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HIGHER ORDER ESTIMATION OF THE SOLUTION OF QUASI-INVERSE PROBLEM

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Abstract: In [1, 2] the authors established a priori estimate for the solution of quasi-inverse problem of the same order but for different weight functions. In this paper we establish a priori estimate for a higher order of the same problem, such a problem play an important role in optimal control theory.

AMS Subject Classification: 35R30
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1. Introduction

In [2, 1] the authors considered the following problem

$$\frac{\partial U_\epsilon}{\partial t} - \frac{\partial^2 U_\epsilon}{\partial x^2} - \epsilon \frac{\partial^3 U_\epsilon}{\partial x^2 \partial t} = 0, \quad (x,t) \in Q,\ t < T,$$
$$U_\epsilon(x,T) = X(x), \quad 0 \le x \le 1,$$
$$U_\epsilon(0,t) = 0,\ \int_0^1 U_\epsilon(x,t)dx = 0, \quad 0 \le t \le T. \tag{1}$$

A priori estimates for the solution of a quasi-inverse problem with different weight functions were proposed there. Here we establish a priori estimate of

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higher order for the above problem (1). We start from the following identity:

$$-\int_{Q_T}((1-x)^2+2(1-x)y)\Big(\frac{\partial U_\epsilon}{\partial t}-\frac{\partial^2 U_\epsilon}{\partial x^2}-\epsilon\frac{\partial^3 U_\epsilon}{\partial x^2\partial t}\Big)\Big(\frac{\partial^3 U_\epsilon}{\partial x^2\partial t}\Big)dxdt=0. \quad (2)$$

Integrating by parts, we establish the following equation

$$-\int_0^\tau\int_0^1((1-x)^2+2(1-x)y)\frac{\partial U_\epsilon}{\partial x}\frac{\partial^3 U_\epsilon}{\partial x\partial t}dxdt$$

$$=\int_0^\tau\int_0^1(1-x)^2\Big(\frac{\partial^2 U_\epsilon}{\partial x\partial t}\Big)^2dxdt+2\int_0^\tau\int_0^1\Big(\frac{\partial U_\epsilon}{\partial t}\Big)^2dxdt, \quad (3)$$

$$-\int_0^\tau\int_0^1((1-x)^2+2(1-x)y)\frac{\partial^2 U_\epsilon}{\partial x^2}\frac{\partial^3 U_\epsilon}{\partial x^2\partial t}dxdt$$

$$=\int_0^\tau\int_0^1(1-x)^2\frac{\partial^2 U_\epsilon}{\partial x^2}\frac{\partial^3 U_\epsilon}{\partial x^2\partial t}dxdt-2\int_0^\tau\int_0^1(1-x)y\frac{\partial^2 U_\epsilon}{\partial x^2}\frac{\partial^3 U_\epsilon}{\partial x^2\partial t}dxdt, \quad (4)$$

$$\epsilon\int_0^\tau\int_0^1((1-x)^2+2(1-x)y)\frac{\partial^3 U_\epsilon}{\partial x^2\partial t}\frac{\partial^3 U_\epsilon}{\partial x^2\partial t}dxdt$$

$$=\epsilon\int_0^\tau\int_0^1(1-x)^2\Big(\frac{\partial^3 U_\epsilon}{\partial x^2\partial t}\Big)^2dxdt+\epsilon\int_0^\tau\int_0^1\Big(y\frac{\partial^3 U_\epsilon}{\partial x^2\partial t}\Big)^2dxdt. \quad (5)$$

Thus, from (2), we have

$$\int_0^\tau\int_0^1(1-x)^2\Big(\frac{\partial^2 U_\epsilon}{\partial x\partial t}\Big)^2dxdt+2\int_0^\tau\int_0^1\Big(\frac{\partial U_\epsilon}{\partial t}\Big)^2dxdt$$

$$+\epsilon\int_0^\tau\int_0^1(1-x)^2\Big(\frac{\partial^3 U_\epsilon}{\partial x^2\partial t}\Big)^2dxdt+\epsilon\int_0^\tau\int_0^1\Big(y\frac{\partial^3 U_\epsilon}{\partial x^2\partial t}\Big)^2dxdt$$

$$=\int_0^\tau\int_0^1(1-x)^2\frac{\partial^2 U_\epsilon}{\partial x^2}\frac{\partial^3 U_\epsilon}{\partial x^2\partial t}dxdt+2\int_0^\tau\int_0^1(1-x)y\frac{\partial^2 U_\epsilon}{\partial x^2}\frac{\partial^3 U_\epsilon}{\partial x^2\partial t}dxdt. \quad (6)$$

Using theorems about geometric and arithmetic mean inequalities, we estimate the integral on the right hand side through those integrals in the left hand side,

$$\int_0^\tau\int_0^1(1-x)^2\frac{\partial^2 U_\epsilon}{\partial x^2}\frac{\partial^3 U_\epsilon}{\partial x^2\partial t}dxdt$$

$$\le\frac{\delta_1}{2}\int_0^\tau\int_0^1(1-x)^2\Big(\frac{\partial^3 U_\epsilon}{\partial x^2\partial t}\Big)^2dxdt+\frac{1}{2\delta_1}\int_0^\tau\int_0^1(1-x)^2\Big(\frac{\partial^2 U_\epsilon}{\partial x^2}\Big)^2dxdt,$$

$$2\int_0^\tau\int_0^1(1-x)y\frac{\partial^2U_\epsilon}{\partial x^2}\frac{\partial^3U_\epsilon}{\partial x^2\partial t}dxdt$$

$$\leq \delta_2\int_0^\tau\int_0^1(1-x)^2\Big(\frac{\partial^3U_\epsilon}{\partial x^2\partial t}\Big)^2dxdt+\frac{1}{\delta_2}\int_0^\tau\int_0^1\Big(y\frac{\partial^2U_\epsilon}{\partial x^2}\Big)^2dxdt.$$

Choosing $\delta_1=\frac{\epsilon}{2}$, and $\delta_2=\frac{\epsilon}{4}$ and using formula (10) from [2], we get

$$\int_0^\tau\int_0^1\Big(y\frac{\partial^2U_\epsilon}{\partial x^2}\Big)^2dxdt\leq 4\int_0^\tau\int_0^1(1-x)^2\Big(\frac{\partial^2U_\epsilon}{\partial x^2}\Big)^2dxdt,$$

and from (6) we have

$$\int_0^\tau\int_0^1(1-x)^2\Big(\frac{\partial^2U_\epsilon}{\partial x\partial t}\Big)^2dxdt+2\int_0^\tau\int_0^1\Big(\frac{\partial U_\epsilon}{\partial t}\Big)^2dxdt$$

$$+\frac{\epsilon}{2}\int_0^\tau\int_0^1(1-x)^2\Big(\frac{\partial^3U_\epsilon}{\partial x^2\partial t}\Big)^2dxdt+\epsilon\int_0^\tau\int_0^1\Big(y\frac{\partial^3U_\epsilon}{\partial x^2\partial t}\Big)^2dxdt$$

$$\leq\frac{17}{\epsilon}\int_0^\tau\int_0^1(1-x)^2\Big(\frac{\partial^2U_\epsilon}{\partial x^2}\Big)^2dxdt. \tag{7}$$

Then we use the identity

$$\int_0^\tau\Big(\frac{\partial U_\epsilon}{\partial x}\Big)^2dt=\int_0^\tau dt\int_0^t\frac{\partial}{\partial\eta}\Big(\frac{\partial U_\epsilon}{\partial x}\Big)^2d\eta+\int_0^\tau\Big(\frac{\partial U_\epsilon}{\partial x}\Big)^2\Big|_{t=0}dt.$$

We estimate the right hand side of (7) as follows:

$$\frac{17}{\epsilon}\int_0^\tau\int_0^1(1-x)^2\Big(\frac{\partial^2U_\epsilon}{\partial x^2}\Big)^2dxdt$$

$$\leq\frac{68T}{\epsilon}\int_0^\tau dt\int_0^t\int_0^1(1-x)^2\Big(\frac{\partial^3U_\epsilon}{\partial x^2\partial\eta}\Big)^2dxd\eta+\frac{34T}{\epsilon}\int_0^1(1-x)^2\Big(\frac{\partial^2U_\epsilon}{\partial x^2}\Big)^2\Big|_{t=0}dx.$$

Then using the Gronwall inequality, we get

$$\int_Q(1-x)^2\Big(\frac{\partial^2U_\epsilon}{\partial x\partial t}\Big)^2dxdt+2\int_Q\Big(\frac{\partial U_\epsilon}{\partial t}\Big)^2dxdt$$

$$+\frac{\epsilon}{2}\int_Q(1-x)^2\Big(\frac{\partial^3U_\epsilon}{\partial x^2\partial t}\Big)^2dxdt+\epsilon\int_Q\Big(y\frac{\partial^3U_\epsilon}{\partial x^2\partial t}\Big)^2dxdt$$

$$\leq\frac{34T}{\epsilon}e^{\frac{136T^2}{\epsilon^2}}\int_0^1(1-x)^2\Big(X''(x)\Big)^2dx. \tag{8}$$

Or we can rewrite our result in the following form:

$$\int_Q (1-x)^2 \Big(\frac{\partial^2 U_\epsilon}{\partial x \partial t}\Big)^2 dxdt + \frac{\epsilon}{2} \int_Q (1-x)^2 \Big(\frac{\partial^3 U_\epsilon}{\partial x^2 \partial t}\Big)^2 dxdt$$

$$\leq \frac{34\tau}{\epsilon} e^{\frac{136\tau^2}{\epsilon^2}} \int_0^1 (1-x)^2 \Big(X''(x)\Big)^2 dx.$$

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