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SOVIET PIONEERS OF FRACTIONAL CALCULUS AND ITS APPLICATIONS: II. MOSES ROZOVSKIY

Olga G. Novozhenova

Mech. Engin. Research Institute
Russian Academy of Sciences
M. Kharitonievsky Per. 4
101990 – Moscow, RUSSIA

Abstract: A short survey of the scientific activity of the famous Soviet mechanician M.I. Rozovskiy is presented, including his contributions to fractional calculus tools.

This is a second part of the historical survey [8].

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Key Words: fractional calculus, operators of the Rabotnov type, integral-differential equations, Mittag-Leffler function, aftereffect, thermal tensions

1. Introduction

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Moses Isaakovich Rozovskiy was born on October 12, 1906. In 1929 he graduated the Physical-Mathematical Faculty of the Dnepropetrovsk Institute of Public Education. The lectures of G.A. Gruzintsev who belonged to the Göttingen scientific school had great influence on Rozovskiy. It should be noted that in the 1930s several professors of the Moscow State University, in particular, the future academician A.N. Kolmogorov read lectures in Dnepropetrovsk.

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Fig. 1: M.I. Rozovskiy (1906–1994)

In 1940 M.I. Rozovskiy got Ph.D. degree. After the end of 1958 he became Doctor of physical-mathematical sciences [21, 25]. In 1945-1989 he worked in the Dnepropetrovsk Mining Institute where he held the position of the Head of the Department of Higher Mathematics.

The first works of Rozovskiy were devoted to the problems of electromagnetic fields described with integral and integral-differential equations [11-12, 17-18]. Since 1946 he started to investigate some temperature problems and the theory of viscoelasticity following the publications of A.N. Gerasimov [13]. Probably they met on May 29, 1947 at the Seminar in the Institute of Mechanics of the USSR Academy of Sciences where Rozovskiy made a report. Academician S.L. Sobolev presented Rozovskiy's papers in print [5, 15].

M.I. Rozovskiy only once has referred to the work of Gerasimov [15]. His pupil I. Krush wrote [7]: "Classical methods for solving the problems of the theory of heredity had received intensive development in the works of V. Volterra, N.H. Arutyunyan, A.N. Gerasimov, A.Yu. Ishlinskiy, G. Myunts, I.E. Prokopovich, Yu.N. Rabotnov, A.R. Rzhanitsyn and M.I. Rozovskiy".

After getting the long-awaited Doctorate degree [21, 25, 29], Moses Isaakovich has no longer demonstrated, as before, his scientific superiority [14, 17] and has been mainly engaged in problems using the operators of Yu.N. Rabotnov's type.

The first found papers [11-12, 18] of Rozovskiy were devoted to the problems of electromagnetic elasticity. Because of the dielectric and magnetic viscosity there is a delay in the manifestation of the electromagnetic properties of the medium. The relations obtained in [11] for solving the integral-differential equation have been used by Rozovskiy while he has treated deformation of real bodies [13, 17].

Solving the problem on the vibrations of a viscoelastic rod, Rozovskiy was given [19] criticism by A.Yu. Ishlinskiy's, A.R. Rzhanitsyn's and A.N. Gerasimov's papers and then he obtained the analytical solution of the considered problem. The problem for a beam supported with an elastic-viscous base had been also solved. In [15] the deformation of an extended uniform cylindrical body in the case of tension with aftereffect and not uniform distribution of an increasing temperature was considered. In Rozovskiy's paper [16], referred by Yu.N. Rabotnov [9], the solution of the problem on forced transverse vibrations of a thread, on radial oscillations of a hollow sphere and of an isotropic membrane was given but without any references to Gerasimov's results. In [18] Rozovskiy proved that the results of Ya.I. Frenkel and his pupil A.I. Gubanov were only special cases of his relations obtained for stretching and longitudinal oscillations of a prismatic rod, torsional vibrations of a cylindrical rod, the results on stability of the transverse vibrations of a rod and also on denting of a cylinder into the horizontal support.

The end impact of a hard right circular cylinder with a surface of medium which mechanical properties changed in time was investigated in [19]. Based on the analysis of the creep curves and of the equations used in the problems with creep and relaxation (I.A. Oding, Yu.N. Rabotnov), Rozovskiy proposed [20] the integral equation which took into account the dependence of the relaxation time on tension. It was used to treat the kinetics of crack propagation in various materials. In [22] the radial deformation of a hollow sphere having anisotropy and elastic aftereffect had been specified. For the case of spatial tension state a generalization of the nonlinear integral dependencies of the elastic module on the degree of deformation or of the relaxation time on tension was obtained in [23]. The possibility of constructing more general equations than the equations in Rabotnov's creep theory was found.

The local disturbances in the half-plane under the action of a constrained force were specified. The deformation of a hollow sphere under internal and external pressure had been considered.

In [24] a semi-symbolic method of solving the integral-differential equations of hereditary elasticity, which did not require the prior introduction of special functions, has been discovered.

In [10], p. 113, Yu. Rossikhin and M. Shitikova wrote: "... in 1966 Rabotnov was the first to show in his book ... [Yu.N. Rabotnov, *Creep of Structural Elements.* Nauka, Moscow (1966), Engl. transl. by North-Holland, Amsterdam in 1969)] the connection between the fractional exponential function (9) (denoted below as $\epsilon_\beta(t)$, eh-function) and the Mittag-Leffler function (see equation (30.3) in the cited book])". However, *this assertion is erroneous.* In fact, the relationship between eh-functions of Rabotnov and the Mittag-Leffler function was established by M.I. Rozovskiy in 1958, [27].Moreover, a year earlier he found the relation between Mittag-Leffler's function and more general class of functions introduced in [26]. Note this more precisely.

In 1957, while solving the Cauchy problem for an integral-differential equation with partial derivations in an unbounded space, Rozovskiy used [26] an exponential function of fractional order derivative $\epsilon_\beta(t)$ which can be expressed approximately in terms of the Mittag-Leffler function $E_\beta(t)$, namely,

$$\frac{d\epsilon_\beta(t)}{dt} \approx -q^2 E_\beta(kt^\beta)\,.$$

A year later, using eh-Rabotnov's function to describe creep and relaxation of an extended hollow cylinder under its rapid uniform rotation about the axis Rozovskiy proved [27] the following formula

$$(eh)^*_\alpha(-k) = \frac{1}{k}\left[1 - E_{1-\alpha}(-kt^{1-\alpha})\right]\,.$$

Here

$$E_\mu(-\zeta) = \sum_{n=0}^{\infty}(-1)^n \frac{\zeta^n}{\Gamma(n\mu+1)}$$

is the Mittag-Leffler function. If $\zeta > 1$, the following approximation

$$E_{1-\alpha}(-\zeta) \approx e^{-\gamma\zeta}, \ \ \gamma = (1-\alpha)^{1-\alpha}$$

is true.

The problem of torsion of a cylinder at large angles of twist staged by Ishlinskiy was considered in eh-functions of Rabotnov basing on [23]. The approximation of action of the operator $(eh)^*_\alpha$ onto the unit was given. The computations for the aluminum alloy and mild steel were given. The way to specify parameters of creep and relaxation with the help of experimental curves of simple aftereffect was proposed in [29]. In [3o] the properties of aggregates, i.e. of the operators $(eh)^*_\alpha$ were established for the case of inhomogeneous and anisotropic bodies. The problem on torsion of a composite shaft was solved.

In the subsequent papers [1-2, 6, 31-39] Rozovskiy was using the operators of Yu.N. Rabotnov widely from illustrations of the new theorem for complex tension state of an anisotropic body up to the theoretical questions on differently-creep and propagation of the deformation waves in nonlinear hereditary-elastic medium.

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