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MATHEMATICAL MODELING OF THE PROCESS OF LIQUID MEDIAS MAGNETIC PURIFICATION FROM MULTICOMPONENT FERROMAGNETIC IMPURITIES

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Abstract: A mathematical model that describes the patterns of magnetic deposition and impurity accumulation in a porous filter load with taking into account the inverse effects of process characteristics (sediment concentration) on the filtration parameters was developed. We proposed an algorithm for solving the corresponding nonlinear perturbed problem for determining the concentration distribution of impurities and sediment, as well as the time of the protective action of the filter loading τ_z, variable parameter $gradP\,(x,t)$, in particular, the limit of pressure loss ΔP. The results of the calculations of the distribution of impurity concentration and mass volume of impurities by the height of the porous filtering loading for different time points, the magnitudes of the filter coefficient at different values of the loading length L, which corresponds to the time of the protective action of loading (filter cycle) are given. This model provides the possibility of automated control of the process of effective deposition of impurities in the magnetized filtering load, depending on the source data of the purified water environment.

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1. Introduction

Deep purification of wastewater and technological waters of various industries, including thermal power engineering and atomic engineering from polluting impurities is a determining factor in the reliability and durability of equipment operation. The main cause of contamination of wastewater and technological waters of thermal and nuclear engineering objects is the continuous and progressive corrosion, wearing due to the operation of technological and communication equipment. The bulk of pollutants consists of ferriferous impurities with ferromagnetic properties [1], [2], [8]. It was investigated that even at the concentration of iron-containing impurities of (0.02-0.2) mg/dm^3 on the steam generating surfaces of the boiler pipes and in the flow part of the turbines are forming ferruginous deposits [1]. These deposits increases thermal and hydraulic resistance, worsen of heat transfer, lead to over-consumption of fuel and heat energy, breakthroughs and pipe breakdowns, reduces the power of turbines, increases the time for equipment to be restored and reduces electricity generation. For example, the deposits in quantities of about 1 kg on the vanes of a high-pressure turbine cylinder of 300 MW units cause a decrease in power by 5...10 MW, which is equivalent to a underproduction of 35...70 million $kWh/year$ of electrical energy. If condensates with a high concentration of iron-containing impurities (especially in launch modes) are discharged into the reservoirs, this leads to pollution of the water basin, the environment and additional heat losses [9]. Actual problem is the deep purification of recycled waters in metallurgy, which are used for hydroshake of cinder, cooling of rolling table of rolling mill and metal sheet. The concentration of dispersed cinder reaches 60-120 mg/dm^3, which causes increased equipment wearing and periodic closure of injectors. Particles of cinder reaches with the recycling water on the metal sheet and worsen the quality of its surface, creating corrosion centers. A promising method for wastewater and technological waters purification is a magnetic purification based on the use of magnetic filters with ferromagnetic filtering nozzles. A mathematical model of this process was developed to increase the efficiency of the magnetic purification of liquid medias and improve the conditions for regeneration of the filtering nozzle.

2. Statement of the problem

This model is implemented in a magnetic filter, which includes the process of precipitation (filtering) of impurities in the ferromagnetic filtering nozzle and the process of nozzle regeneration. The filtration process is characterized by the period of the filtering cycle (τ_z) and the time of regeneration (τ_r). The period of the filtering cycle (τ_z) ranges from several hours to several hundred hours, and the regeneration time can be 3-20 *min*. In this case, depending on the shape, size and weight of the filtering nozzle using different amounts of water for regeneration. There is a need to create such conditions, and to calculate such theoretical recommendations, in which the efficiency of the processes of filtration and regeneration is maximum, and the water consumption during regeneration is minimal.

It is proposed a new design of a magnetic filter [3] which consists of a non-magnetic hull 1 (Fig. 1) filled with a ferromagnetic filtering nozzle 2, magnetizing coils 3, magnetic conductors 4, an additional switch device in the form of elastic elements 5, ferromagnetic rods 6, which fixed to plates 7. For innings and removals of the cleaned water in the hull of filter are provided a special pipes 8, 9. As a filter nozzle can be used a ferromagnetic balls, buckshot, shavings, ferrites, a mixture of granules of ferrite and balls, rods, perforated plates, etc. In the case of fine purifying of the liquid medium, the nozzles are made from a special non-corrosive alloy or covered with a layer of cadmium or nickel. The principle of the magnetic filter operation is as follows. After the activation of the magnetizing coils 3, under the action of magnetic forces, the magnetic conductors 4 are moved and tightly adjacent to the upper and lower plates 7 and together with the rods 6 and magnetized filtering nozzle 2 forms a closed magnetic circuit. Through the tubes 8 receives a polluted technological water, and the purified one from the impurities leading out through the pipes 9. For the regeneration process, the coils 3 are disconnected, the magnetic conductors move under the action of the switch device 5 (the upper goes up and the bottom does down) and they break contact with the ferromagnetic nozzle. In Fig. 1 shows the placement of magnetic conductors in regeneration mode. The water-air mixture through the nozzles 8, 9 washing the deposits impurities from the filtering nozzle into the drainage vessel. After the regeneration process complete the magnetic filter activates for the further work. The presence of additional circuit breakers, which are provided with magnetic conductors, allows in regeneration mode, when the magnetized system is disconnected, to create a gap between the contacts of the magnetic conductors and ferromagnetic nozzle. In this time reducing the residual magnetization of the nozzle and creating conditions for its efficient regeneration. In the cleaning mode, when the magnetizing system is activated, a contact of magnetic conductors with nozzle, greatly reduces the dispersion of the magnetic flow and due to this increases the level of purification of the liquid medium. The using of the proposed magnetic

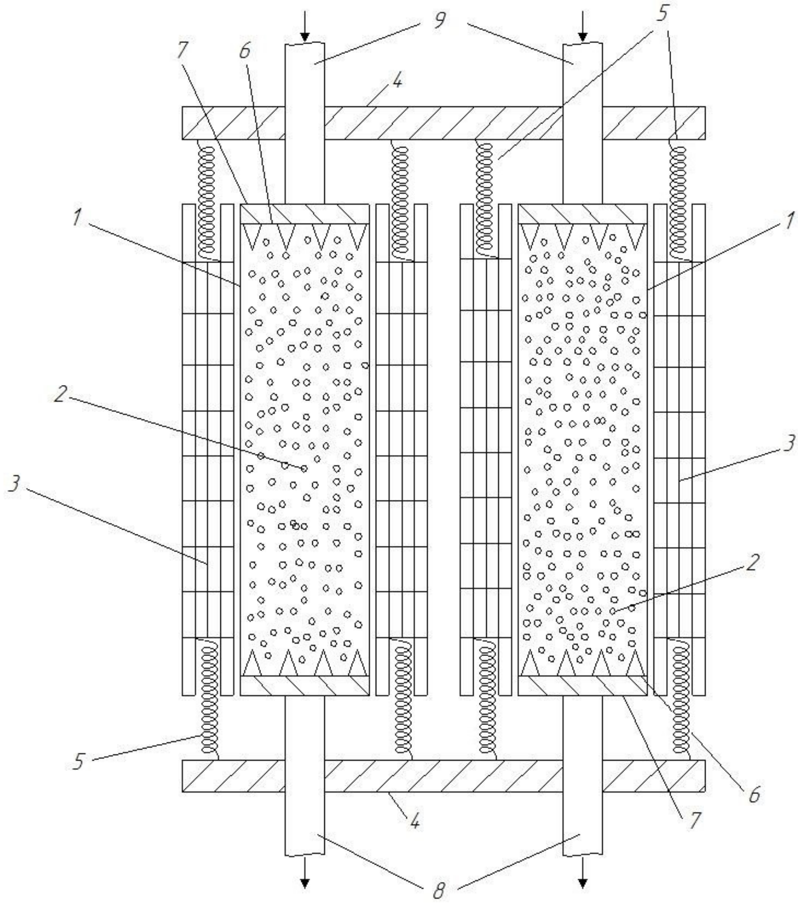


Figure 1: Magnetic filter with an additional device-breaker, made in the form of elastic elements placed between the magnetic conductors and end surfaces of solenoid

filter with an additional spring-loaded device-breaker of contacts of magnetic conductors with filtering nozzle, provides improvement of conditions of filtering nozzle regeneration, by reducing the magnitude of residual magnetization. Furthermore, this increasing the coefficient of magnetic purification by reducing the scattering of magnetic flow and increasing the level of ferromagnetic nozzle magnetization in filtering mode, [3].

In [4] - [5], a mathematical model of a magnetic filter was developed, taking into account the reverse influence of process characteristics on the characteristics of the medium. But this model was single-component and did not take into account the variable filtering velocity, which is essential for the technical implementation of magnetic filters. Let us consider the process of magnetic deposition of impurities which is realized in a magnetic filter ($0 \le x \le L$) with homogeneous granular filtering nozzle. The process is carried out according to the laws the prototype of which is the classical filtration model [6] - [7], taking into account the reciprocal effects of deposited particles on porosity, mass transfer coefficients, as well as the filtration coefficient and variable filtering velocity,

$$\begin{cases} \dfrac{\partial\left(\sigma\left(\rho\right)c_i\left(x,t\right)\right)}{\partial t}+\dfrac{\partial\rho\left(x,t\right)}{\partial t}+\dfrac{\partial\left(v\left(x,t\right)c_i\left(x,t\right)\right)}{\partial x}=0, \\ \dfrac{\partial\rho\left(x,t\right)}{\partial t}=\beta\left(\rho\right)\left(\displaystyle\sum_{i=1}^{m}k_ic_i\left(x,t\right)\right)-\varepsilon\alpha\left(\rho\right)\rho\left(x,t\right), \end{cases} \tag{1}$$

$$c_i|_{x=0}=c_i^*(t), \quad c_i|_{t=0}=0, \quad \rho|_{x=0}=0, \quad \rho|_{t=0}=0, \tag{2}$$

$$v=\ \kappa\left(\rho\right)\cdot grad\ P, \tag{3}$$

where $\alpha\left(\rho\right)$ is coefficient which characterizes mass volumes of separated impurity particles from the granules of the nozzle; $\alpha\left(\rho\right)=\alpha_0+\varepsilon\alpha_*\rho\left(x,t\right)$, v is the velocity of filtration ($v=const$, this is characterizing the closure of the process), $\sigma\left(\rho\right)$ is the porosity of the filtering nozzle (σ_0 is the initial nozzle porosity), $\sigma\left(\rho\right)=\sigma_0-\varepsilon\sigma_*\rho\left(x,t\right)$, $\kappa\left(\rho\right)$ - the coefficient of filtration, ρ_l - the limiting accumulation of impurities, $\kappa\left(\rho\right)=\left[\begin{array}{l}\kappa_0-\varepsilon\gamma\rho\left(x,t\right), \quad \rho<\rho_l \ \left(t<\tau_z\right), \\ \kappa^0, \quad \rho=\rho_l \ \left(t\ge\tau_z\right),\end{array}\right.$ $\kappa^0=\kappa_0-\varepsilon\gamma\rho_l$, α_0, α_*, σ_*, κ_0, γ, ε - hard parameters (they characterize the appropriate coefficients), $\alpha\left(\rho\right)$, $\sigma\left(x,t\right)$, $\kappa\left(x,t\right)$ is soft parameters (can be found by experimental way), P is pressure. This kind of changing of porosity and the coefficient of particles detaching is due to the fact that when the impurity particles increasing in the nozzle, the corresponding filtering parameters are changing. Because of the system is closed, the change of the filtration coefficient leads to a change of the pressure difference $\Delta P = P(L,t) - P(0,t)$ in the porous nozzle.

3. Algorithm of the solution

The solution of the system (1) under conditions (2) is sought in the form of asymptotic series (4), [6] - [7],

$$c_i(x,t) = c_{i,0}(x,t) + \sum_{j=1}^{n} \varepsilon^j c_{i,j}(x,t) + R_c(x,t,\varepsilon),$$

$$\rho(x,t) = \rho_0(x,t) + \sum_{j=1}^{n} \varepsilon^j \rho_j(x,t) + R_\rho(x,t,\varepsilon), \qquad (4)$$

where R_c, R_ρ residual members, $c_{i,j}(x,t)$, $\rho_j(x,t)$ $(j=\overline{0,\ n})$ members of the regular parts of asymptotes. Similar to [6], after substitution (1) in (1)-(2) and the use of the standard "equalization procedure" to find functions $c_{i,j}$ and ρ_j $(j=\overline{0,n})$, we have the following tasks:

$$\begin{cases} \sigma_0 \dfrac{\partial c_{i,0}}{\partial t} + v\dfrac{\partial c_{i,0}}{\partial x} + \dfrac{\partial \rho_0}{\partial t} = 0, \quad \dfrac{\partial \rho_0}{\partial t} = \beta c_{i,0}, \\ c_0|_{x=0} = c_i^*(t), \quad c_0|_{t=0} = 0, \quad \rho_0|_{x=0} = 0, \quad \rho_0|_{t=0} = 0, \end{cases}$$

$$\begin{cases} \sigma_* \rho_{i-1} \dfrac{\partial c_{i,j}}{\partial t} + v\dfrac{\partial c_{i,j}}{\partial x} + \sigma_* \dfrac{\partial \rho_{i-1}}{\partial t} c_{i,j} + \dfrac{\partial \rho_i}{\partial t} = 0, \quad \dfrac{\partial \rho_i}{\partial t} = \beta c_{i,j} - g_i, \\ c_{i,j}|_{x=0} = 0, \quad c_{i,j}|_{t=0} = 0, \quad \rho_i|_{x=0} = 0, \quad \rho_i|_{t=0} = 0, \ i=\overline{1,n}, \ j=\overline{1,m}, \end{cases}$$

As a result of their solution we have:

$$c_{i,0}(x,t) = \begin{cases} c_i^*\left(t - \dfrac{\sigma_0 x}{v}\right) \cdot e^{-\frac{\beta x}{v}}, \ t \geq \dfrac{\sigma_0 x}{v}, \\ 0, \ t < \dfrac{\sigma_0 x}{v}, \end{cases} \quad \rho_0(x,t) = \beta \int_0^t c_{i,0}(x,\tilde{t})d\tilde{t},$$

$$c_{i,j}(x,t) = \begin{cases} \dfrac{e^{-\lambda_i(x,t)}}{v} \cdot \displaystyle\int_0^x g_i(\tilde{x}, f_i(\tilde{x}) + x - f_i(x)) \cdot e^{\lambda_i(\tilde{x},t)} d\tilde{x}, \ \ t \geq f_i(x), \\ 0, \ \ t < f_i(x), \end{cases}$$

$$\rho_i(x,t) = \int_0^t \left(\beta c_{i,j}(x,\tilde{t}) - g_i\left(x,\tilde{t}\right)\right) d\tilde{t},$$

where

$$g_i(x,t) = \sum_{j=1}^{i} \rho_{j-1} \left(\alpha_0 + I(i,j) \sum_{j=2}^{i} (\alpha_* \rho_{i-2}) \right),$$

$$\lambda_i(x,t) = \frac{1}{v} \int_0^x \psi_i(\tilde{x}, f_i(\tilde{x}) + x - f_i(x))\, d\tilde{x},$$

$\psi_i(x,t) = \sigma_* \frac{\partial \rho_{i-1}(x,t)}{\partial t} + \beta$, $(a,b) = \begin{cases} 1, if a \geq b \\ 0, if a < b \end{cases}$. The approximate values of functions $f_i(x)$ are found by interpolating the massif (x_j, t_j), $j = \overline{1,n}$, where $x_j = \Delta x \cdot j$, $t_{j+1} = t_j + \frac{\Delta x}{v} \sigma_* \rho_{i-1}(x_j, t_j)$. The evaluation of residual members are in the same way as [6]. As expected [6], it is enough for calculations to take 2 members from each asymptotic series (4) to obtain approximation with the accuracy of up to 6 meaningful digits in the interval of the calculated time of the filtering cycle. According to [1] - [3] the coefficients of trapped impurity particles and detached particles of sediment are calculated by the following equation: $\beta = \frac{\beta_0 H^{0.75}}{vd^2}$ [1], where β_0 - free parameter, H - magnetic field tension, v - velocity of filtering, d - diameter of the granular filtering nozzle.

4. Results of numerical calculations

Here are the results of calculations by the equations (4) at $c_1^*(t) = 2mg/dm^3$, $c_2^*(t) = 1mg/dm^3$, $v = 200m/h$, $L = 1m$, $\beta_0 = 0.7 \cdot 10^{-9} s^{-1}$, $\alpha_0 = 0.35 s^{-1}$, $H = 60kA/m$, $d = 2.4mm$, $\alpha_* = 1$ (Fig. 2).

It is necessary to mark that in the process of calculation we took the condition of $v = const$, although the coefficient of filtration (as well as porosity) falls due to adhesion of solid particles to the walls (nozzle). This makes it possible to find the gradient of pressure in each cross-cut of the filter (at each point x, $0 \leq x \leq L$). In particular, using the formula it is possible to find the time of receipt greater than the critical value of the gradient of pressure and to take the corresponding "automation decision". Changing the value of *grad P* over time is shown in Fig. 3.

As you can see (Fig. 4), in the case of $c_*(t) = c_* = const$ filter efficiency almost does not change until the time of τ_z after which begins to decline, which is confirmed by the known fact that the filter efficiency changes over time.

Here are the results of calculations by the equations (4) at $n = 2$, $c_1^*(t) = 2mg/dm^3$, $c_2^*(t) = 1mg/dm^3$, $\beta = 60s^{-1}$, $v = 250m/h$, $L = 1m$, $q_1 = q_2 = 1$, $\sigma_0 = 0.5$, $\sigma_* = 1$, $\varepsilon = 0.001$.

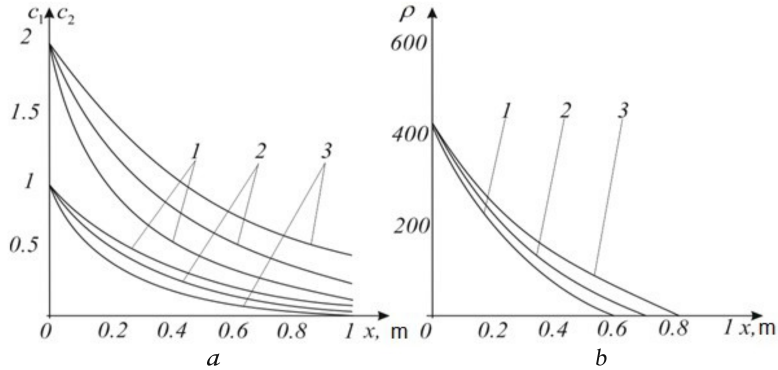


Figure 2: The distribution of the impurities concentration in the liquid (a) and the precipitate (b) along the filter at the time of $t_1 = 10$ hours, $t_2 = 15$ hours, $t_3 = 25$ hours

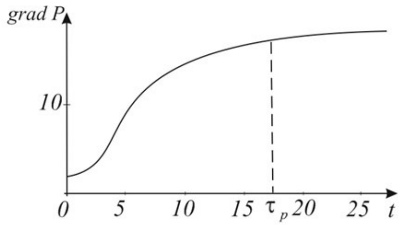


Figure 3: Changing the value of *grad P* in the output of the filter over time

Fig. 5 shows the distribution of the impurities concentration at the filter output at the moment of time t according to equations (4) at $n = 3$ (a) and $n = 1$ (b) (acceptable concentration value $c_1 = c_1 = 0.5mg/dm^3$, $c_2 = c_2 = 0.15mg/dm^3$, [1] - [3]). As can be seen from the Fig. 5, the protective action time of the n-layer filter (for $n = 1 - t_p = 78$ hours and $t_p = 82$ hours) is significantly longer than the protective action time for a single-layer filter (for $n = 3 - t_p = 68$ hours and $t_p = 78$ hours).

5. Conclusions

It was created the mathematical model of the liquids purification process from multicomponent impurities by a n-layers magnetic filter, which takes into account the reverse influence of determining factors of the process (concentra-

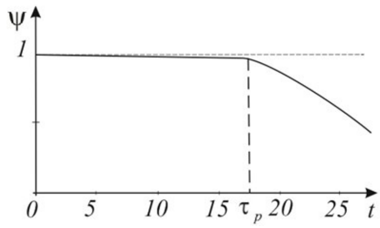
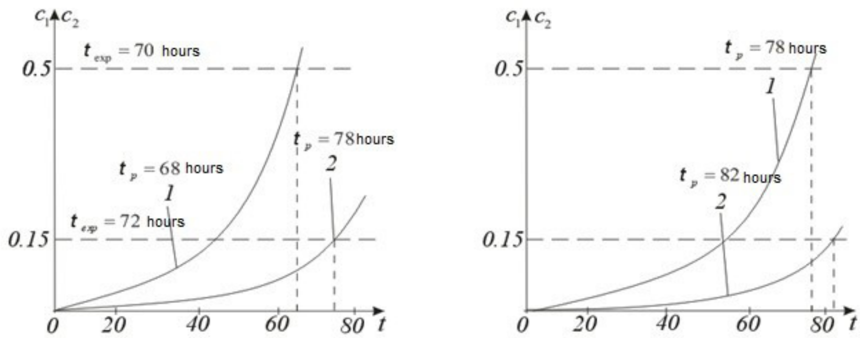


Figure 4: Changing the effectiveness of the filter $\psi = \left(\sum_{i=1}^{m} c_i^* - \sum_{i=1}^{m} c_i\right) / \sum_{i=1}^{m} c_i^*$



a *b*

Figure 5: Graphs of the distribution of the impurities concentration at the filter output at the moment of time t according to the equations (4) at $n = 3$ (a) and $n = 1$ (b)

tion of liquid and precipitate impurities) on the characteristics of the medium (porosity coefficient) includes the possibility of determining an unknown small mass-exchange coefficient. The proposed algorithm for solving the corresponding problem, which suggests the ability to determine the protective action time τ of the filter (based on the set values of permissible concentrations at the filter output). The results of numerical calculations are given. In the framework of this model is foreseen the possibility of automated control of impurities effective deposition process in a magnetized filtering nozzle, depending on the output data of the contaminated aqueous medium.

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