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ON TOPOLOGICAL PROPERTIES OF PLANE GRAPHS BY USING LINE OPERATOR ON THEIR SUBDIVISIONS

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Abstract: In this paper, we will compute some topological indices such as Zagreb indices $M_1(G)$, $M_2(G)$, $M_3(G)$, Zagreb coindices $M_1(\overline{G})$, $\overline{M_1}(G)$, $M_2(\overline{G})$, $\overline{M_2}(G))$, $\overline{M_2}(\overline{G})$, hyper-Zagreb index $HM(G)$, atom-bond connectivity index $ABC(G)$, sum connectivity index $\chi(G)$, augumented Zagreb index AZI and geometric-arithmetic connectivity index $GA(G)$ of line graph of subdivision of some plane graphs.

AMS Subject Classification: 05C07, 05C35, 05C40
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1. Introduction

Topological indices are real numbers that are presented as graph parameters (e.g. the degree of vertices, distances, etc.) introduced during studies conducted on the molecular graphs in chemistry and can describe some physical and chemical properties of molecules. Topological indices are numerical parameters of a graph which are invariant under graph isomorphisms.

Let G be a connected graph with vertex set $V(G)$ and edge set $E(G) \subseteq V(G) \times V(G)$. Let $p = |V(G)|$, the order of G and $q = |E(G)|$, the size of G.

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The complement of a graph G, denoted by $\overline{G}$, is a simple graph having same set of vertices $V(G)$ in which any two vertices that are connected by an edge, if and only if they are not adjacent in G. Consequently, $E(G) \cup E(\overline{G}) = E(K_p)$ and we obtain a complete graph K_p of order p, and $|E(\overline{G})| = \frac{p(p-1)}{2} - q$. The degree d_v of any vertex v is defined as the number of vertices joining to that vertex v and the degree d_e of an edge $e \in E(G)$ is defined as the number of its adjacent vertices in $V(L(G))$, where $L(G)$ is the line graph whose vertices are the edges of G, with $uv \in E(L(G)$ when u and v have a common end point in G. In structural chemistry, line graph of a graph G is very useful. The first topological indices on the basis of line graph was introduced by Bertz in 1981 (see [4]). For more details on line graph (see [8, 16, 17, 18, 20, 22, 25]). The subdivision $S(G)$ of a graph G can be obtained by replacing each edge of G by a path of length 2. Further details on the topological indices of $L(S(G))$ can be viewed in the articles (see [3, 9, 10, 21, 23, 24]). From this motivational work, we calculate the topological indices of line graph of subdivision of Jahangir graph, Banana Tree graph and Firecracker graph.

2. Some Degree-Based Topological Indices

Analyzing the structure-dependency of total π-electron energy, first Zagreb index $M_1(G)$ and second Zagreb index $M_2(G)$ were introduced (see [17]). They are defined as follows:

$$M_1(G) = \sum_{u \in V(G)} d_u^2 \tag{1}$$

$$M_2(G) = \sum_{uv \in E(G)} d_u d_v. \tag{2}$$

For more properties of the Zagreb indices (see [5, 13, 15, 19, 30]). In 1977, Alberton introduced the irregularity of graphs (see [1]). To confirm with the terminology of chemical graph theory Fath-Tabar (see [7]) called Alberton's irregularity the third Zagreb index and is defined as:

$$M_3(G) = \sum_{uv \in E(G)} |d_u - d_v|. \tag{3}$$

In 2013, Shirdel, RezaPour and Sayadi (see [27]) introduced the hyper-Zagreb index. It is defined as follows:

$$HM(G) = \sum_{uv \in E(G)} (d_u + d_v)^2. \tag{4}$$

Randic introduced the so-called Randic index in 1975 (see [26]). It is defined as follows:

$$R(G) = \sum_{uv \in E(G)} \frac{1}{\sqrt{d_u d_v}}. \tag{5}$$

Estrada et al. (see [6]) introduced a new topological index and named it atom-bond connectivity index, defined as follows:

$$ABC(G) = \sum_{uv \in E(G)} \sqrt{\frac{d_u + d_v - 2}{d_u d_v}}. \tag{6}$$

The ABC index is excellently correlated with the thermodynamic properties of alkanes.

Zhou and Trianjstic introduced the general sum-connectivity index $\chi(G)$ (see [29]). It is defined as follows:

$$\chi(G) = \sum_{uv \in E(G)} \frac{1}{\sqrt{d_u + d_v}}. \tag{7}$$

Vukicevic and Furtula defined the first geometric arithmetic index GA (see [28]), defined as follows:

$$GA(G) = \sum_{uv \in E(G)} \frac{2\sqrt{d_u d_v}}{d_u + d_v}. \tag{8}$$

Furtula et al. put forward the following modified version of the ABC index and named it as Augumented Zagreb index AZI (see [11]). It is defined as follows:

$$AZI(G) = \sum_{uv \in E(G)} (\frac{d_u d_v}{d_u + d_v - 2})^3. \tag{9}$$

The first Zagreb coindex is defined as (see [14]):

$$\overline{M_1} = \overline{M_1}(G) = \sum_{uv \notin E(G)} \left[d_u + d_v\right].$$

The second Zagreb coindex is defined as:

$$\overline{M_2} = \overline{M_2}(G) \quad = \quad \sum_{uv \notin E(G)} d_u d_v.$$

Theorem 1. (see [12]) *Let G be a graph of order p and size q. Then,*

$$M_1(\overline{G}) = M_1(G) + p(p-1)^2 - 4q(p-1); \quad (10)$$

$$\overline{M_1}(G) = 2q(p-1) - M_1(G); \quad (11)$$

$$\overline{M_1}(\overline{G}) = 2q(p-1) - M_1(G). \quad (12)$$

Theorem 2. (see [14]) *Let G be a graph of order p and size q. Then,*

$$M_2(\overline{G}) = \frac{1}{2}(p-1)^2(p^2-p-3q) + 2q^2 + \frac{2p-3}{2}M_1(G) - M_2(G); \quad (13)$$

$$\overline{M_2}(G) = 2q^2 - \frac{1}{2}M_1(G) - M_2(G); \quad (14)$$

$$\overline{M_2}(\overline{G}) = q(p-1)^2 - (p-1)M_1(G) + M_2(G). \quad (15)$$

3. Topological indices of line graph of subdivision of the Jahangir graph

Jahangir graph $J_{n,m}$ consisting of a cycle C_{nm} with one additional vertex which is adjacent to m vertices of C_{nm} at distance n to each other on C_{nm} (see [2]). The subdivision of Jahangir graph has additional vertex between each pair of vertices. The subdivision of $J_{n,m}$ has order $m(2n+1)$ and size $2m(n+1)$ as shown in Fig. 1(b).

Theorem 3. *Let $L(S(J_{n,m}))$ be the line graph of subdivision of the Jahangir graph $J_{n,m}$. Then,*

1. $M_1[L(S(J_{n,m}))] = m^2(m-1) + 4m(2n-3) + 28m + m(m+3)$;
2. $M_2[L(S(J_{n,m}))] = \frac{1}{2}m^3(m-1) + 4m(2n-3) + 39m + 3m^2$;
3. $M_3[L(S(J_{n,m}))] = m^2 - m$;
4. $HM[L(S(J_{n,m}))] = 16m(2n-3) + 158m + m(3+m)^2 + 2m(m-1)m^2$;
5. $ABC[L(S(J_{n,m}))] = \frac{1}{2}m(2n-3)\sqrt{2} + m\sqrt{2} + 2m + \frac{1}{\sqrt{3}}m\sqrt{\frac{m+1}{m}} + \frac{1}{2}(m-1)\sqrt{2m-2}$;
6. $GA[L(S(J_{n,m}))] = 2mn + \frac{12m\sqrt{(m+6)}}{15+m} + \frac{2m\sqrt{(m+6)m}}{2m+6} + \frac{1}{2}m(m-1)$;

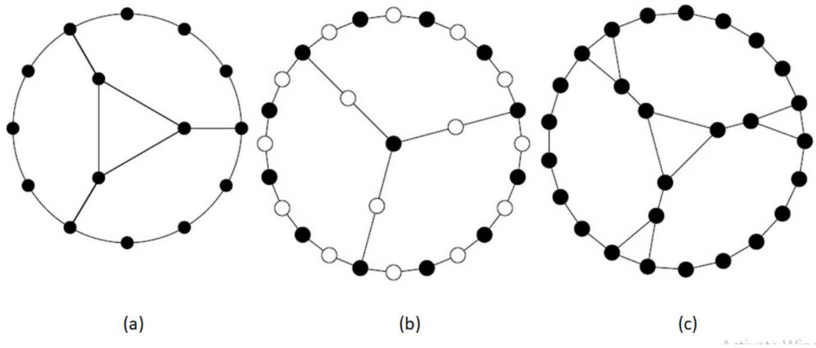

Figure 1: The Jahangir graph, subdivision and line graph of subdivision of Jahangir graph $J_{4,3}$.

(d_u, d_v) where $uv \in E(G)$	$(2,2)$	$(2,3)$	$(3,3)$	$(3,m)$	(m,m)
Number of edges	$m(2n-3)$	$2m$	$3m$	m	$\frac{m(m-1)}{2}$

Table 1: The edge partition of $L(S(J_{n,m}))$.

7. $\chi[L(S(J_{n,m}))] = \frac{1}{2}m(2n-3) + \frac{2}{5}m\sqrt{5} + \frac{1}{2}m\sqrt{6} + \frac{m}{\sqrt{3+m}} + \frac{\sqrt{2}}{4}\sqrt{m}(m-1)$;

8. $AZI[L(S(J_{n,m}))] = 8m(2n-3) + \frac{3211}{64}m + \frac{27m^4}{(1+m)^3} + \frac{1}{2}\frac{m^7(m-1)}{(2m-2)^3}$.

Proof. The graph $L(S(J_{n,m}))$ for $n = 4$ and $m = 3$ is shown in Fig. 1(c). The order of $L(S(J_{n,m}))$ is $2m(n+1)+1$ out of which $2m(n-1)$ vertices are of degree 2, $3m$ vertices are of degree 3 and m vertices are of degree m. The size of $L(S(J_{n,m})))|$ is $\frac{m^2+4mn+5m}{2}$. The edge partition of $L(S(J_{n,m}))$ into the edges of type (d_u, d_v) where uv is an edge of $L(S(J_{n,m}))$ is shown in Table 1.

We obtain the required results by using formulas $(1)-(9)$. □

Theorem 4. *Let $L(S(J_{n,m}))$ be the line graph of subdivision of the Jahangir graph $J_{n,m}$. Then,*

1. $M_1\overline{[L(S(J_{n,m}))]} = 8m^3n^3 + 24m^3n^2 + 203m^3n - 24m^2n^2 + 5m^3 - 53m^2n -$

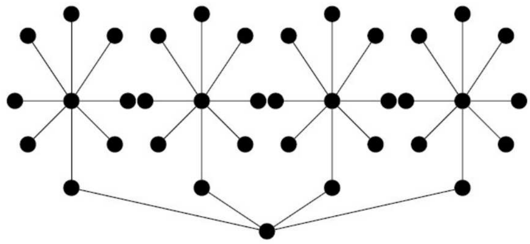

Figure 2: The Banana Tree graph $B_{7,4}$.

$269m^2 + 18mn + 31m$;

2. $\overline{M_1}[L(S(J_{n,m}))] = 4m^3n + 16m^2n^2 + 5m^3 + 36m^2n + 18m^2 - 16mn + 33m$;
3. $M_2\overline{[L(S(J_{n,m}))]} = m(n+1)(2m(n+1)-1)^3 - 3(\frac{1}{2}m^2 + 2mn + \frac{5}{2}m)(2m(n+1)-1)^2 - \frac{5}{2}m^2 + 2mn - \frac{73}{2}m + 8m^2(n+1)(2n-3) + 28m + m(m+3) + m^2(m-1) - \frac{3}{2} - 4m(2n-3) - \frac{1}{2}m^3(m-1)$;
4. $\overline{M_2}[L(S(J_{n,m}))] = \frac{1}{2}(m^2 + 4mn + 5m)^2 - 6m(2n-3) - 53m - \frac{1}{2}m(m+3) - \frac{1}{2}m^2(m-1) - 3m^2 - \frac{1}{2}m^3(m-1)$;
5. $\overline{M_2}(\overline{[L(S(J_{n,m}))]}) = \frac{1}{2}(2m^2(n+1)-1)^2 + 4mn(2m(n+1)-1) + 5m(2m(n+1)-1)^2 - 8m^2(n+1)(2n-3) + 28m + m(3+m) + m^2(m-1) + 1 + 4m(2n-3) + 39m + 3m^2 + \frac{1}{2}m^3(m-1)$.

Proof. It is easily seen from Theorems 1, 2 and Table 1. □

4. Topological indices of line graph of subdivision of Banana Tree graph

The Banana tree graph $B_{n,m}$ is the graph obtained by connecting one leaf of each of n copies of an $m-$star graph with a single root vertex that is distinct from all the stars. The subdivision of $B_{n,m}$ consist of an additional vertex between each pair of vertices, has order $2mn+1$ and size $2mn$ as shown in Fig. 3.

Theorem 5. *Let $L(S(B_{n,m}))$ be the line graph of subdivision of Banana tree graph $B_{n,m}$. Then*

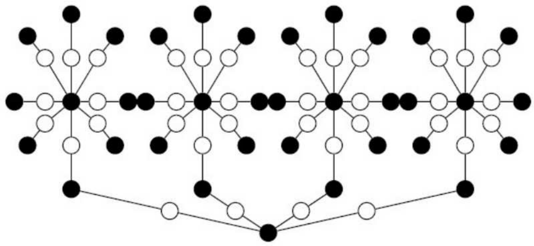

Figure 3: The subdivision of Banana Tree graph $B_{7,4}$.

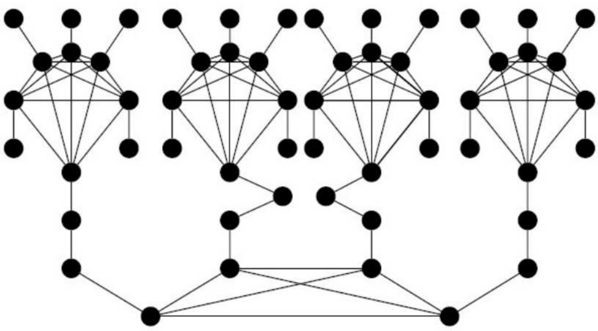

Figure 4: The line graph of subdivision of Banana Tree graph $B_{7,4}$.

1. $M_1[L(S(B_{n,m}))] = m(n-2)n + m(n+1) + m(m+2) + 4m + m^2(m-1) + m(n-1)^2(n-2)$;
2. $M_2[L(S(B_{n,m}))] = m(n-1)(n-2) + 2m(n-1) + 2m^2 + 4m + \frac{1}{2}m^3(m-1) + \frac{1}{2}m(n-1)^3(n-2)$;
3. $M_3[L(S(B_{n,m}))] = -m(n-2)^2 + m(3-n) + m(2-m)$;
4. $HM[L(S(B_{n,m}))] = mn^2(n-2) + m(n+1)^2 + m(m+2)^2 + 16m + 2m^3(m-1) + \frac{1}{2}m(n-1)(n-2)(2n-2)^2$;
5. $ABC[L(S(B_{n,m}))] = m(n-2)\sqrt{\frac{n-2}{n-1}} + \frac{3}{2}m\sqrt{2} + \frac{1}{2}(m-1)\sqrt{2m-2} + \frac{1}{2}m(n-2)\sqrt{2n-4}$;
6. $R[L(S(B_{n,m}))] = \frac{m(n-2)}{\sqrt{n-1}} + \frac{m}{\sqrt{2n-2}} + \frac{1}{2}\sqrt{2m} + \frac{1}{2}m + \frac{1}{2}(m-1) + \frac{1}{2}m(n-2)$;
7. $GA[L(S(B_{n,m}))] = 2m(n-2)\sqrt{\frac{n-1}{n}} + 2m\sqrt{\frac{2n-2}{n+1}} + 2m^{\frac{3}{2}}\sqrt{\frac{2}{m+2}} + m + \frac{1}{2}m(m-1) + \frac{1}{2}m(n-1)(n-2)$;
8. $\chi[L(S(B_{n,m}))] = \frac{m(n-2)}{\sqrt{n}} + \frac{m}{\sqrt{n+1}} + \frac{m}{\sqrt{m+2}} + \frac{1}{2}m + \frac{1}{4}m(m-1)\sqrt{\frac{2}{m}} + \frac{1}{2}\frac{m(n-1)(n-2)}{\sqrt{2n-2}}$;
9. $AZI[L(S(B_{n,m}))] = \frac{m(n-1)^3(n-2)}{(n-2)^3} + 24m + \frac{1}{2}\frac{m(m-1)m^6}{(2m-2)^3} + \frac{1}{2}\frac{m(n-1)^7(n-2)}{(2n-4)^3}$.

Proof. The graph $L(S(B_{n,m}))$ for $n = 7$ and $m = 4$ is shown in Fig. 4. The order of $L(S(B_{n,m}))$ is $2mn$ out of which $m(n-2)$ vertices are of degree 1, $m(n-1)$ vertices are of degree $n-1$, m vertices are of degree m and $2m$ vertices are of degree 2. The size of $L(S(B_{n,m}))$ is $\frac{m^2+3m+mn^2-mn}{2}$. The edge partition of $L(S(B_{n,m}))$ into the edges of type (d_u, d_v) where uv is an edge of $L(S(B_{n,m}))$ is shown in Table 2.

We obtain the required results by using formulas (1) – (9). □

Theorem 6. *Let $L(S(B_{n,m}))$ be the line graph of subdivision of Banana tree graph $B_{n,m}$. Then,*

1. $M_1(\overline{[L(S(B_{n,m}))]}) = mn(n-2) + m(n+1) + m(m+2) + 4m + m(m-1)^2 + m(n-1)(n-2)^2 + 2mn(2mn-1)^2 - 4mn(m^2+3m) + 2$;

(d_u, d_v)	$(1, n-1)$	$(2, n-1)$	$(2, m)$	$(2, 2)$	(m, m)	$(n-1, n-1)$
Number of edges	$m(n-2)$	m	m	m	$\frac{m(m-1)}{2}$	$\frac{m(n-1)(n-2)}{2}$

Table 2: The edge partition of $L(S(B_{n,m}))$.

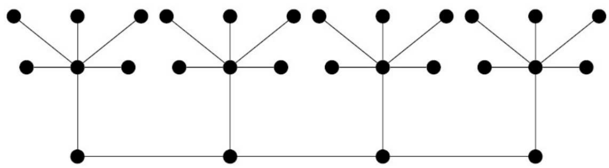

Figure 5: The Firecracker graph $F_{7,4}$.

2. $\overline{M_1}[L(S(B_{n,m}))] = 2mn(m^2 + 3m) - 1 - mn(n-2) - m(n+1) - m(m+2) - 4m - m(m-1)^2 - m(n-1)(n-2)^2$;

3. $\overline{M_2}[L(S(B_{n,m}))] = \frac{1}{2}(m^2 + mn^2 - mn + 3m)^2 - \frac{1}{2}mn(n-2) - \frac{1}{2}m(n+1) - \frac{1}{2}m(m+2) - 6m - \frac{1}{2}m(m-1)^2 - \frac{1}{2}m(n-1)(n-2)^2 - 2m^2n^2(n-2) + m(n+1) + m(m+2) + 4m + m(m-1)^2 + m(n-1)(n-2)^2) + \frac{3}{2} - m(n-1)(n-2) - 2m(n-1) - 2m^2 - \frac{1}{2}m(m-1)^3 - \frac{1}{2}m(n-1)(n-2)^3$;

4. $\overline{M_2}(\overline{[L(S(B_{n,m}))]}) = \frac{1}{2}m(2mn-1)^2 + mn(2mn-1)^2 - mn(2mn-1) + 3m(2mn-1)^2 - 2mn(mn(n-2)) + m(n+1) + m(m+2) + 40m + m(m-1)^2 + m(n-1)(n-2)^2 - 65 + 4n + 3(m-2)(n-1) + \frac{1}{2}m(n-1)(n-2)^3 + 1 + m(n-1)(n-2) + 2m(n-1) + 2m^2 + 4m + \frac{1}{2}m(m-1)^3 + \frac{1}{2}m(n-1)(n-2)^3$.

Proof. We obtain the required results from Theorems 1, 2 and Table 2. □

5. Topological indices of line graph of subdivision of Firecracker graph

The Firecracker graph $F_{n,m}$ is the graph obtained by the concatenation of $nm-$stars by linking one leaf from each. The subdivision of $F_{n,m}$ has order $(2mn-1)$ and size $2(mn-1)$ as shown in the Fig. 6.

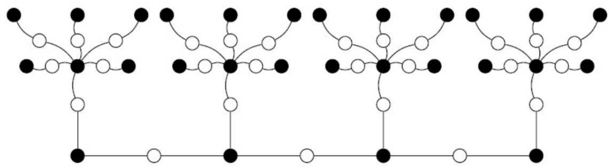

Figure 6: The subdivision of Firecracker graph $F_{7,4}$.

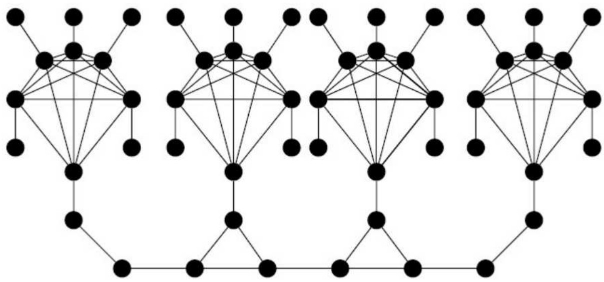

Figure 7: The line graph of subdivision of Firecracker graph $F_{7,4}$.

Theorem 7. *Let $[L(S(F_{n,m}))]$ be the subdivision of line graph of the Firecracker graph $F_{n,m}$. Then*

1. $M_1[L(S(F_{n,m}))] = 24m + 2n + (m-2)(n+2) + m(n-1)^2(n-2) - 34$;
2. $M_2[L(S(F_{n,m}))] = 36m + 4n + 3(m-2)(n-1) + m(n-1)^2(n-2) - 65$;
3. $M_3[L(S(F_{n,m}))] = 4 - 2n - (m-2)(n-4)$;
4. $HM[L(S(F_{n,m}))] = 144m + 2(n+1)^2 + (m-2)(n+2)^2 + 2m(n-1)^3(n-2) - 242$;
5. $ABC[L(S(F_{n,m}))] = 3\sqrt{2} + \frac{8}{3}m - 6 + \frac{1}{3}(m-2)\sqrt{\frac{3n}{n-1}} + \frac{1}{2}m(n-2)\sqrt{2n-4}$;
6. $R[L(S(F_{n,m}))] = \frac{1}{3}\sqrt{6} + \frac{4}{3}m + \frac{2}{\sqrt{2n-2}} + \frac{m-2}{\sqrt{3n-3}} + \frac{1}{2}\frac{m(n-1)(n-2)}{\sqrt{(n-1)^2-2}}$;
7. $GA[L(S(F_{n,m}))] = -7 + \frac{4\sqrt{6}}{5} + 4m + \frac{\sqrt{2n-2}(1+n)}{+}\frac{2(m-2)\sqrt{3n-3)}}{(2+n)} + \frac{m(n-1)^2(n-2)}{(2n-2)}$;
8. $\chi[L(S(F_{n,m}))] = 1 + \frac{2}{\sqrt{5}} + \frac{1}{\sqrt{6}}(4m-9) + \frac{2}{\sqrt{1+n}} + \frac{(m-2)}{\sqrt{2+n}} + \frac{1}{2}\frac{m(n-1)(n-2)}{\sqrt{2n-2}}$;

(d_u, d_v)	$(2,2)$	$(2,3)$	$(3,3)$	$(2,n-1)$	$(3,n-1)$	$(n-1,n-1)$
Number of edges	2	2	$4m-9$	2	$m-2$	$\frac{m(n-1)(n-2)}{2}$

Table 3: The edge partition of $L(S(F_{n,m}))$.

9. $AZI[L(S(F_{n,m}))] = \frac{-3489}{64} + \frac{729}{16}m + \frac{27(m-2)(n-1)^3}{n^3} + \frac{1}{2}\frac{m(n-1)^7(n-2)}{(2n-4)}^3$.

Proof. The graph $L(S(F_{n,m}))$ for $m = 4$ and $n = 7$ is shown in Fig. 7. The order of $L(S(F_{n,m}))$ is $2(mn-1)$ out of which $m(n-2)$ vertices are of degree 1, 4 vertices are of degree 2, $m(n-1)$ vertices are of degree $n-1$, $3(m-2)$ vertices are of degree 3. The size of $L(S(F_{n,m}))$ is $\frac{mn^2-mn+8m-10}{2}$. The edge partition of $L(S(F_{n,m}))$ into edges of the type (d_u, d_v) where uv is an edge of $L(S(F_{n,m}))$ is shown in Table 3.

We obtain the required results by using formulas $(1) - 9)$. □

Theorem 8. *Let $L(S(F_{n,m}))$ be the line graph of subdivision of the Firecracker graph $F_{n,m}$. Then*

1. $M_1\overline{[L(S(F_{n,m}))]} = 24m + 2n + (m-2)(n+2) + m(n-1)^2(n-2) + 2(mn(2mn-3)-1)^2 - 2mn(2mn-3)^2 + 2mn(2mn-3) - 16m(2mn-3) - 14$;
2. $\overline{M_1}[L(S(F_{n,m}))] = \overline{M_1}(\overline{[L(S(F_{n,m}))]}) = 2mn(2mn-3)^2 - 2mn(2mn-3) + 16m(2mn-3) + 14 + 24m + 2n + (m-2)(n+2) + m(n-1)^2(n-2)$;
3. $M_2\overline{[L(S(F_{n,m}))]} = (mn(2mn-3)-1)^3 - 3(\frac{1}{2}mn^2 - \frac{1}{2}mn + 4m - 5)(2mn-3)^2 + \frac{1}{2}mn^2 - \frac{1}{2}mn + 40m + \frac{113}{2} + 2mn(-34 + 24m + 2n) + (m-2)(n+2) + (m(n-1)^2(n-2)) + 4n + 3(m-2)(n-1) + \frac{1}{2}m(n-1)^3(n-2)$;
4. $\overline{M_2}[L(S(F_{n,m}))] = \frac{1}{2}mn^2 - \frac{1}{2}mn + 28m + 77 + 3n - \frac{1}{2}(m-2)(n+2) - \frac{1}{2}m(n-1)^2(n-2) + 3(m-2)(n-1) + \frac{1}{2}m(n-1)^3(n-2)$;
5. $\overline{M_2}\overline{[L(S(F_{n,m}))]} = \frac{1}{2}mn(2mn-3)^2 - mn(2mn-3) + 8m(2mn-3) - 10)^2 - 2mn(-34 + 24m + 2n + (m-2)(n+2) + m(n-1)(n-2)^2 - 62 + 36m + 4n + 3(m-2)(n-1) + \frac{1}{2}m(n-1)(n-2)^3$.

Proof. We obtain the required results from Theorems 1, 2 and Table 3. □

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