CONTROL OF INTERFACE PATTERNING
IN SOFT THIN FILMS

Emily M. Tian

Department of Mathematics and Statistics
Wright State University
Dayton, OH 45435, USA

Abstract: A global feedback control applied on thin film surface pattering induced by the electric field is studied theoretically near the threshold of instability. The growth of the interfacial disturbance between air and polymer film on the unbounded spatial domain is described by the thin film equation, a single nonlinear equation incorporating the competition between surface tension and electric force; A feedback control is applied to suppress the subcritical instability, thus stabilize the stationary patterns developed on the interface.

1. Introduction

In the modern lithographically induced self-assembly process, the electrohydrodynamic instability plays a critical role in guiding the creation and replication of lateral structures in polymer films at submicrometer length scales [1, 9, 12, 13]. The spatially infinite system near the homogeneous equilibrium may be studied by employing simple equations known as amplitude equations [2], which describe the temporal development of the system. We are interested in studying the reduced dynamics contained in these amplitude equations near the instability threshold. The generating and controlling of the regular spatial

patterns can be obtained by determining the stability conditions of these ampli-
tude functions, subjecting to the weakly nonlinearity embedded in the system.

At the long wavelength approximation, a single highly nonlinear, non-
dimensionalized thin film equation is formulated [11] in terms of the local thick-
ness of the film:

$$H_t + \alpha \nabla \cdot [H^3 \nabla (\nabla^2 H)] + \beta \nabla \cdot [H^3 (\epsilon_p \xi + \epsilon_p - (\epsilon_p - 1)H)^{-3} \nabla H] = 0. \quad (1)$$

In the equation (1), H is the non-dimensional thickness of the thin film layer;
$\alpha > 0$ evaluates the surface tension strength; $\beta > 0$ represents the strength of
destabilizing electric field; ξ is the thickness ration that utterly determines the
interfacial morphology [11].

The initial layer of uniform thin film is unconditionally unstable; the system
becomes unstable to infinitesimal perturbations. In a region of ξ values, the
initial exponential growth is incorporated with weakly nonlinear effects when
the amplitudes of linear waves saturate to a finite value near the critical wave
number k_c. The weakly nonlinear selection mechanism indicates that the peri-
odic finite-amplitude disturbances can entrain waves in a neighborhood of the
critical wave number; a field of uniform wave features emerge as a consequence
of this unique selection. A periodic pattern stationary in time centered around
k_c is to arise.The regular finite-amplitude patterns stay on top of the inter-
face in a stationary manner, without visible changes in their shape or speed
for a long time. The regular patterns undergo a subcritical instability [10, 11].
After a long time, when the waves grow close to the top electrode substrate,
the intermolecular forces start to affect the interface morphology, that cause
the neighboring waves to coalesce, creating patches of defects in the otherwise
uniform field of waves.

Much work was done to explore the possibility to control and suppress the
blow up behavior caused by subcritical oscillatory instability, by introducing
the control mechanism either locally [6] or globally [3, 4, 5, 8]. In particular,
the global feedback control is introduced through a single global parameter that
affects the dynamics of the entire system, it can stabilize the unstable modes
and produce noval patterns [7]. Due to the fact that the selection and long
term stability of patterns is associated with growing amplitudes, we choose a
control based on the measurements of the maximum of amplitude. We vary
the control parameter that determines the whole system dynamics to keep the
system within the weakly nonlinear regime. In consideration of the possible
cause by a surface tension varying inversely with the strength of the external
electric field, we would impose the global feedback control on the coefficient α
in equation (1).

2. Global Feedback Control

Consider a small perturbation f near the initial homogeneous layer $H - 1 \sim f$. We assume f in the oscillatory vibration form:

$$f(x, t) = A(t) \cdot \cos(kx - \phi(t)), \tag{2}$$

where the magnitude of amplitude function $|A| \ll 1$, and $\phi(t)$ is the phase variation of the wave number k.

We apply the feedback control to the surface tension coefficient α by measuring the maximum value of the patterns amplitude:

$$\alpha(f) = \alpha_0 - p \cdot \max_x(|f|), \tag{3}$$

in which the control parameter $p > 0$ specifies the feedback intensity.

By substituting the control $\alpha(f)$ into the model equation (1), the perturbation f can be approximated by the powers of its amplitude:

$$\frac{dA}{dt} = (\beta(\epsilon_p \xi + 1)^{-3} k^2 - \alpha_0 k^4) A + (2pk^4) A^2 - (3\alpha_0 k^4) A^3. \tag{4}$$

We introduce σ to represent the linear growth rate of f:

$$\sigma = \beta(\epsilon_p \xi + 1)^{-3} k^2 - \alpha_0 k^4.$$

The control parameter p is contained in the quadratic coefficient to account for the reflection symmetry breaking:

$$\mu = 2pk^4.$$

If μ is nonzero, namely, when the control p is present, the quadratic term in the amplitude equation dominates the nonlinear influence over the disturbance f near the instability threshold. In the light of weakly nonlinear interaction, μ is of order less than unity, so is the control parameter p.

The amplitude $A(t)$ has a long term stationary solution when $\mu \neq 0$:

$$A_e = -\frac{\sigma}{\mu}$$

in the subcritical regime where $\sigma < 0$.

A perturbation around A_e is performed in order to determine the stability conditions of this controlled equilibrium:

$$A = A_e + \widetilde{A},$$

with
$$\widetilde{A} = e^{\gamma t}.$$

Introducing both of them into equation (4), we find that:

$$\gamma = -\sigma(1 + A_e^2) - \frac{9\alpha_0}{4p^2 k^4}\sigma^2,$$

which results in the stability condition:

$$\sigma < -\frac{1 + A_e^2}{9\alpha_0 k^4}\mu^2. \tag{5}$$

The condition (5) ensures $\gamma < 0$, that suppress the growth of $\widetilde{A}$ and stabilize the equilibrium state.

3. Conclusion

We have performed the weakly nonlinear analysis near the threshold of the instability to investigate the effect of applying the global feedback control to stabilize the potential unstable stationary steady state. The stability condition (5) is obtained, which suggests that, applying the control p can stabilize the long term stationary state A_e when the subcritical stability exists in the system. The greater the value of p is, the faster the disturbance $\widetilde{A}$ approaches to zero.

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