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COMPLETE INTERSECTION CALABI-YAU TEN-FOLDS

Vladislav Dumachev

Department of Mathematics
Voronezh Institute of the Ministry
of Internal Affairs of Russia
Voronezh – 394065, RUSSIA

Abstract: In this paper a complete intersection Calabi-Yau 10-folds are considered. Their Hodge diamond, Todd classes and Chern characters for sheaves of differential k-forms are calculated.

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1. Introduction

All definitions in this article are conventional, as in [1]. We work in projective space $\mathbb{P}^n$ over an algebraically closed field of arbitrary characteristic. Let x be divisor degree of hypersurface $X \in \mathbb{P}^n$. We denote $X_m = \bigcap_{i=1}^{k} S_i^{s_i} \in \mathbb{P}^n$ as m-fold, which is a complete intersection of k hypersurfaces S_i degree s_i. Then X is a Calabi-Yau if $x = \sum s_i = n+1$. For a sheaf of differential forms $\Omega_X^i = \Lambda^i \Omega_X$ we introduce Hodge numbers $h^{ij} = \dim \mathrm{H}^{\mathrm{i}}(\Omega_{\mathrm{X}}^{\mathrm{j}})$, which not only are symmetrical: $h^{ij} = h^{ji}$, but Serre symmetrical also: $h^{ij} = h^{n-i,n-j}$. If we have Hodge numbers then Betti numbers may be calculated as

$$b_k = \sum_{i+j=k} h^{ij}.$$

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We can also define the Euler characteristic of X as the alternating sum of the Betti numbers:

$$\chi(X) = \sum_k (-)^k b_k.$$

For clarity, we rotate the matrix h^{ij} on 45° and call it as the Hodge diamond. So, for $n = \dim X_n$, we will have:

$$\begin{array}{ccccccccc} & & & h^{0,0} & & & & & b_0 = h^{0,0} \\ & & h^{1,0} & & h^{0,1} & & & & b_1 = h^{1,0} + h^{0,1} \\ & ... & & ... & & ... & & & ... \\ h^{n,0} & & h^{(n-1),1} & ... & h^{1,(n-1)} & & h^{0,n} & & b_n = \sum_k h^{(n-k),k} \\ & ... & & ... & & ... & & & ... \\ & & h^{n,(n-1)} & & h^{(n-1),3} & & & & ... \\ & & & h^{n,n} & & & & & b_{2n} = h^{n,n} \end{array} .$$

If $X = \bigcap_{k=1}^{t} S_k$ is complete intersection of t-hypersurfaces of degree s_k, then $N_X = \bigoplus_{k=1}^{t} \mathcal{O}_X(-s_k)$ and the sheaf $\mathcal{O}_X$ can be determined with used the following recurrence relations:

$$\begin{array}{c} 0 \to \mathcal{O}_{S_1 \cap S_2 \cap ... \cap S_{k-1}}(-s_k) \to \mathcal{O}_{S_1 \cap S_2 \cap ... \cap S_{k-1}} \to \mathcal{O}_{S_1 \cap S_2 \cap ... \cap S_k} \to 0 \\ 0 \to \mathcal{O}_{S_1 \cap S_2 \cap ... \cap S_{k-2}}(-s_{k-1}) \to \mathcal{O}_{S_1 \cap S_2 \cap ... \cap S_{k-2}} \to \mathcal{O}_{S_1 \cap S_2 \cap ... \cap S_{k-1}} \to 0 \\ ... \\ 0 \to \mathcal{O}_{S_1}(-s_2) \to \mathcal{O}_{S_1} \to \mathcal{O}_{S_1 \cap S_2} \to 0 \\ 0 \to \mathcal{O}_{\mathbb{P}^n}(-s_1) \to \mathcal{O}_{\mathbb{P}^n} \to \mathcal{O}_{S_1} \to 0 \end{array} \quad c.$$

Now, for calculations of $H^i(\Omega_X^k)$ we can get the power series

$$0 \to Sym^k N_X \to Sym^{k-1} N_X \otimes \Omega_{\mathbb{P}^n|X} \to ... \to \Omega^k_{\mathbb{P}^n|X} \to \Omega^k_X \to 0.$$

2. Todd and Chern classes

As shown in [2] for any rational complete intersection Calabi-Yau X all non-zero entries of the Hodge diamond are always lying on its equator or on the central column. Also, $h^{ii} = 1$ if $i \neq m/2$. Therefore, we can simplify all calculation with use of characteristic classes theory.

We take the Riemann-Roch-Hirzebruch equation

$$\chi(E,X)=\int_X \mathrm{ch(E)}\wedge \mathrm{td(T_X)}, \tag{1}$$

attach it to $E=\bigwedge^q \Omega_X=\Omega_X^q$ and rewrite over Chern classes of tangent bundle $c(T_X)=\sum\limits_i c_i(T_X)=\prod\limits_i(1+\alpha_i)$:

$$\mathrm{Td(T_X)}=\prod_{\mathrm{i}}\frac{\alpha_\mathrm{i}}{1-\mathrm{e}^{-\alpha_\mathrm{i}}}=\sum \mathrm{td_i},$$

$$\mathrm{Ch}(\Omega_X^q)=\sum_{i_1<i_2<\ldots<i_q} e^{\alpha_{i_1}}\ldots e^{\alpha_{i_q}}=\sum \mathrm{ch_i}. \tag{2}$$

Here α_i are the Chern roots of $T_{\mathbb{P}^n}$.

For the first time the classes $\mathrm{td(T_X)}$ up to td_6 were obtained in [3] (p.14). In the original paper [4] (p.221) Eq.(61) coincides with the td_6 in our notation. All classes up to td_9 were obtained in [5], up to td_{10} in [2].

The Taylor series for (2) is

$$\mathrm{ch_m}(\Omega_n^p)=\sum_{s=1}^{\#I} A_{I_s}X^{I_s}, \tag{3}$$

where according to [11]

$$A_{I_s}=\frac{1}{I_s!}\binom{n-\#I_s}{p-\#I_s}, \qquad X^{I_s}=\sum_k^n \bigwedge^{\#I_s} x_k^{I_s}.$$

The multi-index

$$I=\{m,(m-1,1),(m-2,2),\ldots,(2,\underbrace{1,\ldots,1}_{m-2}),(\underbrace{1,1,\ldots,1}_{m})\} \tag{4}$$

is a tuple of a partition of integer m and $\#I$ is length of a tuple I.

For Chern characters of n-folds X from (2) we have the following theorems.

Theorem 1.

$$\begin{aligned}\mathrm{ch}_5(\Omega_n^p)=\frac{(-1)^m}{5!}&\Big[\tbinom{n-1}{p-1}c_1^5+\Big[\tbinom{n-2}{p-2}-\tbinom{n-1}{p-1}\Big]5c_2c_1^3\\&+\Big[\tbinom{n-1}{p-1}-\tbinom{n-2}{p-2}\Big]5c_2^2c_1\\&+\Big[4\tbinom{n-3}{p-3}-5\tbinom{n-2}{p-2}+\tbinom{n-1}{p-1}\Big]5c_3c_1^2\\&+\Big[-2\tbinom{n-3}{p-3}+3\tbinom{n-2}{p-2}-\tbinom{n-1}{p-1}\Big]5c_2c_3\\&+\Big[12\tbinom{n-4}{p-4}-22\tbinom{n-3}{p-3}+11\tbinom{n-2}{p-2}-\tbinom{n-1}{p-1}\Big]5c_4c_1\end{aligned} \tag{5}$$

$$+\left[24\tbinom{n-5}{p-5}-60\tbinom{n-4}{p-4}+50\tbinom{n-3}{p-3}-15\tbinom{n-2}{p-2}+\tbinom{n-1}{p-1}\right]5c_5\Big].$$

Proof. Let the multi-index (4)

$$I=\{5,(4,1),(3,2),(3,1,1),(2,2,1),(2,1,1,1),(1,1,1,1,1)\}$$

be a tuple of a integer partition. Since $m=5$ then we have the following expansion.

For $s=1$: $I_1=5, \#I_1=1$,

$$A_{I_1}=A_5=\frac{1}{5!}\binom{n-\#I_1}{p-\#I_1}=\frac{1}{5!}\binom{n-1}{p-1},$$

$$X^{I_1}=\sum_k^n\bigwedge^{\#I_1}x_k^{I_1}=\sum_{k=1}^n x_k^5$$
$$=c_1^5-5c_2c_1^3+5c_3c_1^2+5c_2^2c_1-5c_4c_1-5c_2c_3+5c_5.$$

For $s=2$: $I_2=(4,1), \#I_1=2$,

$$A_{I_2}=A_{4,1}=\frac{1}{4!1!}\binom{n-\#I_2}{p-\#I_2}=\frac{1}{4!}\binom{n-2}{p-2},$$

$$X^{I_2}=\sum_k^n\bigwedge^{\#I_2}x_k^{I_2}=\sum_{k_1<k_2}^n x_{k_1}^4x_{k_2}^1$$
$$=c_2c_1^3-c_3c_1^2-3c_2^2c_1+c_4c_1+5c_2c_3-5c_5.$$

For $s=3$: $I_3=(3,2), \#I_3=2$,

$$A_{I_3}=A_{3,2}=\frac{1}{3!2!}\binom{n-\#I_3}{p-\#I_3}=\frac{1}{3!2!}\binom{n-2}{p-2},$$

$$X^{I_3}=\sum_k^n\bigwedge^{\#I_3}x_k^{I_3}=\sum_{k_1<k_2}^n x_{k_1}^3x_{k_2}^2=-2c_3c_1^2+c_2^2c_1+5c_4c_1-c_2c_3-5c_5.$$

For $s=4$: $I_4=(3,1,1), \#I_4=3$,

$$A_{I_4}=A_{3,1,1}=\frac{1}{3!1!1!}\binom{n-\#I_4}{p-\#I_4}=\frac{1}{3!}\binom{n-3}{p-3},$$

$$X^{I_4} = \sum_k^n \bigwedge^{\#I_4} x_k^{I_4} = \sum_{k_1<k_2<k_3}^n x_{k_1}^3 x_{k_2}^1 x_{k_2}^1 = c_3c_1^2 - c_4c_1 - 2c_2c_3 + 5c_5.$$

For $s = 5$: $I_5 = (2, 2, 1), \#I_5 = 3$,

$$A_{I_5} = A_{2,2,1} = \frac{1}{2!2!1!}\binom{n - \#I_5}{p - \#I_5} = \frac{1}{4}\binom{n-3}{p-3},$$

$$X^{I_5} = \sum_k^n \bigwedge^{\#I_5} x_k^{I_5} = \sum_{k_1<k_2<k_3}^n x_{k_1}^2 x_{k_2}^2 x_{k_2}^1 = c_2c_3 - 3c_1c_4 + 5c_5.$$

For $s = 6$: $I_6 = (2, 1, 1, 1), \#I_6 = 4$,

$$A_{I_6} = A_{2,1,1,1} = \frac{1}{2!1!1!1!}\binom{n - \#I_6}{p - \#I_6} = \frac{1}{2!1!1!1!}\binom{n-4}{p-4},$$

$$X^{I_6} = \sum_k^n \bigwedge^{\#I_6} x_k^{I_6} = \sum_{k_1<k_2<k_3<k_4}^n x_{k_1}^2 x_{k_2}^1 x_{k_3}^1 x_{k_4}^1 = c_1c_4 - 5c_5.$$

For $s = 7$: $I_7 = (1, 1, 1, 1, 1), \#I_7 = 5$,

$$A_{I_7} = A_{1,1,1,1,1} = \frac{1}{1!1!1!1!1!}\binom{n - \#I_7}{p - \#I_7} = \binom{n-5}{p-5},$$

$$X^{I_7} = \sum_k^n \bigwedge^{\#I_7} x_k^{I_7} = \sum_{k_1<k_2<k_3<k_4<k_5}^n x_{k_1}^1 x_{k_2}^1 x_{k_3}^1 x_{k_4}^1 x_{k_5}^1 = c_5.$$

Substituting this into (3), we obtain (5). □

Equations from $\mathrm{ch}_0(\Omega_n^p)$ to $\mathrm{ch}_4(\Omega_n^p)$ are in work [11]. Taking into account that h_{ij} is symmetric, we do not write out the highest values for ch_i. The complete formulas for $\mathrm{ch_m}(\Omega_{10}^p)$ are given in Appendix.

3. Hodge diamond

The Hodge diamond of 2-fold Calabi-Yau may be seen in [1] (p. 590), for 3-folds in [6] (p. 45). Four to nine-folds were considered in [7]–[11].

For example, we find the Hodge diamond of 10-fold Calabi-Yau $X_{10} \in \mathbb{P}^{15}$, which is a complete intersection of two quadrics, two cubic and sextic $X_{10} = S^6 \cap S^3 \cap S^3 \cap S^2 \cap S^2$ (Fig.1).

Summing the Betti numbers

$$\mathbf{b} = (1,0,1,0,1,0,1,0,1,0,6354857582,0,1,0,1,0,1,0,1,0,1),$$

we obtain $\chi = 6354857592$. $\triangle$

This result is easily verified using the theory of characteristic classes. For any morphism $X \xrightarrow{f} \mathbb{P}^n$ with using the bundle

$$0 \to T_X \to f^* T_{\mathbb{P}^n} \to N_f \to 0, \tag{6}$$

we find Chern class $c(T_X)$. Defining $c(\mathcal{O}_{\mathbb{P}^n}(d))) = 1 + dt$, for a complete intersection k hypersurfaces $X = \bigcap_{i=1}^{k} S_i$ of degree s_i we obtain $c(N_f) = \prod_{i=1}^{k} (1 + s_i t)$. The Euler sequence dual to (6) has the form

$$0 \to \mathcal{O}_{\mathbb{P}^n} \to \mathcal{O}_{\mathbb{P}^n}(1)^{\oplus(n+1)} \to T_{\mathbb{P}^n} \to 0.$$

										1
									0	
								0		1
							0		0	
						0		0		1
					0		0		0	
				0		0		0		1
			0		0		0		0	
		0		0		0		0		1
	0		0		0		0		0	
1		46658		9860950		254711095		1539517822		2746584538
	0		0		0		0		0	
		0		0		0		0		1
			0		0		0		0	
				0		0		0		1
					0		0		0	
						0		0		1
							0		0	
								0		1
									0	
										1

Fig 1. Half Hodge diamond for $X_{10} = S^6 \cap S^3 \cap S^3 \cap S^2 \cap S^2 \in \mathbb{P}^{15}$

From this $c(T_{\mathbb{P}^n}) = (1+t)^{n+1}$, hence

$$c(T_X) = \frac{(1+t)^{n+1}}{\prod_{i=1}^{k} (1 + s_i t)}.$$

The Euler characteristic of m-fold is

$$\chi = \int_X c_m(X) = \int_{\mathbb{P}^n} c_{n-m}(N_f) \wedge c_m(T_X) = c_{n-m}(N_f) \cdot c_m(T_X). \tag{7}$$

For the intersection of $X_{10} = S^6 \cap S^3 \cap S^3 \cap S^2 \cap S^2 \in \mathbb{P}^{15}$ we have

$$\begin{aligned} c(N_f) &= (1+2t)^2(1+3t)^2(1+6t) \\ &= 1 + 16t + ... + 216t^5, \quad \text{i.e.} \quad c_5(N_f) = 216; \\ c(T_X) &= \frac{(1+t)^{16}}{(1+2t)^2(1+3t)^2(1+6t)} \\ &\approx 1 + 23t^2 - 90t^3 + ... + 29420637t^{10} + ... \\ &\quad \text{i.e.} \quad c_{10}(T_X) = 29420637. \end{aligned}$$

Therefore the Euler characteristic from (7) has the same value χ:

$$\chi = \int_X c_{10}(X) = \int_{\mathbb{P}^{15}} c_5(N_f) \wedge c_{10}(T_X) = 6354857592. \qquad \triangle$$

In the following table we show the Hodge numbers of the 10-fold Calabi-Yay that are complete intersections in ordinary projective spaces.

N	$\mathbb{P}^n \| X_{10}$	$h^{19}+1$	$h^{28}+1$	$h^{37}+1$	$h^{46}+1$	h^{55}	χ
1	11|12	1351935	387277321	10501578386	63459767087	112835748610	261535698072
2	12|11 2	1058070	299664796	8111318371	49015898362	87164322830	202020202032
3	12|10 3	596544	162512872	4373000879	26426295482	47012291482	108937103040
4	12|9 4	289393	73627803	1960861422	11850301257	21096306270	48866466024
5	12|8 5	131534	30752880	808129348	4884193394	8703004804	20149419120
6	12|7 6	68770	15076856	391715181	2367483432	4222038470	9770726952
7	13|10 2 2	764229	212652976	5740234801	34687936648	61696775528	142979952840
8	13|932	395602	104608116	2801884659	16932261018	30131946918	69810245712
9	13|842	176266	42869502	1133563160	6850811216	12202062940	28256903232
10	13|833	187380	46211960	1224444300	7399928290	13178323160	30519867024
11	13|752	76764	17040272	443823979	2682451162	4782826362	11069610720
12	13|743	77310	17209063	448520426	2710861649	4833211732	11186548632
13	13|662	49410	10496126	271147941	1638789922	2923763030	6764729832
14	13|653	35384	7192420	184451493	1114857104	1990033714	4603106520
15	13|644	31483	6272228	160344972	969186642	1730378210	4002048864
16	13|554	19289	3620803	91502942	553072081	988321166	2284751400
17	14|9222	502681	136161240	3660098253	22118092152	39351041592	91180750248
18	14|8322	235688	59673238	1587491884	9593761744	17080617640	39562942752
19	14|7422	95886	21937280	574296078	3470961362	6186495792	14321077008

N	$\mathbb{P}^n \| X_{10}$	$h^{19}+1$	$h^{28}+1$	$h^{37}+1$	$h^{46}+1$	h^{55}	χ
20	14|7332	100008	23114108	606066279	3662937364	6527955590	15112391112
21	14|6522	43039	8998032	231860759	1401372052	2500618752	5785166520
22	14|6432	38399	7881154	202538442	1224198392	2184839974	5054152752
23	14|6333	38530	7920660	203638425	1230856290	2196642090	5081549904
24	14|5532	22214	4265428	108246067	654265906	1168787566	2702386800
25	14|5442	16689	3071896	77361956	467607438	835810598	1931926560
26	14|5433	15534	2825939	71033730	429366685	767556980	1774040760
27	14|4443	9582	1626046	40313192	243672656	436068724	1007311680
28	15|82222	296244	77053898	2058180880	12437961524	22138399096	51285384192
29	15|73222	123904	29460438	775921645	4689373994	8354688906	19344448872
30	15|64222	46524	9819074	253534016	1532391356	2733962504	6325544448
31	15|63322	46658	9860950	254711095	1539517822	2746584538	6354857592
32	15|55222	26349	5198688	132557517	801191856	1430767376	3308716200
33	15|54322	18122	3378568	85305198	515622794	921448312	2130097680
34	15|53332	16554	3038336	76523471	462557228	826755338	1911026520
35	15|44422	10820	1874874	46665072	282062916	504621208	1165848576
36	15|44332	8889	1490552	36874020	222887278	398935958	921457440
37	15|43333	6805	1091415	26768310	161806005	289801110	669146184
38	16|722222	153384	37547847	993366254	6003343207	10692441908	24761263296

N	$\mathbb{P}^n\|X_{10}$	$h^{19}+1$	$h^{28}+1$	$h^{37}+1$	$h^{46}+1$	h^{55}	χ
39	16|632222	56454	12274403	318540030	1925259019	3433641176	7945900992
40	16|542222	21104	4032790	102276008	618191144	1104385104	2553427200
41	16|533222	19187	3604810	91179609	551140982	984774220	2276663400
42	16|443222	9820	1674918	41571352	251279000	449639832	1038710016
43	16|433322	7299	1184720	29127002	176063618	315276974	728042256
44	16|333332	4600	690555	16706445	100983735	181064910	417835584
45	17|6222222	68249	15276362	398310604	2407322932	4291990838	9933947136
46	17|5322222	22204	4270387	108471654	655654587	1171154332	2707992000
47	17|4422222	10784	1868762	46525568	281223716	503104192	1162361856
48	17|4332222	7748	1270210	31293600	189160892	338673976	782138880
49	17|3333222	4510	674400	16302921	98544816	176702646	407755944
50	18|52222222	25659	5051946	128859556	778877268	1390835778	3216464640
51	18|43222222	8130	1343098	33145048	200354436	358664908	828366336
52	18|33322222	4262	630265	15202306	91892629	164803024	380261952
53	$19\|42_1...2_8$	8420	1398170	34547760	208837220	373808600	863391744
54	$19\|332_1...2_7$	3819	552682	13272932	80231060	143938402	332059392
55	$20\|32_1...2_9$	3140	437210	10415920	62961700	113029400	260665344
56	$21\|2_1...2_{11}$	2180	281690	6601520	39903140	71733400	165310464

Appendix

Taylor series coefficients for

$$\mathrm{Ch}(\Omega_{10}^{p}) = \sum \mathrm{ch}_i.$$

$\mathbf{ch}(\Omega_{10}^{0})$:

$$\mathrm{ch}_0 = 1; \qquad \mathrm{ch}_k = 0, \text{ if } k > 0.$$

$\mathbf{ch}(\Omega_{10}^{1})$:

$$\mathrm{ch}_0 = 10; \quad \mathrm{ch}_1 = -c_1; \quad \mathrm{ch}_2 = \tfrac{1}{2}\left(c_1^2 - 2c_2\right);$$

$$\mathrm{ch}_3 = \tfrac{1}{3!}\left(-c_1^3 + 3c_2c_1 - 3c_3\right);$$

$$\mathrm{ch}_4 = \tfrac{1}{4!}\left(c_1^4 - 4c_2c_1^2 + 4c_3c_1 + 2c_2^2 - 4c_4\right);$$

$$\mathrm{ch}_5 = \tfrac{1}{5!}\left(-c_1^5 + 5c_2c_1^3 - 5c_3c_1^2 - 5c_2^2c_1 + 5c_4c_1 + 5c_2c_3 - 5c_5\right);$$

$$\mathrm{ch}_6 = \tfrac{1}{6!}\left(\begin{array}{l} c_1^6 - 6c_2c_1^4 + 6c_3c_1^3 + 9c_2^2c_1^2 - 6c_4c_1^2 \\ -12c_2c_3c_1 + 6c_5c_1 - 2c_2^3 + 3c_3^2 + 6c_2c_4 - 6c_6 \end{array}\right);$$

$$\mathrm{ch}_7 = \tfrac{1}{7!}\left(\begin{array}{l} -c_1^7 + 7c_2c_1^5 - 7c_3c_1^4 - 14c_2^2c_1^3 + 7c_4c_1^3 + 21c_2c_3c_1^2 \\ -7c_5c_1^2 + 7c_2^3c_1 - 7c_3^2c_1 - 14c_2c_4c_1 \\ +7c_6c_1 - 7c_2^2c_3 + 7c_3c_4 + 7c_2c_5 - 7c_7 \end{array}\right);$$

$$\mathrm{ch}_8 = \tfrac{1}{8!}\left(\begin{array}{l} c_1^8 - 8c_2c_1^6 + 8c_3c_1^5 + 20c_2^2c_1^4 - 8c_4c_1^4 - 32c_2c_3c_1^3 \\ +8c_5c_1^3 - 16c_2^3c_1^2 + 12c_3^2c_1^2 + 24c_2c_4c_1^2 - 8c_6c_1^2 \\ +24c_2^2c_3c_1 - 16c_3c_4c_1 - 16c_2c_5c_1 + 8c_7c_1 + 2c_2^4 \\ -8c_2c_3^2 + 4c_4^2 - 8c_2^2c_4 + 8c_3c_5 + 8c_2c_6 - 8c_8 \end{array}\right);$$

$$\mathrm{ch}_9 = \tfrac{1}{9!}\left(\begin{array}{l} -c_1^9 + 9c_2c_1^7 - 9c_3c_1^6 - 27c_2^2c_1^5 + 9c_4c_1^5 + 45c_2c_3c_1^4 \\ -9c_5c_1^4 + 30c_2^3c_1^3 - 18c_3^2c_1^3 - 36c_2c_4c_1^3 + 9c_6c_1^3 \\ -54c_2^2c_3c_1^2 + 27c_3c_4c_1^2 + 27c_2c_5c_1^2 - 9c_9 - 9c_7c_1^2 \\ -9c_2^4c_1 + 27c_2c_3^2c_1 - 9c_4^2c_1 + 27c_2^2c_4c_1 - 18c_3c_5c_1 \\ -18c_2c_6c_1 + 9c_8c_1 - 3c_3^3 + 9c_2^3c_3 \\ -18c_2c_3c_4 - 9c_2^2c_5 + 9c_4c_5 + 9c_3c_6 + 9c_2c_7 \end{array}\right);$$

$$\mathrm{ch}_{10} = \frac{1}{10!}\begin{pmatrix} c_1^{10} - 10c_2c_1^8 + 10c_3c_1^7 + 35c_2^2c_1^6 - 10c_4c_1^6 - 60c_2c_3c_1^5 \\ 10c_5c_1^5 - 50c_2^3c_1^4 + 25c_3^2c_1^4 + 50c_2c_4c_1^4 - 10c_6c_1^4 \\ 100c_2^2c_3c_1^3 - 40c_3c_4c_1^3 - 40c_2c_5c_1^3 + 10c_7c_1^3 + 25c_2^4c_1^2 \\ 15c_4^2c_1^2 - 60c_2c_3^2c_1^2 - 60c_2^2c_4c_1^2 + 30c_3c_5c_1^2 + 30c_2c_6c_1^2 \\ 10c_3^3c_1 - 10c_8c_1^2 - 40c_2^3c_3c_1 + 60c_2c_3c_4c_1 + 30c_2^2c_5c_1 \\ 10c_9c_1 - 20c_4c_5c_1 - 20c_3c_6c_1 - 20c_2c_7c_1 - 2c_2^5 \\ 15c_2^2c_3^2 - 10c_2c_4^2 + 5c_5^2 + 10c_2^3c_4 - 10c_3^2c_4 - 20c_2c_3c_5 \\ 10c_4c_6 - 10c_2^2c_6 + 10c_3c_7 + 10c_2c_8 - 10c_{10} \end{pmatrix}.$$

$\mathbf{ch}(\Omega^2_{10})$:

$$\mathrm{ch}_0 = 45; \quad \mathrm{ch}_1 = -9c_1; \quad \mathrm{ch}_2 = \frac{1}{2}\left(9c_1^2 - 16c_2\right);$$

$$\mathrm{ch}_3 = \frac{1}{2}\left(-3c_1^3 + 8c_2c_1 - 6c_3\right);$$

$$\mathrm{ch}_4 = \frac{1}{4!}\left(9c_1^4 - 32c_2c_1^2 + 20c_3c_1 + 16c_2^2 - 8c_4\right);$$

$$\mathrm{ch}_5 = \frac{1}{5!}\begin{pmatrix} -9c_1^5 + 40c_2c_1^3 - 20c_3c_1^2 - 40c_2^2c_1 \\ -10c_4c_1 + 30c_2c_3 + 30c_5 \end{pmatrix};$$

$$\mathrm{ch}_6 = \frac{1}{6!}\begin{pmatrix} 9c_1^6 - 48c_2c_1^4 + 18c_3c_1^3 + 72c_2^2c_1^2 + 42c_4c_1^2 - 102c_5c_1 \\ -66c_2c_3c_1 - 16c_2^3 + 24c_3^2 - 12c_2c_4 + 132c_6 \end{pmatrix};$$

$$\mathrm{ch}_7 = \frac{1}{7!}\begin{pmatrix} 56c_2c_1^5 - 9c_1^7 - 14c_3c_1^4 - 112c_2^2c_1^3 - 91c_4c_1^3 \\ 105c_2c_3c_1^2 + 231c_5c_1^2 + 56c_2^3c_1 - 63c_3^2c_1 + 84c_2c_4c_1 \\ 42c_3c_4 - 336c_6c_1 - 42c_2^2c_3 - 168c_2c_5 + 378c_7 \end{pmatrix};$$

$$\mathrm{ch}_8 = \frac{1}{8!}\begin{pmatrix} 9c_1^8 - 64c_2c_1^6 + 8c_3c_1^5 + 160c_2^2c_1^4 + 160c_4c_1^4 \\ -144c_2c_3c_1^3 - 440c_5c_1^3 - 128c_2^3c_1^2 + 124c_3^2c_1^2 \\ -256c_2c_4c_1^2 + 720c_6c_1^2 + 136c_2^2c_3c_1 - 128c_3c_4c_1 \\ +656c_2c_5c_1 - 888c_7c_1 + 16c_2^4 - 64c_2c_3^2 + 88c_4^2 \\ +48c_2^2c_4 - 104c_3c_5 - 608c_2c_6 + 944c_8 \end{pmatrix};$$

$$\mathrm{ch}_9 = \frac{3}{9!}\begin{pmatrix} -3c_1^9 + 24c_2c_1^7 - 72c_2^2c_1^5 - 84c_4c_1^5 + 60c_2c_3c_1^4 \\ +252c_5c_1^4 + 80c_2^3c_1^3 - 72c_3^2c_1^3 + 192c_2c_4c_1^3 \\ -462c_6c_1^3 - 96c_2^2c_3c_1^2 + 102c_3c_4c_1^2 - 570c_2c_5c_1^2 \\ +630c_7c_1^2 - 24c_2^4c_1 + 78c_2c_3^2c_1 - 114c_4^2c_1 \\ -90c_2^2c_4c_1 + 108c_3c_5c_1 + 780c_2c_6c_1 - 714c_8c_1 \\ -6c_3^3 + 18c_2^3c_3 - 36c_2c_3c_4 + 150c_2^2c_5 + 102c_4c_5 \\ -234c_3c_6 - 570c_2c_7 + 738c_9 \end{pmatrix};$$

$$\mathrm{ch}_{10} = \frac{1}{10!}\begin{pmatrix} 9c_1^{10} - 80c_2c_1^8 - 10c_3c_1^7 + 280c_2^2c_1^6 + 370c_4c_1^6 \\ -210c_2c_3c_1^5 - 400c_2^3c_1^4 + 350c_3^2c_1^4 - 1100c_2c_4c_1^4 \\ +2470c_6c_1^4 + 500c_2^2c_3c_1^3 - 650c_3c_4c_1^3 + 3670c_2c_5c_1^3 \\ -3730c_7c_1^3 + 200c_2^4c_1^2 - 570c_2c_3^2c_1^2 + 930c_4^2c_1^2 \\ +870c_2^2c_4c_1^2 - 660c_3c_5c_1^2 - 6240c_2c_6c_1^2 + 4570c_8c_1^2 \\ +50c_3^3c_1 - 230c_2^3c_3c_1 + 480c_2c_3c_4c_1 - 2190c_2^2c_5c_1 \\ -1390c_4c_5c_1 + 2390c_3c_6c_1 + 6710c_2c_7c_1 + 20c_4c_6 \\ -4930c_9c_1 - 16c_2^5 + 120c_2^2c_3^2 - 380c_2c_4^2 + 640c_5^2 \\ -100c_2^3c_4 - 20c_3^2c_4 + 500c_2c_3c_5 + 1780c_2^2c_6 \\ -2500c_3c_7 - 4300c_2c_8 - 1210c_5c_1^5 + 5020c_{10} \end{pmatrix}.$$

$\mathbf{ch}(\Omega^3_{10})$:

$\mathrm{ch}_0 = 120; \quad \mathrm{ch}_1 = -36c_1; \quad \mathrm{ch}_2 = 2\left(9c_1^2 - 14c_2\right);$

$\mathrm{ch}_3 = -6c_1^3 + 14c_2c_1 - 7c_3;$

$\mathrm{ch}_4 = \frac{4}{4!}\left(9c_1^4 - 28c_2c_1^2 + 7c_3c_1 + 14c_2^2 + 8c_4\right);$

$\mathrm{ch}_5 = \frac{2}{5!}\left(-18c_1^5 + 70c_2c_1^3 - 70c_2^2c_1 - 75c_4c_1 + 35c_2c_3 + 85c_5\right);$

$$\mathrm{ch}_6 = \frac{2}{6!}\begin{pmatrix} 18c_1^6 - 84c_2c_1^4 - 21c_3c_1^3 + 126c_2^2c_1^2 + 171c_4c_1^2 \\ 42c_3^2 - 63c_2c_3c_1 - 201c_5c_1 - 28c_2^3 - 96c_2c_4 + 96c_6 \end{pmatrix};$$

$$\mathrm{ch}_7 = \frac{1}{7!}\begin{pmatrix} -36c_1^7 + 196c_2c_1^5 + 98c_3c_1^4 - 392c_2^2c_1^3 - 623c_4c_1^3 \\ +147c_2c_3c_1^2 + 763c_5c_1^2 + 196c_2^3c_1 - 245c_3^2c_1 \\ +770c_2c_4c_1 - 28c_6c_1 - 98c_2^2c_3 + 98c_3c_4 \\ -742c_2c_5 - 938c_7 \end{pmatrix};$$

$$\mathrm{ch}_8 = \frac{4}{8!}\begin{pmatrix} 9c_1^8 - 56c_2c_1^6 - 42c_3c_1^5 + 140c_2^2c_1^4 + 252c_4c_1^4 \\ -28c_2c_3c_1^3 - 322c_5c_1^3 - 112c_2^3c_1^2 + 133c_3^2c_1^2 \\ -490c_2c_4c_1^2 - 168c_6c_1^2 + 70c_2^2c_3c_1 - 98c_3c_4c_1 \\ +658c_2c_5c_1 + 1134c_7c_1 + 14c_2^4 - 56c_2c_3^2 + 112c_4^2 \\ +112c_2^2c_4 - 196c_3c_5 - 112c_2c_6 - 1904c_8 \end{pmatrix};$$

$$\mathrm{ch}_9 = \frac{6}{9!}\begin{pmatrix} -6c_1^9 + 42c_2c_1^7 + 42c_3c_1^6 - 126c_2^2c_1^5 - 252c_4c_1^5 \\ +336c_5c_1^4 + 140c_2^3c_1^3 - 168c_3^2c_1^3 + 672c_2c_4c_1^3 \\ +399c_6c_1^3 - 84c_2^2c_3c_1^2 + 189c_3c_4c_1^2 - 1071c_2c_5c_1^2 \\ -2331c_7c_1^2 - 42c_2^4c_1 + 147c_2c_3^2c_1 - 315c_4^2c_1 \\ -357c_2^2c_4c_1 + 462c_3c_5c_1 + 42c_2c_6c_1 + 4641c_8c_1 \\ -7c_3^3 + 21c_2^3c_3 - 42c_2c_3c_4 + 315c_2^2c_5 + 189c_4c_5 \\ -483c_3c_6 + 1365c_2c_7 - 6069c_9 \end{pmatrix};$$

$$\mathrm{ch}_{10}=\frac{2}{10!}\begin{pmatrix}18c_1^{10}-140c_2c_1^8-175c_3c_1^7+490c_2^2c_1^6+1075c_4c_1^6\\105c_2c_3c_1^5-1495c_5c_1^5-700c_2^3c_1^4+875c_3^2c_1^4\\350c_2^2c_3c_1^3-3650c_2c_4c_1^4-2915c_6c_1^4-1355c_3c_4c_1^3\\6565c_2c_5c_1^3+17405c_7c_1^3+350c_2^4c_1^2-1155c_2c_3^2c_1^2\\2685c_4^2c_1^2+3165c_2^2c_4c_1^2-3450c_3c_5c_1^2\\1230c_2c_6c_1^2-40505c_8c_1^2+35c_3^3c_1-245c_2^3c_3c_1\\750c_2c_3c_4c_1-4395c_2^2c_5c_1-2665c_4c_5c_1\\6155c_3c_6c_1-21205c_2c_7c_1+61925c_9c_1-28c_2^5\\210c_2^2c_3^2-1040c_2c_4^2+1870c_5^2-400c_2^3c_4\\40c_3^2c_4+1700c_2c_3c_5+400c_2^2c_6-2560c_4c_6\\29840c_2c_8-40c_3c_7-73040c_{10}\end{pmatrix}.$$

ch(Ω_{10}^4)

$\mathrm{ch}_0=210;\quad \mathrm{ch}_1=-84c_1;\quad \mathrm{ch}_2=14\left(3c_1^2-4c_2\right);$

$\mathrm{ch}_3=-7\left(2c_1^3-4c_2c_1+c_3\right);$

$\mathrm{ch}_4=\frac{4}{4!}\left(21c_1^4-56c_2c_1^2-7c_3c_1+28c_2^2+34c_4\right);$

$$\mathrm{ch}_5=\frac{2}{5!}\begin{pmatrix}-42c_1^5+140c_2c_1^3+70c_3c_1^2-140c_2^2c_1\\-205c_4c_1+35c_2c_3+115c_5\end{pmatrix};$$

$$\mathrm{ch}_6=\frac{2}{6!}\begin{pmatrix}42c_1^6-168c_2c_1^4-147c_3c_1^3+252c_2^2c_1^2+417c_4c_1^2\\84c_3^2-21c_2c_3c_1-147c_5c_1-56c_2^3-282c_2c_4-258c_6\end{pmatrix};$$

$$\mathrm{ch}_7=\frac{7}{7!}\begin{pmatrix}-12c_1^7+56c_2c_1^5+70c_3c_1^4-112c_2^2c_1^3-205c_4c_1^3\\-21c_2c_3c_1^2+25c_5c_1^2+56c_2^3c_1-77c_3^2c_1+296c_2c_4c_1\\+380c_6c_1-14c_2^2c_3+14c_3c_4-136c_2c_5-434c_7\end{pmatrix};$$

$$\mathrm{ch}_8=\frac{28}{8!}\begin{pmatrix}3c_1^8-16c_2c_1^6-26c_3c_1^5+40c_2^2c_1^4+80c_4c_1^4+20c_2c_3c_1^3\\+10c_5c_1^3-32c_2^3c_1^2+45c_3^2c_1^2-180c_2c_4c_1^2-280c_6c_1^2\\+6c_2^2c_3c_1-20c_3c_4c_1+100c_2c_5c_1+334c_7c_1+4c_2^4\\-16c_2c_3^2+38c_4^2+44c_2^2c_4-74c_3c_5+136c_2c_6+44c_8\end{pmatrix};$$

$$\mathrm{ch}_9=\frac{42}{9!}\begin{pmatrix}-2c_1^9+12c_2c_1^7+24c_3c_1^6-36c_2^2c_1^5-78c_4c_1^5-30c_2c_3c_1^4\\-30c_5c_1^4+40c_2^3c_1^3-60c_3^2c_1^3+240c_2c_4c_1^3+435c_6c_1^3\\+45c_3c_4c_1^2-135c_2c_5c_1^2-543c_7c_1^2-12c_2^4c_1+45c_2c_3^2c_1\\-111c_4^2c_1-135c_2^2c_4c_1+198c_3c_5c_1-462c_2c_6c_1\\-591c_8c_1-c_3^3+3c_2^3c_3-6c_2c_3c_4+57c_2^2c_5\\+33c_4c_5-87c_3c_6+513c_2c_7+2427c_9\end{pmatrix};$$

$$\mathrm{ch}_{10} = \frac{2}{10!}\left(\begin{array}{l} 42c_1^{10} - 280c_2c_1^8 - 665c_3c_1^7 + 980c_2^2c_1^6 + 2285c_4c_1^6 \\ +1155c_2c_3c_1^5 + 1495c_5c_1^5 - 1400c_2^3c_1^4 + 2275c_3^2c_1^4 \\ -8950c_2c_4c_1^4 + 420c_2^2c_3^2 - 350c_2^2c_3c_1^3 - 2425c_3c_4c_1^3 \\ +4775c_2c_5c_1^3 + 24175c_7c_1^3 + 700c_2^4c_1^2 - 2625c_2c_3^2c_1^2 \\ +6765c_4^2c_1^2 + 8175c_2^2c_4c_1^2 - 11670c_3c_5c_1^2 \\ +55205c_8c_1^2 - 35c_3^3c_1 - 175c_2^3c_3c_1 + 30900c_2c_6c_1^2 \\ -5055c_2^2c_5c_1 - 3215c_4c_5c_1 + 10015c_3c_6c_1 \\ -48305c_2c_7c_1 - 247985c_9c_1 - 18505c_6c_1^4 \\ -2530c_2c_4^2 + 4640c_5^2 - 1070c_2^3c_4 + 170c_3^2c_4 \\ +4390c_2c_3c_5 - 56c_2^5 - 10250c_4c_6 + 1140c_2c_3c_4c_1 \\ +11170c_3c_7 - 6490c_2^2c_6 - 12410c_2c_8 + 441170c_{10} \end{array}\right).$$

$\mathbf{ch}(\Omega^5_{10})$:

$$\mathrm{ch}_0 = 252; \quad \mathrm{ch}_1 = -126c_1; \quad \mathrm{ch}_2 = 7\left(9c_1^2 - 10c_2\right);$$

$$\mathrm{ch}_3 = -7\left(3c_1^3 - 5c_1c_2\right);$$

$$\mathrm{ch}_4 = \frac{2}{4!}\left(63c_1^4 - 140c_2c_1^2 - 70c_3c_1 + 70c_2^2 + 100c_4\right);$$

$$\mathrm{ch}_5 = \frac{2}{5!}\left(-63c_1^5 + 175c_2c_1^3 + 175c_3c_1^2 - 175c_2^2c_1 - 250c_4c_1\right);$$

$$\mathrm{ch}_6 = \frac{2}{6!}\left(\begin{array}{l} 63c_1^6 - 210c_2c_1^4 - 315c_3c_1^3 + 315c_2^2c_1^2 + 465c_4c_1^2 - 70c_2^3 \\ +105c_2c_3c_1 + 285c_5c_1 + 105c_3^2 - 390c_2c_4 - 570c_6 \end{array}\right);$$

$$\mathrm{ch}_7 = \frac{7}{7!}\left(\begin{array}{l} 70c_2c_1^5 - 18c_1^7 + 140c_3c_1^4 - 140c_2^2c_1^3 - 215c_4c_1^3 + 70c_2^3c_1 \\ 390c_2c_4c_1 - 285c_5c_1^2 - 105c_3^2c_1 + 570c_6c_1 - 105c_2c_3c_1^2 \end{array}\right);$$

$$\mathrm{ch}_8 = \frac{14}{8!}\left(\begin{array}{l} 9c_1^8 - 40c_2c_1^6 - 100c_3c_1^5 + 100c_2^2c_1^4 + 160c_4c_1^4 \\ +120c_2c_3c_1^3 + 340c_5c_1^3 - 80c_2^3c_1^2 + 130c_3^2c_1^2 \\ -460c_2c_4c_1^2 - 720c_6c_1^2 - 20c_2^2c_3c_1 - 20c_3c_4c_1 \\ -100c_2c_5c_1 - 420c_7c_1 + 10c_2^4 - 40c_2c_3^2 + 100c_4^2 \\ +120c_2^2c_4 - 200c_3c_5 + 520c_2c_6 + 1400c_8 \end{array}\right);$$

$$\mathrm{ch}_9 = \frac{42}{9!}\left(\begin{array}{l} 15c_2c_1^7 - 3c_1^9 + 45c_3c_1^6 - 45c_2^2c_1^5 - 75c_4c_1^5 \\ 50c_2^3c_1^3 - 75c_2c_3c_1^4 - 225c_5c_1^4 - 90c_3^2c_1^3 + 300c_2c_4c_1^3 \\ 510c_6c_1^3 + 30c_2^2c_3c_1^2 + 30c_3c_4c_1^2 \\ 150c_2c_5c_1^2 + 630c_7c_1^2 - 15c_2^4c_1 + 60c_2c_3^2c_1 - 150c_4^2c_1 \\ 300c_3c_5c_1 - 180c_2^2c_4c_1 - 780c_2c_6c_1 - 2100c_8c_1 \end{array}\right);$$

$$\mathrm{ch}_{10}=\frac{2}{10!}\begin{pmatrix} 63c_1^{10}-350c_2c_1^8-1225c_3c_1^7+1225c_2^2c_1^6+2125c_4c_1^6\\ +2625c_2c_3c_1^5+8375c_5c_1^5-1750c_2^3c_1^4+3500c_3^2c_1^4\\ -11000c_2c_4c_1^4-20345c_6c_1^4-1750c_2^2c_3c_1^3\\ -1775c_3c_4c_1^3-8975c_2c_5c_1^3-39505c_7c_1^3+875c_2^4c_1^2\\ -3675c_2c_3^2c_1^2+9300c_4^2c_1^2+10725c_2^2c_4c_1^2\\ -19200c_3c_5c_1^2+48480c_2c_6c_1^2+142405c_8c_1^2\\ -175c_3^3c_1+175c_2^3c_3c_1+750c_2c_3c_4c_1\\ +1275c_2^2c_5c_1+275c_4c_5c_1+2795c_3c_6c_1\\ +10715c_2c_7c_1+78095c_9c_1-70c_2^5+525c_2^2c_3^2\\ -3350c_2c_4^2+6175c_5^2-1450c_2^3c_4+250c_3^2c_4\\ +5900c_2c_3c_5-11990c_2^2c_6-15370c_4c_6\\ +19910c_3c_7-88810c_2c_8-780950c_{10}\end{pmatrix}.$$

$\mathbf{ch}(\Omega_{10}^6)$:

$\mathrm{ch}_0=210;\quad \mathrm{ch}_1=-126c_1;\quad \mathrm{ch}_2=7\left(9c_1^2-8c_2\right);$

$\mathrm{ch}_3=-7\left(3c_1^3-4c_2c_1-c_3\right);$

$\mathrm{ch}_4=\frac{2}{4!}\left(63c_1^4-112c_2c_1^2-98c_3c_1+56c_2^2+68c_4\right);$

$$\mathrm{ch}_5=\frac{2}{5!}\begin{pmatrix} -63c_1^5+140c_2c_1^3+210c_3c_1^2-140c_2^2c_1\\ -135c_4c_1-35c_2c_3-115c_5\end{pmatrix};$$

$$\mathrm{ch}_6=\frac{2}{6!}\begin{pmatrix} 63c_1^6-168c_2c_1^4-357c_3c_1^3+252c_2^2c_1^2+207c_4c_1^2-56c_2^3\\ +189c_2c_3c_1+543c_5c_1+84c_3^2-282c_2c_4-258c_6\end{pmatrix};$$

$$\mathrm{ch}_7=\frac{7}{7!}\begin{pmatrix} -18c_1^7+56c_2c_1^5+154c_3c_1^4-112c_2^2c_1^3-79c_4c_1^3\\ -421c_5c_1^2+56c_2^3c_1-91c_3^2c_1+268c_2c_4c_1-147c_2c_3c_1^2\\ +136c_6c_1+14c_2^2c_3-14c_3c_4+136c_2c_5+434c_7\end{pmatrix};$$

$$\mathrm{ch}_8=\frac{14}{8!}\begin{pmatrix} 9c_1^8-32c_2c_1^6-108c_3c_1^5+80c_2^2c_1^4+48c_4c_1^4-72c_6c_1^2\\ 452c_5c_1^3-64c_2^3c_1^2+118c_3^2c_1^2-304c_2c_4c_1^2+152c_2c_3c_1^3\\ 16c_3c_4c_1-44c_2^2c_3c_1-344c_2c_5c_1-1068c_7c_1+8c_2^4\\ 76c_4^2-32c_2c_3^2+88c_2^2c_4-148c_3c_5+272c_2c_6+88c_8\end{pmatrix};$$

$$\mathrm{ch}_9=\frac{42}{9!}\begin{pmatrix} -3c_1^9+12c_2c_1^7+48c_3c_1^6-36c_2^2c_1^5-18c_4c_1^5\\ -282c_5c_1^4+40c_2^3c_1^3-84c_3^2c_1^3+192c_2c_4c_1^3-3c_6c_1^3\\ +48c_2^2c_3c_1^2-9c_3c_4c_1^2+351c_2c_5c_1^2+1143c_7c_1^2\\ -12c_2^4c_1+51c_2c_3^2c_1-117c_4^2c_1-129c_2^2c_4c_1\\ +246c_3c_5c_1-354c_2c_6c_1+327c_8c_1+c_3^3-3c_2^3c_3\\ +6c_2c_3c_4-57c_2^2c_5-33c_4c_5-90c_2c_3c_1^4\\ +87c_3c_6-513c_2c_7-2427c_9\end{pmatrix};$$

$$\mathrm{ch}_{10} = \frac{2}{10!}\left(\begin{array}{l} 63c_1^{10} - 280c_2c_1^8 - 1295c_3c_1^7 + 980c_2^2c_1^6 + 395c_4c_1^6 \\ +3045c_2c_3c_1^5 + 10105c_5c_1^5 - 1400c_2^3c_1^4 + 3325c_3^2c_1^4 \\ -6850c_2c_4c_1^4 + 1865c_6c_1^4 - 2450c_2^2c_3c_1^3 + 305c_3c_4c_1^3 \\ -17695c_2c_5c_1^3 - 61715c_7c_1^3 + 700c_2^4c_1^2 - 3255c_2c_3^2c_1^2 \\ +7395c_4^2c_1^2 + 7545c_2^2c_4c_1^2 - 16710c_3c_5c_1^2 \\ +19560c_2c_6c_1^2 - 41185c_8c_1^2 - 245c_3^3c_1 + 455c_2^3c_3c_1 \\ -120c_2c_3c_4c_1 + 6915c_2^2c_5c_1 + 3715c_4c_5c_1 \\ -8255c_3c_6c_1 + 59425c_2c_7c_1 + 261685c_9c_1 - 56c_2^5 \\ +420c_2^2c_3^2 - 2530c_2c_4^2 + 4640c_5^2 - 1070c_2^3c_4 \\ +4390c_2c_3c_5 - 6490c_2^2c_6 - 10250c_4c_6 + 170c_3^2c_4 \\ +11170c_3c_7 - 12410c_2c_8 + 441170c_{10} \end{array}\right).$$

$\mathbf{ch}(\Omega^7_{10})$:

$$\mathrm{ch}_0 = 120; \quad \mathrm{ch}_1 = -84c_1; \quad \mathrm{ch}_2 = 14\left(3c_1^2 - 2c_2\right);$$

$$\mathrm{ch}_3 = \frac{1}{3!}\left(-84c_1^3 + 84c_2c_1 + 42c_3\right);$$

$$\mathrm{ch}_4 = \frac{4}{4!}\left(21c_1^4 - 28c_2c_1^2 - 35c_3c_1 + 14c_2^2 + 8c_4\right);$$

$$\mathrm{ch}_5 = \frac{2}{5!}\left(\begin{array}{l} -42c_1^5 + 70c_2c_1^3 + 140c_3c_1^2 - 70c_2^2c_1 \\ -5c_4c_1 - 35c_2c_3 - 85c_5 \end{array}\right);$$

$$\mathrm{ch}_6 = \frac{2}{6!}\left(\begin{array}{l} 42c_1^6 - 84c_2c_1^4 - 231c_3c_1^3 + 126c_2^2c_1^2 - 39c_4c_1^2 \\ +147c_2c_3c_1 + 309c_5c_1 - 28c_2^3 + 42c_3^2 - 96c_2c_4 + 96c_6 \end{array}\right);$$

$$\mathrm{ch}_7 = \frac{7}{7!}\left(\begin{array}{l} -12c_1^7 + 28c_2c_1^5 + 98c_3c_1^4 - 56c_2^2c_1^3 + 37c_4c_1^3 \\ -217c_5c_1^2 + 28c_2^3c_1 - 49c_3^2c_1 + 82c_2c_4c_1 - 188c_6c_1 \\ +14c_2^2c_3 - 14c_3c_4 + 106c_2c_5 - 105c_2c_3c_1^2 + 134c_7 \end{array}\right);$$

$$\mathrm{ch}_8 = \frac{28}{8!}\left(\begin{array}{l} 3c_1^8 - 8c_2c_1^6 - 34c_3c_1^5 + 20c_2^2c_1^4 - 20c_4c_1^4 + 52c_2c_3c_1^3 \\ +110c_5c_1^3 - 16c_2^3c_1^2 + 33c_3^2c_1^2 - 42c_2c_4c_1^2 + 160c_6c_1^2 \\ -18c_2^2c_3c_1 + 14c_3c_4c_1 - 118c_2c_5c_1 - 106c_7c_1 + 2c_2^4 \\ -8c_2c_3^2 + 16c_4^2 + 16c_2^2c_4 - 28c_3c_5 - 16c_2c_6 - 272c_8 \end{array}\right);$$

$$\mathrm{ch}_9 = \frac{42}{9!}\left(\begin{array}{l} -2c_1^9 + 6c_2c_1^7 + 30c_3c_1^6 - 18c_2^2c_1^5 + 24c_4c_1^5 - 60c_2c_3c_1^4 \\ -132c_5c_1^4 + 20c_2^3c_1^3 - 48c_3^2c_1^3 + 48c_2c_4c_1^3 - 273c_6c_1^3 \\ +36c_2^2c_3c_1^2 - 27c_3c_4c_1^2 + 225c_2c_5c_1^2 + 165c_7c_1^2 - 6c_2^4c_1 \\ +27c_2c_3^2c_1 - 51c_4^2c_1 - 45c_2^2c_4c_1 + 102c_3c_5c_1 \\ +90c_2c_6c_1 + 969c_8c_1 + c_3^3 - 3c_2^3c_3 + 6c_2c_3c_4 \\ -45c_2^2c_5 - 27c_4c_5 + 69c_3c_6 - 195c_2c_7 + 867c_9 \end{array}\right);$$

$$\mathrm{ch}_{10}=\frac{2}{10!}\begin{pmatrix} 42c_1^{10}-140c_2c_1^8-805c_3c_1^7+490c_2^2c_1^6-815c_4c_1^6\\ +1995c_2c_3c_1^5+4595c_5c_1^5-700c_2^3c_1^4+1925c_3^2c_1^4\\ -1550c_2c_4c_1^4+12415c_6c_1^4-1750c_2^2c_3c_1^3\\ +1375c_3c_4c_1^3-10865c_2c_5c_1^3-6745c_7c_1^3+350c_2^4c_1^2\\ -1785c_2c_3^2c_1^2+3315c_4^2c_1^2+2535c_2^2c_4c_1^2\\ -7590c_2c_6c_1^2-72635c_8c_1^2-175c_3^3c_1+385c_2^3c_3c_1\\ -510c_2c_3c_4c_1+5055c_2^2c_5c_1+3005c_4c_5c_1\\ -8335c_3c_6c_1+19745c_2c_7c_1-120145c_9c_1-28c_2^5\\ +210c_2^2c_3^2-1040c_2c_4^2+1870c_5^2-400c_2^3c_4+40c_3^2c_4\\ +1700c_2c_3c_5+400c_2^2c_6-7230c_3c_5c_1^2\\ -2560c_4c_6-40c_3c_7+29840c_2c_8-73040c_{10} \end{pmatrix}.$$

ch(Ω^8_{10}):

$\mathrm{ch}_0=45;\quad \mathrm{ch}_1=-36c_1;\quad \mathrm{ch}_2=\frac{1}{2}\left(36c_1^2-16c_2\right);$

$$\mathrm{ch}_3=\frac{1}{3!}\left(-36c_1^3+24c_2c_1+18c_3\right);$$

$$\mathrm{ch}_4=\frac{4}{4!}\left(9c_1^4-8c_2c_1^2-13c_3c_1+4c_2^2-2c_4\right);$$

$$\mathrm{ch}_5=\frac{2}{5!}\begin{pmatrix} -18c_1^5+20c_2c_1^3+50c_3c_1^2-20c_2^2c_1\\ +25c_4c_1-15c_2c_3-15c_5 \end{pmatrix};$$

$$\mathrm{ch}_6=\frac{2}{6!}\begin{pmatrix} 18c_1^6-24c_2c_1^4-81c_3c_1^3+36c_2^2c_1^2-69c_4c_1^2\\ +57c_2c_3c_1+39c_5c_1-8c_2^3+12c_3^2-6c_2c_4+66c_6 \end{pmatrix};$$

$$\mathrm{ch}_7=\frac{1}{7!}\begin{pmatrix} -36c_1^7+56c_2c_1^5+238c_3c_1^4-112c_2^2c_1^3+287c_4c_1^3\\ -147c_5c_1^2+56c_2^3c_1-105c_3^2c_1-588c_6c_1+42c_2^2c_3\\ -42c_3c_4+168c_2c_5-273c_2c_3c_1^2-378c_7 \end{pmatrix};$$

$$\mathrm{ch}_8=\frac{4}{8!}\begin{pmatrix} 9c_1^8-16c_2c_1^6-82c_3c_1^5+40c_2^2c_1^4-128c_4c_1^4+52c_3c_4c_1\\ 73c_3^2c_1^2-32c_2^3c_1^2+20c_2c_4c_1^2+432c_6c_1^2-50c_2^2c_3c_1\\ 534c_7c_1-172c_2c_5c_1+4c_2^4-16c_2c_3^2+22c_4^2+58c_5c_1^3\\ 12c_2^2c_4-26c_3c_5-152c_2c_6+132c_2c_3c_1^3+236c_8 \end{pmatrix};$$

$$\mathrm{ch}_9=\frac{6}{9!}\begin{pmatrix} -6c_1^9+12c_2c_1^7+72c_3c_1^6-36c_2^2c_1^5+138c_4c_1^5\\ -150c_2c_3c_1^4-54c_5c_1^4+40c_2^3c_1^3-108c_3^2c_1^3\\ -48c_2c_4c_1^3-681c_6c_1^3+96c_2^2c_3c_1^2-111c_3c_4c_1^2\\ +309c_2c_5c_1^2-1251c_7c_1^2-12c_2^4c_1+57c_2c_3^2c_1-75c_4^2c_1\\ -27c_2^2c_4c_1+102c_3c_5c_1+522c_2c_6c_1-1059c_8c_1+3c_3^3\\ -9c_2^3c_3+18c_2c_3c_4-75c_2^2c_5\\ -51c_4c_5+117c_3c_6+285c_2c_7-369c_9 \end{pmatrix};$$

$$\mathrm{ch}_{10} = \frac{2}{10!}\left(\begin{array}{l} 18c_1^{10} - 40c_2c_1^8 - 275c_3c_1^7 + 140c_2^2c_1^6 - 625c_4c_1^6 \\ +705c_2c_3c_1^5 + 205c_5c_1^5 - 200c_2^3c_1^4 + 625c_3^2c_1^4 \\ +350c_2c_4c_1^4 + 4205c_6c_1^4 - 650c_2^2c_3c_1^3 + 845c_3c_4c_1^3 \\ -2035c_2c_5c_1^3 + 10285c_7c_1^3 + 100c_2^4c_1^2 - 555c_2c_3^2c_1^2 \\ +735c_4^2c_1^2 + 165c_2^2c_4c_1^2 - 1050c_3c_5c_1^2 - 5100c_2c_6c_1^2 \\ +12815c_8c_1^2 - 65c_3^3c_1 + 155c_2^3c_3c_1 - 300c_2c_3c_4c_1 \\ +1155c_2^2c_5c_1 + 835c_4c_5c_1 - 2315c_3c_6c_1 + 10c_4c_6 \\ -5195c_2c_7c_1 + 8605c_9c_1 - 8c_2^5 + 60c_2^2c_3^2 \\ -190c_2c_4^2 + 320c_5^2 - 50c_2^3c_4 - 10c_3^2c_4 + 250c_2c_3c_5 \\ +890c_2^2c_6 - 1250c_3c_7 - 2150c_2c_8 + 2510c_{10} \end{array}\right).$$

$\mathbf{ch}(\Omega_{10}^9)$:

$$\mathrm{ch}_0 = 10; \quad \mathrm{ch}_1 = -9c_1; \quad \mathrm{ch}_2 = \frac{1}{2}\left(9c_1^2 - 2c_2\right);$$

$$\mathrm{ch}_3 = \frac{1}{2}\left(-3c_1^3 + c_2c_1 + c_3\right);$$

$$\mathrm{ch}_4 = \frac{1}{4!}\left(9c_1^4 - 4c_2c_1^2 - 8c_3c_1 + 2c_2^2 - 4c_4\right);$$

$$\mathrm{ch}_5 = \frac{1}{5!}\left(-9c_1^5 + 5c_2c_1^3 + 15c_3c_1^2 - 5c_2^2c_1 + 15c_4c_1 - 5c_2c_3 + 5c_5\right);$$

$$\mathrm{ch}_6 = \frac{1}{6!}\left(\begin{array}{l} 9c_1^6 - 6c_2c_1^4 - 24c_3c_1^3 + 9c_2^2c_1^2 - 36c_4c_1^2 + 18c_2c_3c_1 \\ -24c_5c_1 - 2c_2^3 + 3c_3^2 + 6c_2c_4 - 6c_6 \end{array}\right);$$

$$\mathrm{ch}_7 = \frac{1}{7!}\left(\begin{array}{l} -9c_1^7 + 7c_2c_1^5 + 35c_3c_1^4 - 14c_2^2c_1^3 + 70c_4c_1^3 - 42c_2c_3c_1^2 \\ +70c_5c_1^2 + 7c_2^3c_1 - 14c_3^2c_1 - 28c_2c_4c_1 \\ +35c_6c_1 + 7c_2^2c_3 - 7c_3c_4 - 7c_2c_5 + 7c_7 \end{array}\right);$$

$$\mathrm{ch}_8 = \frac{1}{8!}\left(\begin{array}{l} 9c_1^8 - 8c_2c_1^6 - 48c_3c_1^5 + 20c_2^2c_1^4 - 120c_4c_1^4 + 80c_2c_3c_1^3 \\ -160c_5c_1^3 - 16c_2^3c_1^2 + 40c_3^2c_1^2 + 80c_2c_4c_1^2 - 120c_6c_1^2 \\ -32c_2^2c_3c_1 + 40c_3c_4c_1 + 40c_2c_5c_1 - 48c_7c_1 + 2c_2^4 \\ -8c_2c_3^2 + 4c_4^2 - 8c_2^2c_4 + 8c_3c_5 + 8c_2c_6 - 8c_8 \end{array}\right);$$

$$\mathrm{ch}_9 = \frac{3}{9!}\left(\begin{array}{l} -3c_1^9 + 3c_2c_1^7 + 21c_3c_1^6 - 9c_2^2c_1^5 + 63c_4c_1^5 - 45c_2c_3c_1^4 \\ +105c_5c_1^4 + 10c_2^3c_1^3 - 30c_3^2c_1^3 - 60c_2c_4c_1^3 + 105c_6c_1^3 \\ +30c_2^2c_3c_1^2 - 45c_3c_4c_1^2 - 45c_2c_5c_1^2 + 63c_7c_1^2 - 3c_2^4c_1 \\ +15c_2c_3^2c_1 - 9c_4^2c_1 + 15c_2^2c_4c_1 - 18c_3c_5c_1 - 18c_2c_6c_1 \\ +21c_8c_1 + c_3^3 - 3c_2^3c_3 + 6c_2c_3c_4 \\ +3c_2^2c_5 - 3c_4c_5 - 3c_3c_6 - 3c_2c_7 + 3c_9 \end{array}\right);$$

$$\mathrm{ch}_{10} = \frac{1}{10!}\left(\begin{array}{l} 9c_1^{10} - 10c_2c_1^8 - 80c_3c_1^7 + 35c_2^2c_1^6 - 280c_4c_1^6 \\ +210c_2c_3c_1^5 - 560c_5c_1^5 - 50c_2^3c_1^4 + 175c_3^2c_1^4 \\ +350c_2c_4c_1^4 - 700c_6c_1^4 - 200c_2^2c_3c_1^3 + 350c_3c_4c_1^3 \\ +350c_2c_5c_1^3 - 560c_7c_1^3 + 25c_2^4c_1^2 - 150c_2c_3^2c_1^2 \\ +105c_4^2c_1^2 - 150c_2^2c_4c_1^2 + 210c_3c_5c_1^2 + 210c_2c_6c_1^2 \\ -280c_8c_1^2 - 20c_3^3c_1 + 50c_2^3c_3c_1 - 120c_2c_3c_4c_1 \\ -60c_2^2c_5c_1 + 70c_4c_5c_1 + 70c_3c_6c_1 + 70c_2c_7c_1 \\ -80c_9c_1 - 2c_2^5 + 15c_2^2c_3^2 - 10c_2c_4^2 + 5c_5^2 + 10c_2^3c_4 \\ -10c_3^2c_4 - 20c_2c_3c_5 - 10c_2^2c_6 \\ +10c_4c_6 + 10c_3c_7 + 10c_2c_8 - 10c_{10} \end{array}\right).$$

$\mathbf{ch}(\Omega_{10}^{10})$:

$$\mathrm{ch}_k = (-)^k \frac{c_1^k}{k!}.$$

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