A PHENOMENOLOGICAL MODEL OF
SUSPENSION FILTRATION IN POROUS MEDIUM

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Abstract: A suspension filtration problem with modified deposition kinetics in a porous medium is considered. A new model developed, in which "aging" and "charging" phenomena are taken into account in the kinetics of deposition. It is suggested that the process of deposition forming happens with "charging", transient, "aging" and breakthrough stages and stops when the capacity of filter fills with deposition. To solve the formed system of partial differential equations an algorithm based on finite difference schemes is developed. Based on numerical results the influences of "aging" and "charging" phenomena on suspended particle transport, attachment and detachment characteristics are analysed.

1. Introduction

The process of suspensions flow through porous media in which suspended particles removed from a fluid is called deep bed filtration. The process is

usable in different industries such as environmental, pharmaceutical, oil, water and wastewater treatment. In the oil industry, suspensions are applied in well drilling and oil production with secondary and tertiary recovery. In water filtration the fluid is water, the suspended particles can be inorganic, organic or bacterial matter [1].

In deep bed filtration the suspension flows through the granular medium, some of the particles, under the influence of a number of forces, may come into contact with the filter grains and deposit on the grain surface or already deposited particles. As a result, the geometrical and structural characteristics of the media, and the surface characteristics of the filter grains may be modified significantly. Which, can affect the transport and deposition of suspended particles through the medium.

Deep bed filtration usually viewed on microscopic and macroscopic levels [2, 3]. In microscopic approach process described as a sequence of following removal steps of the suspended particles: a transport step, which is a physical-hydraulic process, an attachment step, which is a physical-chemical process and a third is a detachment step [4, 5].

As mentioned in [4], the main mechanisms of transport of suspended particles are: interception, diffusion, inertia, sedimentation and hydrodynamic effect. In [3, 6] these mechanisms are shown schematically. When the distance between suspended particles and media grain becomes close in the order of nanometers the attachment mechanisms affects to removal particles. The forces affecting to attachment can divide into two: the first group is van der Waals forces and electric double-layer force called also long-term forces, as their influence can be dominant in distances up to 100 nm, the Born repulsion forces and hydration forces, whose influence is important in distances up to 5 nm, also called short-term forces [2, 3, 4, 5, 6].

Detachment was first given in works of Mints [7] who has supported his views by producing experiments. Mints [7] mentioned that by increasing the headloss the hydrodynamic forces may partially destroy the structure of porous media. Some of the less strongly linked previously deposited particles may be detached from porous media grains and join to flow again.

In the macroscopic approach, the mathematical model suspension filtration in porous medium is consists of mass balance, kinetic equations, and Darcy's law which described as a system of partial differential equations [2, 8]. The macroscopic approach to suspension filtration in porous medium begins with the work of Iwasaki [9]. Iwasaki [9] described the process with a first-order kinetic equation.

$$\frac{\partial \rho}{\partial t} = \lambda c, \tag{1}$$

where ρ is the concentration of deposited particles, t is time, c is the particle concentration of the suspension, λ is filter coefficient. This expression was confirmed with experiments in [10].

The fact that λ varies with time and its value at the beginning of the filtration λ_0 is changes, Tien [11] gives following relation:

$$\frac{\partial \rho}{\partial t} = \lambda_0 F(\alpha, \rho)c \tag{2}$$

with

$$F(\alpha, 0) = 1, \tag{3}$$

where α vector of parameters.

Tien [11] suggested that there is three types of functions used for $F(\alpha, \rho)$: increasing function, which means the filter's ability to collect particles improves during the process; decreasing function, means filter's performance increases during filtration; non-monotone function, or a combination of both kinds of behaviour mentioned above, it first increases with the increase of deposition and then decreases after reaching maximum.

The use of (2) impossible in many cases, however, in these expressions does not allow for possible detachment of deposited particles. For including the detachment of deposited particles, used the following type of expression [4, 11]

$$\frac{d\rho}{dt} = \beta_a vc - \beta_d \rho, \tag{4}$$

where, v is filtration velocity (m/s), β_a and β_d are phenomenological attachment and detachment coefficients, respectively.

In expression (4) the filtration rate is estimated as sum of the attachment and detachment rates. It seems not correct, however, that detachment would occur from the beginning of filtration. In [11] given more reliable expression, one may argue that detachment occurs only after deposition reaches a certain value ρ_1

$$\frac{d\rho}{dt} = \beta_a vc - \beta_d(\rho - \rho_1). \tag{5}$$

In [2] filtration rate described with the following multistage kinetic equation

$$\frac{d\rho}{dt} = \begin{cases} \beta_r vc & \text{for} & 0 < \rho < \rho_1, \\ \beta_a vc - \beta_d \rho & \text{for} & \rho_1 < \rho < \rho_0, \\ 0 & \text{for} & \rho = \rho_0, \end{cases} \tag{6}$$

where β_r is the attachment rate coefficient at the ripening stage, ρ_0 indicates the filter retention capacity limit. In [13] this model is compared with experiments. Various kinetic equation also given in [14, 15, 16, 17].

To describe the flow of suspension in porous media many phenomenological relationships have been obtained, but "aging" and "charging" phenomena not taken account together. In this paper we developed a phenomenological mathematical model of suspension filtration in a porous medium with tacking into account all stages of filtration in kinetic equation. The problem is solved numerically, by using the method of finite difference. Influences of suspension "aging" and "charging" phenomena on filtration characteristics are studied.

2. Statement of the problem

Here we consider a mathematical model of suspension filtration in a porous medium with multi-stage deposition kinetics. The model is a modification of the known models [2, 12, 17, 18, 19]. Let, from a certain moment in time $t > 0$, a suspension with a concentration of suspended solids c_0, with a filtration velocity $v(t) = v_0 = \text{const}$ begins to flow into a homogeneous formation of length L, with initial porosity m_0, saturated with a clean (without suspended particles) fluid.

The mass balance equation of suspended particles in a suspension flow in the one-dimensional case has the form

$$m_0 \frac{\partial c}{\partial t} + v \frac{\partial c}{\partial x} + \frac{\partial \rho}{\partial t} = D \frac{\partial^2 c}{\partial x^2}, \tag{7}$$

where, m_0 is initial deposition, D is diffusion coefficient.

The kinetics of deposition in which is taken in the form

$$\frac{\partial \rho}{\partial t} = \begin{cases} \beta_1 vc & \text{for} \quad 0 < \rho \le \rho_1, \\ \beta_2 vc - \beta_3(1 + \gamma |\nabla p|)\rho & \text{for} \quad \rho_1 < \rho \le \rho_2, \\ \beta_2 \frac{\rho_0}{\rho} vc - \beta_3(1 + \gamma |\nabla p|)\rho & \text{for} \quad \rho_2 < \rho < \rho_0, \\ 0 & \text{for} \quad \rho = \rho_0, \end{cases} \tag{8}$$

where, ρ_0 the total filter capacity, ρ_1 is the value of ρ at which "charging" is completed, β_1 is the coefficient associated with the effect of "charging", β_2 is the coefficient associated with the deposition of particles, β_3 is the coefficient associated with the detachment of particles, $|\nabla p|$ is the pressure gradient modulus, γ is constant coefficient.

We use following physical representation to develop the model. Filtration of the suspension takes place in a clean filter that does not initially contain deposition of particles. During the initial particle deposition, an increase in the specific surface and, accordingly, kinetic coefficients occurs. On the other

hand, upon enveloping the loading surface with a monolayer of deposition, the surface potential may change, and both an improvement and impairment in the mass transfer efficiency can occur. The effect of this effect, called "charging" the filter, is significant at the initial stages of filtering [2, 12]. "Charging" phenomenon delays until the monolayer of particles covers surface of the porous media grain. Then particles begin to interact mainly with the previously deposited particles, the filtration cycle enter the next transition stage. In this stage, attachment of particles and detachment by a fluid flow occur in parallel [2]. The transition stage, in which irreversible deposition begins to be reversible, occurs when the deposition concentration reaches a predetermined value ρ_1. When deposition concentration reaches the value ρ_2 begins "aging" (compaction) stage [12]. In this stage intensity of deposition increases, due to deposition begins compaction. The volume of particles that can be captured by the filter is finite. In the final stage, the deposition of particles reaches its maximum possible value ρ_0, i.e. the filter is completely saturated with deposition [2, 12]. From this moment it is assumed that further particle deposition is stopped. First term on the right hand side of equation (8) characterizes the intensity of particle attachment, and the second, detachment. In accordance with [18, 19], we assume that, detachment of particles depend on the module of the pressure gradient $|\nabla p|$, and the bigger $|\nabla p|$, and the greater the probability of detachment of particles from pores.

For $|\nabla p|$ we use Darcy's law

$$v = K\left(m\right)|\nabla p|, \tag{9}$$

where $K\left(m\right)$ is filtration coefficient, $m = m_0 - \rho$ is current porosity.

For $K\left(m\right)$ we use Carmen-Kozeny equation [11]

$$K\left(m\right) = k_0 m^3 / \left(1 - m\right)^2, \quad k_0 = \text{const.}, \tag{10}$$

where k_0 is initial permeability coefficient.

Initial and boundary conditions are in following form

$$c(0,t) = c_0, \ \left.\frac{\partial c}{\partial x}\right|_{x=L} = 0, \ c(x,0) = 0, \ \rho(x,0) = 0. \tag{11}$$

3. Solution method

Problem (7)-(11) solved by using finite difference method [20]. In the area $D = \{0 \leq x \leq L, \ 0 \leq t \leq T\}$ we introduce a net, where T is the time during

which the filtration is studied. For this, we divide the interval $[0, L]$ with step h, and $[0, T]$ with step τ. As a result we have net

$$\omega_{h\tau} = \left\{ (x_i, t_j), \ x_i = ih, \ i = \overline{0, I} \ t_j = j\tau, \ j = \overline{0, J}, \ \tau = T/J \right\}.$$

Instead of functions $c(t, x)$, $\rho(t, x)$ we consider the net functions whose values at the nodes (x_i, t_j) respectively, denote by c_i^j, ρ_i^j.

The equations (7)-(10) is approximated on a grid $\omega_{h\tau}$ in the following form

$$m_0 \frac{c_i^{j+1} - c_i^j}{\tau} + v_0 \frac{c_i^{j+1} - c_{i-1}^{j+1}}{h} + \frac{\rho_i^{j+1} - \rho_i^j}{\tau}$$

$$= D \frac{c_{i-1}^{j+1} - 2c_i^{j+1} + c_{i+1}^{j+1}}{h^2}, \quad (12)$$

$$\frac{\rho_i^{j+1} - \rho_i^j}{\tau}$$

$$= \begin{cases} \beta_1 v c_i^j, & 0 < \rho_i^j \le \rho_1, \\ \beta_2 v c_i^j - \beta_3 \left(1 + \gamma \left| \nabla p \right|_i^{j+1} \right) \rho_i^j, & \rho_1 < \rho_i^j \le \rho_2, \\ \beta_2 \frac{\rho_0}{\rho_i^j} v c_i^j - \beta_3 \left(1 + \gamma \left| \nabla p \right|_i^{j+1} \right) \rho_i^j, & \rho_2 < \rho_i^j < \rho_0, \\ 0, & \rho_i^j = \rho_0, \end{cases}, \quad (13)$$

$$\left| \nabla p \right|_i^{j+1} = \frac{v_0 \left(1 - m_0 + \left(\rho_{a,i}^j + \rho_{n,i}^j \right) \right)^2}{k_0 \left(m_0 - \left(\rho_{a,i}^j + \rho_{n,i}^j \right) \right)^3}. \quad (14)$$

The initial and boundary conditions (11) are also presented in the net form

$$c_i^0 = 0, \ \rho_i^0 = 0, \ i = \overline{0, I}, c_0^j = c_0, \ c_I^j - c_{I-1}^j = 0, \ j = \overline{0, J}. \quad (15)$$

The stability of difference schemes (12), (13) analyzed by methodologies used in [20, 21]. The schemes are stable in used values of parameters and have first order accurate in both time and space.

The calculation scheme is as follows. According to conditions (15), the values of c_i^j and ρ_i^j are determined at all points of the zero layer. From (14) value of $\left| \nabla p \right|_i^{j+1}$ are determined through the known quantities of ρ_i^j at the lower layer at the corresponding points, substituting those found $\left| \nabla p \right|_i^{j+1}$ in (13) ρ_i^{j+1} are found. For c_i^{j+1} in (12) we use Thomas algorithm [22].
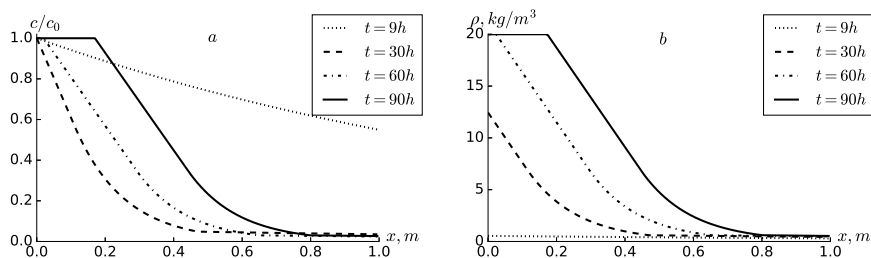
Figure 1: Profiles of $c/c_0(a)$, $\rho(b)$, at $\rho_1 = 0,6$ kg/m^3, $\rho_2 = 7$ kg/m^3.

4. Results and discussion

As the values of the initial parameters for the calculations, we take the following quantities: $\rho_0 = 20$ kg/m^3, $c_0 = 0,01$ kg/m^3, $m_0 = 0,44$, $v_0 = 1/360$ m/s, $\beta_1 = 0,8$ m^{-1}, $\beta_2 = 7,5$ m^{-1}, $\beta_3 = 7,2$ s^{-1}, $k_0 = 0,01$ $m^2/(MPas)$.

Figures 1 - 4 show profiles of concentrations of suspended particles c/c_0 and deposited particles ρ at different time instants. At $t = 9\,h$ (Fig. 1), advancement of the profile is observed, which corresponds to a period of almost complete absence of particle deposition. This character of the profile is consistent with the results of [2]. It can be explained that, during "charging" stage intensity of deposition is very low. With increasing current time, the process of particle deposition covers the entire filter zone, reaches its maximum capacity (Fig. 2).

The concentration profiles coincide at various values until about $t \le 9$ h and at this time, "aging" has not yet begun (Fig. 3). At large values, the deposition concentration ρ_2 increases near $x{=}0$ and leads to a decrease of c/c_0 in the corresponding points of the formation. However, with some x, the deposition rate slows down at large ρ_2 (Fig. 4). Over time, the filter is completely saturated with deposition near $x = 0$, and this process starts faster at higher values of ρ_2 (Fig. 4).

The analysis of concentrations of suspended and deposited particles at fixed points in the reservoir shows (Fig. 5) that sharp changes are observed in their dynamics at points of achievement, ρ_1, $\rho_2$and ρ_0 and. This feature is characteristic of multistage deposition kinetics.
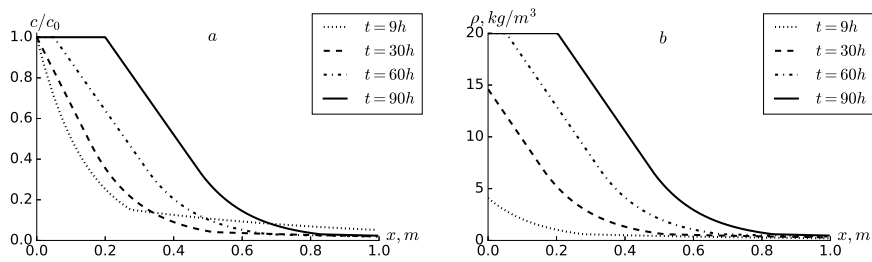
Figure 2: Profiles of $c/c_0(a)$, $\rho(b)$, at $\rho_1 = 1.5$ kg/m^3, $\rho_2 = 7$ kg/m^3.

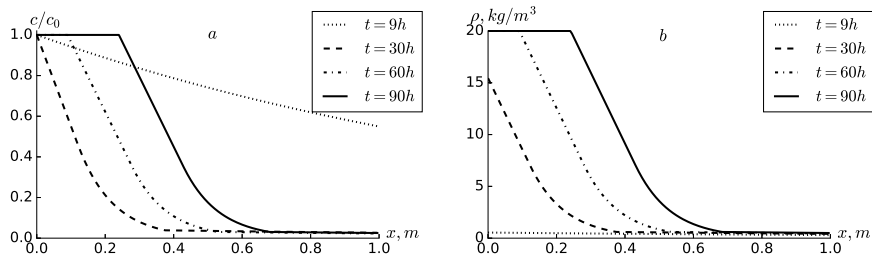Figure 3: Profiles of $c/c_0(a)$, $\rho(b)$, at $\rho_1 = 0.6$ kg/m^3, $\rho_2 = 10$ kg/m^3.
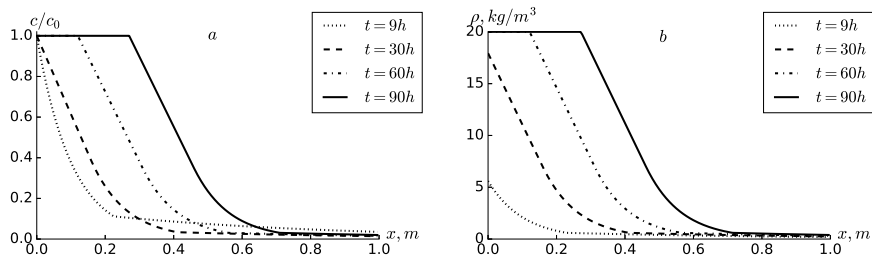

Figure 4: Profiles of $c/c_0(a)$, $\rho(b)$, at $\rho_1 = 1.5$ kg/m^3, $\rho_2 = 10$ kg/m^3.
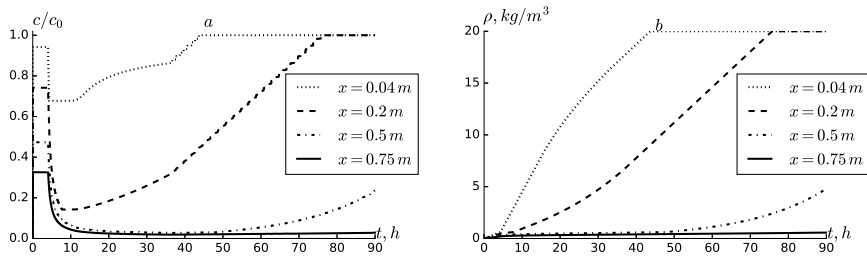
Figure 5: Dynamics of $c/c_0(a)$ and $\rho(b)$ at different fixed points.

5. Conclusions

In the paper the problem of suspension filtration in a porous medium is posed. Phenomenological model of suspension filtration in porous medium is considered as a system of partial differential equations, which consists of mass balance equation, kinetic equation, Darcy's law and Carman–Kozeny equation. A mathematical model is developed and the problem of filtration a single-component suspension in a porous medium is numerically solved taking into "charging" and "aging" phenomenon. Based on the proposed generalized model, which takes into account all stages of filtration process. The influence of module of pressure gradient have been taken into account in kinetic equations directly. It gave an opportunity to estimate how the changing in porous media characteristics affect on particle deposition and release processes. Numerical calculation of the solution is performed, distribution of concentrations of suspended and deposited particles depending on time and coordinate are presented in graphical form. With an increase in time, the suspended particles concentration tends to a constant concentration at the entrance of the porous medium. The analysis of concentrations at fixed points in the reservoir showed that sharp changes are observed in their dynamics at points of achievement, this feature is characteristic of multistage deposition kinetics.

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