots$	C_c
A_1	$\bar{P}_{11}$	$\bar{P}_{12}$	$\bar{P}_{13}$	$\ldots$	$\bar{P}_{1c}$
A_2	$\bar{P}_{21}$	$\bar{P}_{22}$	$\bar{P}_{23}$	$\ldots$	$\bar{P}_{2c}$
A_3	$\bar{P}_{31}$	$\bar{P}_{32}$	$\bar{P}_{33}$	$\ldots$	$\bar{P}_{3c}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
A_a	P_{a1}	P_{a2}	P_{a3}	$\ldots$	P_{ac}

$$\bar{w} = [\varpi_1, \varpi_2, ..., \varpi_c]$$

Table 3: The fuzzy trade-off matrix form.

$(\varpi_{j1}, \varpi_{j2}, \varpi_{j3})$ denotes the weight of the criterion, where both are using triangular fuzzy numbers, for $i = 1, 2, ..., a, j = 1, 2, ..., c$. The fuzzy trade-off matrix form for the MCDM problem is in Table 3.

This paper considers group decision-making of K numbers of DMs. Each DM is needed to evaluate the weight of each criterion and the performance of every alternative using the linguistic scales as in Tables 1 and 2. The evaluation results for criteria weights and the performance of the alternatives are considered as the mean values from the scales obtained from DMs. The formula by using an addition operator $\oplus$ of fuzzy numbers is as follows:

$$\bar{P}_{ij} = \frac{1}{K} \left(\bar{P}_{ij}^1 \oplus \bar{P}_{ij}^2 \oplus \cdots \oplus \bar{P}_{ij}^K \right), \tag{2}$$

$$\bar{w}_j = \frac{1}{K} \left(\bar{w}_j^1 \oplus \bar{w}_j^2 \oplus \cdots \oplus \bar{w}_j^K \right). \tag{3}$$

The definitions and notations of fuzzy sets are also introduced in this section. The membership function $\lambda_{\bar{f}}(x)$ is defined as follows:
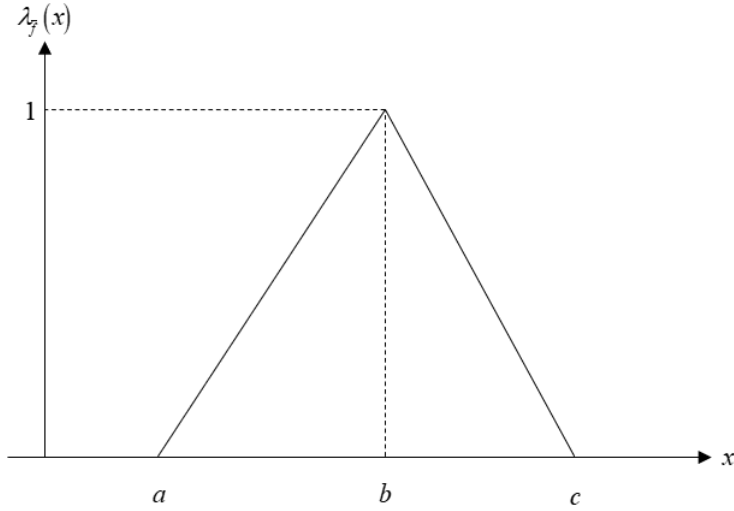
Figure 1: The triangular fuzzy number $\bar{f} = (a, b, c)$.

$$\lambda_{\bar{f}}(x) \begin{cases} \frac{x-a}{b-a}, & a \leq x \leq b, \\ \frac{c-x}{c-b}, & b \leq x \leq c, \\ 0, & \text{otherwise.} \end{cases}$$

The triangular fuzzy number $\bar{f} = (a, b, c)$, where a, b and c are real numbers is shown in Figure 1.

The fuzzy numbers arithmetic operations for any real number is defined as follows:

Addition:

$$\sum_{s=1}^{m} \oplus \bar{f}_1 = \left(\sum_{s=1}^{m} a_1, \sum_{s=1}^{m} b_1, \sum_{s=1}^{m} c_1 \right). \tag{4}$$

Scalar addition:

$$\bar{f} \oplus r = (a + r, b + r, c + r). \tag{5}$$

Subtraction:

$$\bar{f}_1 \ominus \bar{f}_2 = (a_1 - c_2, b_1 - b_2, c_1 - a_2). \tag{6}$$

Multiplication:

$$\bar{f}_1 \otimes \bar{f}_2 = (a_1 \times a_2, b_1 \times b_2, c_1 \times c_2). \tag{7}$$

Scalar multiplication:

$$r \times \bar{f} = (r \times a, r \times b, r \times c). \tag{8}$$

Scalar division:

$$\bar{f}/r = (a/r, b/r, c/r)\,, r > 0. \tag{9}$$

Operator max:

$$\max_s \bar{f}_s = \left(\max_s a_s, \max_s b_s, \max_s c_s\right). \tag{10}$$

Operator min:

$$\min_s \bar{f}_s = \left(\min_s a_s, \min_s b_s, \min_s c_s\right). \tag{11}$$

Distance:

$$d\left(\bar{f}_1, \bar{f}_2\right) = \sqrt{\frac{1}{3}\left[(a_1 - a_2)^2 + (b_1 - b_2)^2 + (c_1 - c_2)^2\right]}. \tag{12}$$

Defuzzification:

$$crisp\left(\overline{f}\right) = \frac{a + 2b + c}{4}. \tag{13}$$

4.2. TOR method with defuzzification

The TOR method [5] is built to solve the MCDM method in conflicting criteria based on the problem with a set of Pareto solutions. Here, the TOR method is focussing on one decision-maker (DM) preference only from a set of solutions. The idea of this method is to suggest the best solution with the least compromise. Fundamentally, the $L_2 - metric$ distance formula is implemented to measure the trade-off between the alternatives as shown in the algorithm of the TOR method. However, the easier way to solve a fuzzy MCDM problem is using the defuzzification process. In the decision-making process, the defuzzification calculation is done at the first step to change the fuzzy numbers into crisp values.

Therefore, the alternative performance $\bar{P}_{ij} = (a_{ij}, b_{ij}, c_{ij})\,, i = 1, 2, ..., a, j = 1, 2, ..., c$ and the criteria weights $\bar{w}_j = (\varpi_{j1}, \varpi_{j2}, \varpi_{j3})\,,$
$j = 1, 2, ..., c$ defuzzification are calculated using formula 13. After this, the defuzzification performance and weight of each criterion are denoted as P_{ij} and w_j, respectively. The TOR method algorithm is as follows:

a. The calculation starts with the normalization of P_{ij} and w_j. The normalization of the performance of criterion in the alternative j, P_{ij} using the equation:

$$f_{ij} = \frac{P_{ij} - \min_j P_{ij}}{\max_j P_{ij} - \min_j P_{ij}}, i = 1, 2, ..., a, j = 1, 2, ..., c, \tag{14}$$

$$w' = \frac{w_j}{\sum\limits_{j=1}^{c} w_j}, j = 1, 2, ..., c. \tag{15}$$

The weightage used in the conventional TOR method is based on the average.

b. Determination of the extreme solutions, $ES_k^*, k = 1, 2, ..., c,$ using the formula as follows:

$$ES_k^* = \left\{ \min_{1 \leq i \leq q} f_{ij} \right\}, \; j = 1, 2, ..., c, \text{ for the cost criteria, or}$$

$$ES_k^* = \left\{ \max_{1 \leq i \leq q} f_{ij} \right\}, \; j = 1, 2, ..., c, \text{ for the benefit criteria.} \tag{16}$$

c. The TOR method has two stages of selection. The first stage is the calculation of the distance between an alternative to an extreme solution while the second stage is the calculation between an alternative with other alternatives if the value DT^1 is the same.

i. The first stage of TOR method selection:
• Calculate the distance between an alternative a to an extreme solution ES_k^*, denoted as $d_{TOR}(ES_k^*, ES_a)$, using the equation as follows:

$$d_{TOR1}(ES_k^*, ES_\alpha) = \left[\sum_{j=1}^{c} (f_{kj}, f_{\alpha j})^2 \right]^{1/2}, \alpha = 1, 2, ..., a, k = 1, 2, ..., c. \tag{17}$$

• Calculate the degree of trade-off, DT between all extreme solutions with an alternative using the formula as follows,

$$DT_{A_\alpha}^1 = \sum_{j=1}^{c} \left[w'_j \times d_{TOR1}(A_k^*, A_\alpha) \right], \alpha = 1, 2, ..., a, k = 1, 2, ..., c. \tag{18}$$

ii. The second stage of TOR method selection:
• Calculate the distance between the alternatives denoted as $d_{TOR2}(ES_\alpha, ES_\beta)$, using the equation as follows:

$$d_{TOR2}(ES_\alpha, ES_\beta) = \left[\sum_{j=1}^{c} \left(\overline{P}_{\alpha j} - \overline{P}_{\beta j} \right)^2 \right]^{1/2}, \alpha, \beta = 1, 2, ..., a,$$

where the weighted performance of an alternative i in criterion j.

$$\overline{P_{ij}} = w'_j \times f_{ij}, i = 1, 2, ..., a, j = 1, 2, ..., c.$$

• Calculate the degree of trade-off, DT between the alternatives using the formula as follows,

$$DT^2_{A_\alpha} = \sum_{i=1}^{a} [d_{TOR^2}(A_\alpha, A_i)], \alpha = 1, 2, ..., a.$$

d. Rank the best alternative with the lowest value of DT^2, if DT^1 is the same.

5. Shapley TOR method

In this section, the Shapley value solution concept is used to combine with the TOR method. The Shapley value in the TOR method is developed to give a new contribution to the fairness-based weightage allocation. This proposed methodology is named acronym the S-TOR method.

The weights of the criteria are determined in terms of Shapley values. Therefore, the weight formula will be changed to the Shapley value term. The Shapley value formula in terms of the combination of criteria is as follows:

$$w_j = \sum_{C \subseteq N, j \in C} \frac{(|N| - |C|)! \, (|C| - 1)!}{|N|!} [v(C) - v(C \backslash \{j\})], \qquad (19)$$

where $j = 1, 2, ..., c$, C is the combination of the criteria, N is the grand criteria combination, $v(C)$ is the value of the criteria combination and $v(C \backslash \{j\})$ is the value of the criteria combination without criteria j. Due to the additivity axiom, the criteria combination must be higher than or equal to their value. The challenging part of the Shapley value solution concept is finding the characteristic function. Most of the recent studies use fuzzy numbers to represent the characteristic function. The characteristic function of this paper will use the triangular fuzzy number for the evaluation process. However, the characteristic function will be obtained after the fuzzy scale turns to crisp value by the defuzzification process.

The combination criteria after the defuzzification process are considered. There is a condition where the combination of more than one criterion must take into account since the linguistic fuzzy number is used to represent the criteria independently. Sometimes, DMs want to achieve their preference with numerous important criteria or preference by the third party. However, the different preferences among the DMs or third parties will lead to unfair allocation. There is a combination of criteria where there are difficulties to compare two or more alternatives with one is better while the others are worse, or vise versa [19]. Thus, the fairness allocation in the Shapley value solution concept
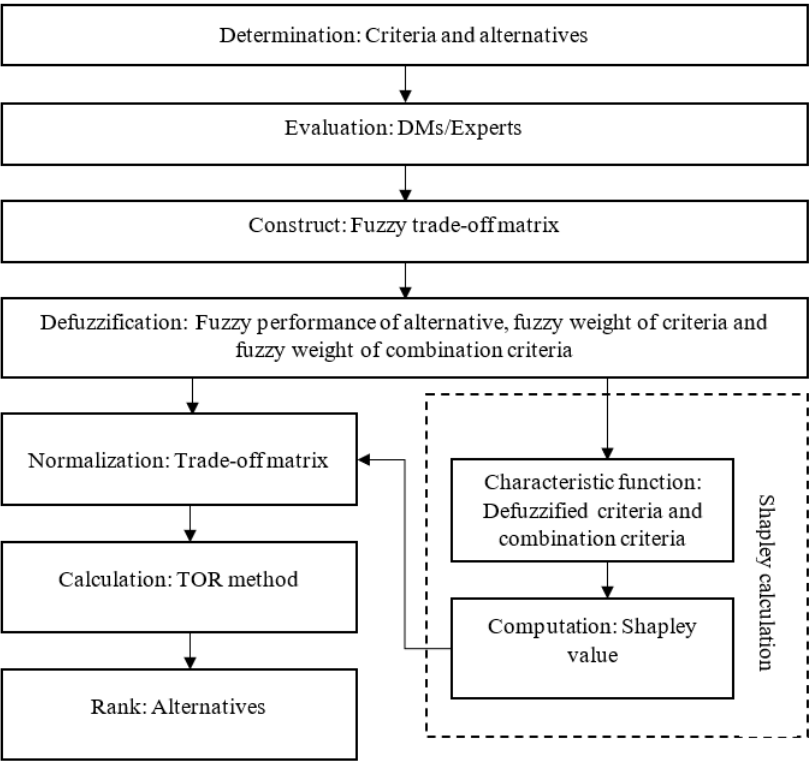
Figure 2: The flowchart of the proposed S-TOR method.

is suitable to aid the group of DMs in making a decision fairly. Therefore, the combination of defuzzification weight of each criterion is needed to evaluate the criteria. Figure 2 shows the flowchart of the proposed S-TOR method.

6. Numerical example: Personnel selection problem

The problem considered here is the extension of the personnel selection problem in [3, 5] to hire a system analysis engineer. Here, the modifications are made to the criteria weights to suit the proposed methodology. Otherwise are maintain the same. The criteria involve are emotional steadiness (C_1), oral communication skill (C_2), personality (C_3), past experience (C_4) and self-confidence (C_5). These five benefit criteria are evaluated by the three DMs from the human

Criterion	Top management
$C_1 C_4$	VH, VH

Table 4: The linguistic score of the top management's combination criteria weight.

Criterion	DM_1	DM_2	DM_3
C_1	H	H	H
C_2	VH	VH	VH
C_3	VH	H	H
C_4	VH	VH	VH
C_5	M	MH	MH

Table 5: The linguistic score for the criteria weights by DMs.

resource department, DM_1, DM_2 and DM_3 to select three alternatives A_1, A_2 and A_3, for the system analysis engineer position.

Suppose that, the top management has its preference of criteria to select new personnel. They need new personnel that can handle a new project urgently. Therefore, the new personnel must be potentially very high in emotional steadiness (C_1) and past experience (C_4) as a priority. The evaluation process of the criteria weights from top management is proposed by the board of directors with a "Very High" linguistic score while the evaluation process of the criteria weights and the performance of the alternatives are made by the DMs from the human resource department. The evaluation process of the top management and the DMs from the human resource department is independent of each other. The DMs are not influenced by the top management during the evaluation process so the Shapley value solution concept will allocate the result fairly. By referring to Tables 1 and 2, the evaluation results are based on the linguistic terms represented in the triangular fuzzy form, which are shown in Tables 4 to 6.

i. Triangular fuzzy number.

The mean calculation of the alternative's performances and the criteria weights by top management and DMs from the human resource department used the formulae 1 and 2 respectively. Then, the fuzzy score for the combination criteria weights is doubled using the formula 8 as shown in Table 7. The fuzzy decision and the defuzzified decision matrices for the personnel selection problem are shown in Tables 8 and 9.

ii. Shapley value solution concept.

Criterion	A_1			A_2			A_3		
	DM_1	DM_2	DM_3	DM_1	DM_2	DM_3	DM_1	DM_2	DM_3
C_1	MG	G	MG	G	G	MG	VG	G	F
C_2	VG	VG	VG	MG	MG	MG	G	G	G
C_3	G	G	G	F	G	G	VG	VG	G
C_4	G	G	G	VG	VG	VG	VG	G	VG
C_5	G	G	G	F	F	F	G	G	MG

Table 6: The linguistic score for alternatives performance by DMs.

Criterion	Top management
C_1C_4	$= 2 \times (0.9, 1.0, 1.0)$
	$= (1.8, 2.0, 2.0)$

Table 7: The fuzzy score for the combination criteria weights by top management.

Alternative	C_1	C_2	C_3	C_4	C_5
	$(0.7, 0.9, 1.0)$	$(0.9, 1.0, 1.0)$	$(0.77, 0.93, 1.0)$	$(0.9, 1.0, 1.0)$	$(0.43, 0.63, 0.83)$
A_1	$(5.7, 7.7, 9.3)$	$(9, 10, 10)$	$(7, 9, 10)$	$(7, 9, 10)$	$(7, 9, 10)$
A_2	$(6.3, 8.3, 9.7)$	$(5, 7, 9)$	$(5.7, 7.7, 9)$	$(9, 10, 10)$	$(3, 5, 7)$
A_3	$(6.3, 8, 9)$	$(7, 9, 10)$	$(8.3, 9.7, 10)$	$(8.3, 9.7, 10)$	$(6.3, 8.3, 9.7)$

Table 8: The fuzzy decision matrix for the personnel selection problem.

Alternative	C_1	C_2	C_3	C_4	C_5
	0.875	0.975	0.908	0.975	0.630
A_1	7.60	9.75	8.75	8.75	8.75
A_2	8.15	7.00	7.53	9.75	5.00
A_3	7.83	8.75	9.43	9.42	8.15

Table 9: The defuzzified decision matrix.

Next, the characteristic function of the Shapley value solution concept is obtained from the result in Table 9. The characteristic function for each combination criteria is calculated by adding two or more criteria weight values together except the value of C_1C_4 since the linguistic scale is given by the top management and defuzzification has been done to obtain it. Table 10 shows the characteristic function values from defuzzified and combination criteria.

Characteristic function	Value
$v\left(\{C_1\}\right)$	0.875
$v\left(\{C_2\}\right)$	0.975
$v\left(\{C_3\}\right)$	0.908
$v\left(\{C_4\}\right)$	0.975
$v\left(\{C_5\}\right)$	0.630
$v\left(\{C_1C_2\}\right)$	1.850
$v\left(\{C_1C_3\}\right)$	1.783
$v\left(\{C_1C_4\}\right)$	1.950
$v\left(\{C_1C_5\}\right)$	1.505
$v\left(\{C_2C_3\}\right)$	1.883
$v\left(\{C_2C_4\}\right)$	1.950
$v\left(\{C_2C_5\}\right)$	1.605
$v\left(\{C_3C_4\}\right)$	1.883
$v\left(\{C_3C_5\}\right)$	1.538
$v\left(\{C_4C_5\}\right)$	1.605
$v\left(\{C_1C_2C_3\}\right)$	2.758
$v\left(\{C_1C_2C_4\}\right)$	2.925
$v\left(\{C_1C_2C_5\}\right)$	2.480
$v\left(\{C_1C_3C_4\}\right)$	2.858
$v\left(\{C_1C_3C_5\}\right)$	2.413
$v\left(\{C_1C_4C_5\}\right)$	2.580
$v\left(\{C_2C_3C_4\}\right)$	2.858
$v\left(\{C_2C_3C_5\}\right)$	2.513
$v\left(\{C_2C_4C_5\}\right)$	2.580
$v\left(\{C_3C_4C_5\}\right)$	2.513
$v\left(\{C_1C_2C_3C_4\}\right)$	3.833
$v\left(\{C_1C_2C_3C_5\}\right)$	3.388
$v\left(\{C_1C_2C_4C_5\}\right)$	3.555
$v\left(\{C_1C_3C_4C_5\}\right)$	3.488
$v\left(\{C_2C_3C_4C_5\}\right)$	3.488
$v\left(\{C_1C_2C_3C_4C_5\}\right)$	4.463

Table 10: The characteristic function values from defuzzified and combination criteria.

The result of the Shapley value using formula 19 and normalized Shapley value using formula 3 for criteria are obtained and calculated in Tables 11 and

Criterion	Shapley value
Criteria 1	0.925
Criteria 2	0.975
Criteria 3	0.908
Criteria 4	1.025
Criteria 5	0.630

Table 11: The Shapley value for criteria.

Criterion	Normalized Shapley value
Criteria 1	0.207
Criteria 2	0.218
Criteria 3	0.203
Criteria 4	0.230
Criteria 5	0.141

Table 12: The normalized Shapley value for criteria.

Alternative	C_1 0.207	C_2 0.218	C_3 0.203	C_4 0.230	C_5 0.141
A_1	0	1	0.64	0	1
A_2	1	0	0	1	0
A_3	0.42	0.64	1	0.68	0.84

Table 13: The normalized defuzzified decision matrix using normalized Shapley values for criteria weights.

12 respectively. The calculation of the Shapley values is done using R software (GameTheory Package) as shown in Appendix A.

iii. Trade-off Ranking method.

The TOR method starts with the normalization of the performance of each criterion using formula 14. The normalized Shapley value weight for each criterion from Table 12 is used in Table 13. Table 13 shows a normalized defuzzified decision matrix using normalized Shapley values for criteria weights.

The extreme solutions of the TOR method are obtained using formula 16. After calculating the data in Table 13 and using formulae 17 and 18, the result of the S-TOR method with defuzzification is given in Table 14. While, the ranking for the case is given in Table 16

Now, this paper considers another type of criteria weights calculated in [5]

Trade-off	A_1	A_2	A_3
DT^1	1.113	1.080	1.044

Table 14: The S-TOR method with defuzzification result.

New weight	A_1	A_2	A_3
Similar	1.826	0.331	1.359
Dissimilar	1.067	1.103	1.093

Table 15: The results by the S-TOR with defuzzification method using similar and dissimilar new weights of criteria.

concerning DMs' preferences in choosing emotional steadiness (C_1) and past experience (C_4). Note that, the DMs' preferences are similar to the top management. Thus, the criteria weights and results from [5] are obtained and then the proposed S-TOR method is used. The new DMs' weights of C_1, C_2, C_3, C_4 and C_5 are $0.82, 0.06, 0.30, 0.91$ and 0.09 respectively.

Also, this paper studies a condition when there are dissimilar criteria interest between the top management and DMs from the human resource department. The top management prefers emotional steadiness (C_1) and past experience (C_4) but DMs prefer oral communication skill (C_2), personality (C_3) and self-confidence (C_5). Assume that the DMs' new weights of C_1, C_2, C_3, C_4 and C_5 are $0.15, 0.94, 0.68, 0.19$ and 0.88 respectively. Table 15 shows the results by the S-TOR with defuzzification method using similar and dissimilar new weights of criteria.

Table 16 shows that all cases have different results of first ranking using the S-TOR method since the rankings for all cases depend on the weights.

For case i, A_3 is the best candidate followed by the second and third ranks which are candidates A_2 and A_1 respectively. The criteria weights are almost equal in this case. Thus, A_3 is the best option since it has the most balanced traits. For case ii, A_2 is the best candidate for the S-TOR method when the DMs have a similar preference with the top management followed by the second and third ranks which are candidates A_3 and A_1 respectively. Candidate A_2 has a balancing characteristic that shows its least compromise to all criteria compared to A_3 and A_1 in similar criteria weights. In this case, the important criteria are C_1 and C_4, thus A_2 holds the best rank as it has the best value in both criteria, eventhough it has the worst criteria in all others.

However, for case iii, A_1 is the best candidate when there is a dissimilar criteria weight between top management and DMs followed by the second and

Rank	i	ii	iii
1	A_3	A_2	A_1
2	A_2	A_3	A_3
3	A_1	A_1	A_2

Note. i - Result from Table 14, ii - Similar preferences between top management and DMs, and iii - Dissimilar preferences between top management and DMs.

Table 16: The ranking of alternatives by S-TOR method.

third ranks which are candidates A_3 and A_2. Candidate A_1 has balance characteristic that shows its least compromise to all criteria compared to A_2 and A_3. In this case, the important criteria are C_2, C_3 and C_5. Hence, candidate A_1 is the best option to be hired since the candidate possess the best value in these three criteria. Notes that, for both types of criteria weights for cases ii and iii, the candidate A_3 is ranked second. The distance position of the candidate A_3 among all alternatives is consistent in all criteria between similar and dissimilar criteria weights of top management and DMs.

7. Conclusion

A Shapley trade-off ranking (S-TOR) method has been proposed in this paper. The proposed methodology has been used to obtain the best solution to the fuzzy conflicting MCDM using the Shapley value solution concept in the personnel selection problem. Generally, the MCDM problem uses fuzzy set theory to represent the uncertain, vague and imprecise data. Thus, the triangular fuzzy number is used in this paper to represent the DMs evaluation on criteria and alternatives performance using a linguistic scale realistically. Each criterion is evaluated by DMs independently using the triangular fuzzy number. This paper turned the fuzzy number to crisp values by the defuzzification process. Shapley value solution concept is used to take into consideration the interaction among the criteria for an adequate fuzzy MCDM problem. To adapt independent criteria, this paper proposed that the grand combination criteria are the entire evaluation of the decision-making process. Shapley value calculation is used to calculate the marginal criteria combination into overall consideration among criteria and DMs' preferences during the decision-making process. The future suggestions are the fuzzy system may be changed to real data for more

practical problems instead of using defuzzification, attempt to incorporate a comprehensive method to increase sharing-profit and decrease sharing-loss in the economy or financial problems, and other types of fuzzy number may be used to represent an evaluation of the DMs.

Appendix A

- Shapley value calculation in R language (GameTheory Package) [2].

```
# Start defining the game

CharacFunc<-c(0.875,0.975,0.908,0.975,0.630,1.850,
1.783,1.950,1.505,1.883,1.950,1.605,1.883,1.538,1.605,
2.758,2.925,2.480,2.858,2.413,2.580,2.858,2.513,2.580,
2.513,3.833,3.388,3.555,3.488,3.488,4.463)
Personnel<-DefineGame(5,CharacFunc)
summary(Personnel)

# End defining the game

NAMES <- c("Criteria 1","Criteria 2","Criteria 3",
"Criteria 4","Criteria 5")
PersonnelShapley <- ShapleyValue(Personnel,NAMES)
summary(PersonnelShapley)
```

- Similar preference between DMs with top management.

```
# Start defining the game

CharacFunc<-c(0.82,0.06,0.30,0.91,0.09,0.88,1.12,1.95,
0.91,0.36,0.97,0.15,1.21,0.39,1.00,1.18,2.01,0.97,2.25,
1.21,2.04,1.27,0.45,1.06,1.30,2.31,1.27,2.10,2.34,1.36,
2.40)
Personnel<-DefineGame(5,CharacFunc)
summary(Personnel)

# End defining the game

NAMES <- c("Criteria 1","Criteria 2","Criteria 3",
"Criteria 4","Criteria 5")
```

```
PersonnelShapley <- ShapleyValue(Personnel,NAMES)
summary(PersonnelShapley)
```

- Dissimilar preference between DMs with top management.

```
# Start defining the game

CharacFunc<-c(0.15,0.94,0.68,0.19,0.88,1.09,0.83,1.95,
1.03,1.62,1.13,1.82,0.87,1.56,1.07,1.77,2.89,1.97,2.63,
1.71,2.83,1.81,2.50,2.01,1.75,3.57,2.65,3.77,3.51,2.69,
4.45)
Personnel<-DefineGame(5,CharacFunc)
summary(Personnel)

# End defining the game

NAMES <- c("Criteria 1","Criteria 2","Criteria 3",
"Criteria 4","Criteria 5")
PersonnelShapley <- ShapleyValue(Personnel,NAMES)
summary(PersonnelShapley)
```

Acknowledgments

References

[1] E. Abu-taieh, A. Al-Rasheed, M. Alohali, Multi-criteria decision-making using an extended Gale and Shapley (E-GS) algorithm, In: *Education Excellence and Innovation Management: A 2025 Vision to Sustain Economic Development during Global Challenges* (2020), 862-876.

[2] S. Cano-berlanga, J. Giménez-gómez, C. Vilella, Enjoying cooperative games: The R package Game Theory, *Appl. Math. Comput.*, **305** (2017), 381-393.

[3] C.-T. Chen, Extensions of the TOPSIS for group decision-making under fuzzy environment, *Fuzzy Sets Syst.*, **114** (2000), 1-9.

[4] M.N. Hindia, A.W. Reza, K.A. Noordin, A novel scheduling algorithm based on game theory and multicriteria decision making in LTE network, *Int. J. Distrib. Sens. Networks*, **2015** (2014), 1-8.

[5] N.I. Jaini, S.V. Utyuzhnikov, A fuzzy trade-off ranking method for multi-criteria decision-making, *Axioms*, **7** (2018), 1-21.

[6] L. Jing, Z. Li, X. Peng, J. Li, S. Jiang, A relative equilibrium decision approach for concept design through fuzzy cooperative game theory, *J. Comput. Inf. Sci. Eng.*, **19** (2019), 1-24.

[7] X. Lv, S. Zhao, Research on profit distribution of software outsourcing alliances based on the improved Shapley value model, *Cybern. Inf. Technol.*, **13** (2013), 100-109.

[8] A.R. Mishra, P. Rani, Shapley divergence measures with VIKOR method for multi-attribute decision-making problems, *Neural Comput. Appl.*, **31** (2019), 1299-1316.

[9] A.R. Mishra, P. Rani, A. Mardani, R. Kumari, E. K. Zavadskas, D.K. Sharma, An extended Shapley TODIM approach using novel exponential fuzzy divergence measures for multi-criteria service quality in vehicle insurance firms, *Symmetry (Basel).*, **12** (2020), 1-19.

[10] A.R. Mishra, P. Rani, K.R. Pardasani, Multiple-criteria decision-making for service quality selection based on Shapley COPRAS method under hesitant fuzzy sets, *Granul. Comput.*, **4** (2018), 435-449.

[11] S.H. Mousavi-Nasab, Resource allocation based on overall equipment effectiveness using cooperative game, *Kybernetes*, **49** (2019), 819-834.

[12] J. von Neumann, O. Morgenstern, *Theory of Games and Economic Behavior*, Princeton University Press, New York (1944).

[13] S. Opricović, Compromise in cooperative game and the VIKOR method, *Yugosl. J. Oper. Res.*, **19** (2009), 225-238.

[14] P. Phochanikorn, C. Tan, An integrated multi-criteria decision-making model based on prospect theory for green supplier selection under uncertain environment: A case study of the Thailand palm oil products industry, *Sustainability*, **11** (2019), 1-22.

[15] P. Rani, D. Jain, D.S. Hooda, Shapley function based interval-valued intuitionistic fuzzy VIKOR technique for correlative multi-criteria decision making problems, *Iran. J. Fuzzy Syst.*, **15** (2018), 25-54.

[16] L.S. Shapley, A value for n-person games, *Contrib. to Theory Games*, **2** (1953), 307-317.

[17] O.F. Shyr, Y.P. Kuo, Applying TOPSIS and cooperative game theory in airline merging and coalition decisions, *J. Mar. Sci. Technol.*, **16** (2008), 8-18.

[18] H.J. Shyur, H.S. Shih, A hybrid MCDM model for strategic vendor selection, *Math. Comput. Model.*, **44** (2006), 749-761.

[19] D. Singh, R.L.K. Tiong, A fuzzy decision framework for contractor selection, *J. Constr. Eng. Manag.*, **131** (2005), 62-70.

[20] L. Sun, Y. Liu, B. Zhang, Y. Shang, H. Yuan, Z. Ma, An integrated decision-making model for transformer condition assessment using game theory and modified evidence combination extended by D numbers, *Energies*, **9** (2016), 1-22.

[21] Y. Teng, X. Li, P. Wu, X. Wang, Using cooperative game theory to determine profit distribution in IPD projects, *Int. J. Constr. Manag.*, **19** (2019), 32-45.

[22] G. Wei, N. Zhang, A multiple criteria hesitant fuzzy decision making with Shapley value-based VIKOR method, *J. Intell. Fuzzy Syst.*, **26** (2014), 1065-1075.