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A NEW MODEL OF HARVESTING FISH POPULATION

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Abstract: In this paper we propose a new model of harvesting fish population. This model is particularly applicable to those types of fish population which reproduce at a particular time of the year and this cycles continue throughout the life of the fish population. Many of the species including many types of fish follow such cyclic behaviour of reproduction. The model assumes the minimum and the maximum fraction of harvesting and guarantee the survival of the species and at the same time optimal harvesting. The model is analyzed both graphically and algebraically.

AMS Subject Classification: 34H05, 49J15
Key Words: fish harvesting, modelling fish population, applications of ordinary differential equations

1. Introduction

In a recent United Nation survey, it is pointed out that more than 200 million people depends on fishing as their source of food and income [1]. However, uncontrolled fishing practice can threaten the security of marine life and can

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have a drastic effect on the ecosystem. For example, the collapse of the cod fish in Newfoundland in the early 1900 is an example of such dramatic effect of the over fishing [5]. It is important to balance between the economic needs and ecological consideration. There are various mathematical models that was established for this purposes (see [2], [3], [4] and [6]). For a more comprehensive study, please see [7]. In particular, the well studied harvesting model known as "Schaefer model" can be found in [8]. None of the models mentioned above deals directly with those particular fish population which reproduce at a certain time of the year and that time is typically fixed throughout their life cycle. In this research paper, we propose a model of fish harvesting that will best fit the particular species of fish that obey this particular behaviour of reproduction mentioned before.

2. Some Preliminary Assumption and Notation

We will assume that the particular fish population will reproduce in the month of t_0 every year throughout their life cycle. So $1 \leq t_0 \leq 12$. This t_0 is fixed for a particular fish but may be different depending on different fish population. The unit for time will be month. Let us denote by $y(t)$ the amount of fish at any moment of time. We assume the maximum harvest will never exceed above $My(t)$ and never decreased below $my(t)$, where $0 < m < M < 1$, are some predetermined numbers. These numbers may be determined based on the birth and death rate, the rate of the consumption of this fish population by some other predatory population beside human, condition of the local habitat where the fish is populated, etc. In general, the logistic model of harvesting takes the following form:

$$\frac{dy}{dt} = r\left(1 - \frac{y(t)}{k}\right) y(t) - H(t, y(t)).$$

Here k is the carrying capacity and $H(t, y)$ is the harvesting factor. In our case, we let $H(t, y(t)) = h(t)y(t)$ and we are looking for function $h(t)$ which is always between m and M, $m \leq h(t) \leq M$, and takes the minimum value in the month of breeding, that is $h(t_0) = m$ and gradually oscillate through out the year between m and M.

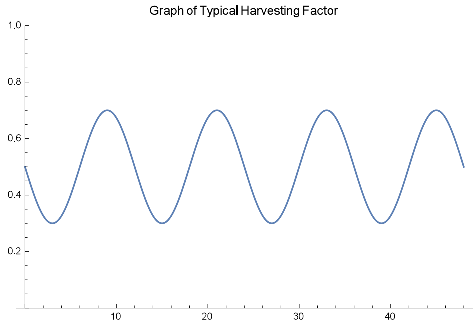

Figure 1: Graph of $h(t)$

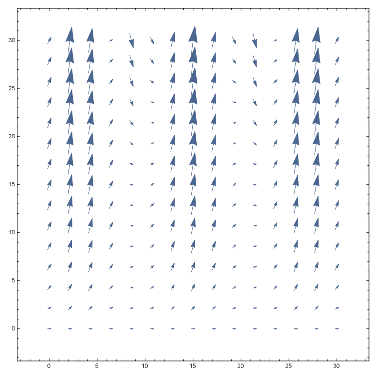

Figure 2: Directional field of our Model for $r = 0.8$ and $k = 100$

3. The Function $h(t)$

We proposed $h(t)$ to be the following function:

$$h(t) = \frac{(M-m)}{2} \cos\left(\frac{\pi}{2}(t+6-t_0)\right) + \frac{(M+m)}{2}.$$

This function $h(t)$ satisfies all the conditions mentioned in the previous section. Please see Figure 1 for a typical plot of $h(t)$ for $M = 0.7, m = 0.3$ and $t_0 = 3$.

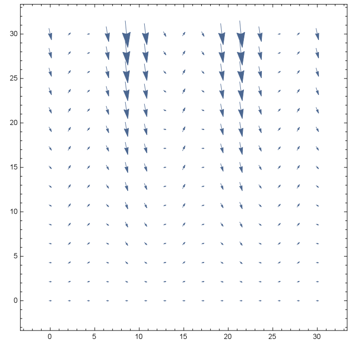

Figure 3: Directional field of our Model for $r = 0.5$ and $k = 100$

Out[21]=
$$\left\{\left\{y[t] \to e^{-\frac{m t}{2}-\frac{M t}{2}+r t+\frac{3 m \operatorname{Sin}\left[\frac{\pi t}{6}-\frac{1}{6}\pi(-6+t0)\right]}{\pi}-\frac{3 M \operatorname{Sin}\left[\frac{\pi t}{6}-\frac{1}{6}\pi(-6+t0)\right]}{\pi}} \Big/ \left(C[1]-\int_1^t -\frac{1}{k} e^{-\frac{1}{2} m K[1]-\frac{1}{2} M K[1]+r K[1]-\frac{3 m \operatorname{Cos}\left[\frac{1}{6}\pi K[1]\right] \operatorname{Sin}\left[\frac{1}{6}\pi(-6+t0)\right]}{\pi}+\frac{3 M \operatorname{Cos}\left[\frac{1}{6}\pi K[1]\right] \operatorname{Sin}\left[\frac{1}{6}\pi(-6+t0)\right]}{\pi}+\frac{3 m \operatorname{Cos}\left[\frac{1}{6}\pi(-6+t0)\right] \operatorname{Sin}\left[\frac{1}{6}\pi K[1]\right]}{\pi}-\frac{3 M \operatorname{Cos}\left[\frac{1}{6}\pi(-6+t0)\right] \operatorname{Sin}\left[\frac{1}{6}\pi K[1]\right]}{\pi}} r \, dK[1]\right)\right\}\right\}$$

Figure 4: Solution of our Model written in terms of integral

4. Our Proposed Model

Now the model that we are proposing takes the following format:

$$\frac{dy}{dt} = r\left(1 - \frac{y(t)}{k}\right) y(t) - h(t)y(t),$$

where

$$h(t) = \frac{(M-m)}{2} \cos\left(\frac{\pi}{2}(t+6-t_0)\right) + \frac{(M+m)}{2}.$$

So combining them together, it gives the final result:

$$\frac{dy}{dt} = r\left(1 - \frac{y(t)}{k}\right) y(t) - \left[\frac{(M-m)}{2} \cos\left(\frac{\pi}{2}(t+6-t_0)\right) + \frac{(M+m)}{2}\right] y(t).$$

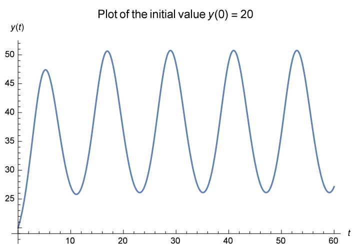

Figure 5: Graph of the solution of the initial value $y(0) = 20$

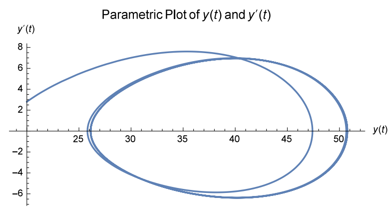

Figure 6: Graph of the corresponding parametric plot of $(y(t), y'(t))$

5. Analysis of the Model

We use "Mathematica" software to algebraically and numerically analyze our model. First take a look at the directional field of our model in Figure 2. Here we choose $r = 0.8$ and $k = 100$. In Figure 3, we choose $r = 0.8$ and $k = 100$. We can clearly see the oscillating behaviour of $y(t)$. This guarantee that the fish will never be extinct and in fact flourish through out the year (of course, assuming all the other variables stay inside their limited boundary). Although we cannot find the solution in closed format, it can be written using integral and this can be used to numerically approximate the solution. See Figure 4 for the explicit solution written in terms of integral.

Finally Figure 5 shows what a typical solution curve looks like (in this case for the initial condition $y(0) = 20$) and Figure 6 shows the corresponding parametric plot of $(y(t), y'(t))$.

6. Conclusion

The function $h(t)$ that is considered in this paper is not unique. There are various other functions which behave very similarly. Moreover, the $h(t)$ can be changed or modified to fit the reproductive behaviour of particular fish. For example, some species of fish may reproduce every third year instead of every year. In that case, we change the domain of t_0 as $1 \leq t_0 \leq 36$ and replace the number 6 inside the definition of $h(t)$ by the number $\frac{36}{2} = 18$.

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