

**ON A MATHEMATICAL INVESTIGATION OF  
A SEDIMENTATION MODEL OF LAKES**

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**Abstract:** The study of the influence of sediments brought on the sedimentary bottom formation and evolution of rivers or lakes presents a big scientific interest both by its interdisciplinary aspect and by its importance. In this work, we propose to associate the hydrodynamic and geological points with a view to establish a continuous model of the sedimentary bottom evolution processes based on conservation mass law. This model, general and progressive, make it possible to consider the flocculation phenomenon, geological activities of the bottom, and other. We establish later on the conditions of existence of admissible solutions where sedimentation is more important than the erosion process.

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## 1. Introduction

Water through different basins (rivers, lakes, aquifers, and so on) has always

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been for centuries a socio-economic development issue. In the context of climate change, several rivers are disappearing from their bed, leading to a short, medium and long terms drying process of great lakes and rivers that once were important water systems. Also, modelling and numerical simulation of sedimentation process of these constitutes a real challenge in the forthcoming years.

Despite this real scientific challenge, very few papers devoted to rivers and lakes sedimentation phenomena in particular are reported in the literature. Among the most important papers one can refer to [3, 4, 7, 9, 10]. Unfortunately, in most of these articles, the study of the dynamics of lakes is only addressed under the hydrodynamic point of view, neglecting the geological activity of the bottom. To understand a process of lakes drying up in the long term, hydrodynamic models are not satisfactory.

We propose here a model described by means of a system of partial differential equations, governing the evolution of the sedimentary bottom of a lake which explains its drying process. We make then a qualitative analysis of the proposed model.

## 2. Formulation of the model

Let  $\Omega$  be a bounded domain of  $\mathbb{R}^2$  representing the basis of a sedimentary basin at the zero topographic level which we assume to be horizontal. Its boundary will be denoted by  $\partial\Omega$ . We design by  $S(t, x, y)$  the sedimentary bottom height at the point  $(x, y)$  at time  $t$ . We will sometimes denote by  $S(t)$  a function defined by  $S(t)(x, y) = S(t, x, y)$ .

To describe the sedimentary bottom evolution process, we formulate following assumptions:

- the sedimentary bottom height variation, consequence of diffusive flux, is proportional to the directional variations of the granulometric gradients;
- the variation of the bottom height due to convective flux is consequence of a gravitational effect according to the sedimentary bottom slope.

Considering the mass conservation law on sediments flux during the erosion, sedimentation phenomena or sediments transport, the fundamental equation can be written as follows

$$\frac{\partial S}{\partial t} = a \frac{\partial^2 S}{\partial x^2} + b \frac{\partial^2 S}{\partial y^2} + \left( \left| \frac{\partial S}{\partial x} \right|^p + \left| \frac{\partial S}{\partial y} \right|^p \right) \left( \lambda \frac{\partial S}{\partial x} + \alpha \frac{\partial S}{\partial y} \right) \quad \text{on } ]t_0, +\infty[ \times \Omega, \quad (1)$$

where the terms  $a = a(x, y), b = b(x, y)$  indicate the diffusion coefficients following respectively  $x$  and  $y$  directions,  $\lambda = \lambda(x, y)$ , and  $\alpha = \alpha(x, y)$  are the convection coefficients following the  $x$  and  $y$  directions,  $P$  indicates the relief slope. We shall set

$$F(S) = - \left( \left| \frac{\partial S}{\partial x} \right|^p + \left| \frac{\partial S}{\partial y} \right|^p \right) \left( \lambda \frac{\partial S}{\partial x} + \alpha \frac{\partial S}{\partial y} \right) + \frac{\partial a}{\partial x} \frac{\partial S}{\partial x} + \frac{\partial b}{\partial y} \tag{2}$$

and  $A = \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}$  so that the equation (1) can be rewritten as

$$\frac{\partial S}{\partial t} = \text{div}(A \nabla S) - F(S) \text{ on } ]t_0, +\infty[ \times \Omega. \tag{3}$$

Observe that the term  $F(S)$  well reflects the nonlinear and convective phenomenon complex character. Such a term has already been considered in a model closed to our elaborated in [2],[5] for the study of sedimentary basin evolution of multilithologic strates under constraint of a maximum erosion rate. But this one does not take into account the convective flux by the slip phenomena. The initial and boundary conditions accompanying the equation (3) are given by

$$A \nabla S \cdot \mathbf{n} = g \text{ on } ]t_0, +\infty[ \times \partial \Omega, \tag{4}$$

$$S(t_0) = S_0 \text{ on } \Omega. \tag{5}$$

where  $g$  and  $S_0$  are given functions and, where  $\mathbf{n}$  denotes the outward normal vector to the boundary  $\partial \Omega$ .

In the sedimentation-erosion phenomenon, the phase of unconsolidated sediments production by the climatic and biological attack precedes their transport by gravitating erosion [1]. Thus the sedimentary bottom evolves if the sediments deposited rate remains higher than an erosion rate  $-E \geq 0$  [6]. Therefore the so-called sedimentation constraint can be expressed as follows

$$\frac{\partial S}{\partial t} + E \geq 0 \text{ on } ]t_0, +\infty[ \times \Omega. \tag{6}$$

In particular if  $\frac{\partial S}{\partial t} + E > 0$ , the maximum sedimentation rate is not reached, and thus the flux is not limited. Otherwise if in  $(t, x, y), \frac{\partial S}{\partial t} + E = 0$ , the maximum sedimentation rate is reached, the flux is then limited. The constraint (6) thus makes it possible to give a condition being able to lead to the drying up of lakes

in shorter or longer term. Then we agree to call a physically admissible solution any couple  $(\Lambda, S)$  where we have set  $\Lambda = (a, b, \lambda, \alpha)$  and, where the associate function  $S$  satisfies the equations system (3)-(6). Furthermore we shall set

$$\mathcal{E} = \{(\Lambda, S) \in (L^\infty(\Omega))^4 \times C^1(]t_0, +\infty[; H^1(\Omega)), (\Lambda, S) \text{ is admissible.}\} \quad (7)$$

One can easily see that this set is non-empty. To be convinced itself, it is enough to see that the couple  $(S, \Lambda)$ , with  $\Lambda = (1, 1, 1, 1)$  and where  $S(t, x, y) = \frac{1}{2}(x + y)E^2 + \frac{t \times E^{2(p+1)}}{2^{p-1}}$  is a physically admissible solution.

On the other hand, as in [5], one can introduce the concept of maximum solutions in such a way that a couple  $(\Lambda_{max}, S_{max}) \in \mathcal{E}$  is said to be a maximum solution if for all  $(\Lambda, S) \in \mathcal{E}$ ,

$$\|\Lambda\|_{(L^\infty(\Omega))^4} \leq \|\Lambda_{max}\|_{(L^\infty(\Omega))^4}.$$

Then analysis of physically admissible solutions can be brought back to the study of the two following problems:

$(P_1)$  : Given  $\Lambda \in (L^\infty(\Omega))^4$ , find  $S$  satisfying the equations (3)-(5).

$(P_2)$  : Find a necessary condition on  $\Lambda \in (L^\infty(\Omega))^4$  for which the couple  $(\Lambda, S)$  of solution of (3)-(5) is physically acceptable.

### 3. Existence study of admissible solutions

We investigate here on the existence and the uniqueness of solutions of the system (3)-(5) as formulated by the problem  $(P_1)$ . We have:

**Proposition 3.1.** *Suppose  $S_0 \in H^1(\Omega)$ . Under the following hypotheses:*

- i) the functions  $F$  and  $g$  are of class  $C^1$  relatively to each of their arguments,*
- ii)  $F$  is increasing,*
- iii) the matrix  $A$  is positive definite, then there is a sequence  $(S^k(t))_{k \in \mathbb{N}} \subset H^1(\Omega)$  for  $i = 1, \dots, k$  such that:*

$$\frac{d}{dt} \int_{\Omega} S^k(t) \phi_i + \int_{\Omega} A \nabla S^k(t) \cdot \nabla \phi_i = \int_{\partial\Omega} g \phi_i - \int_{\Omega} F(S^k(t)) \cdot \phi_i, \quad (8)$$

$$\int_{\Omega} S^k(t_0) \phi_i = \int_{\Omega} S_0 \phi_i. \quad (9)$$

To prove this proposition we need the following.

**Remark 3.1.** From (2) we notice that  $F(0) = 0$ . Thus, as  $F$  is increasing then, it is easy to establish that:

1.  $F(v).v \geq 0 \quad \forall v,$
2.  $(F(v_1) - F(v_2)).(v_1 - v_2) \geq 0 \quad \forall v_1, v_2.$

We now prove Proposition 3.1.

*Proof.* Let  $(\phi_i)_{i \in \mathbb{N}}$  be a total family of  $H^1(\Omega)$  satisfying  $\int_{\Omega} \phi_i \phi_j = \delta_{ij}$ . Then, we look for a solution of (8)-(9) under the form  $S^k(t) = \sum_{i=1}^k S_{ki}(t)\phi_i$ . From

(8)-(9) it follows

$$\left\{ \begin{array}{l} \frac{dS_{ki}(t)}{dt} + \sum_{i=1}^k S_{ki}(t)a(\phi_i, \phi_j) = \int_{\partial\Omega} g\phi_j - \int_{\Omega} F\left(\sum_{i=1}^k S_{ki}\phi_i\right)\phi_j \\ S_{kj}(t_0) = \int_{\Omega} S_0\phi_j \end{array} \right., \quad (10)$$

where we have set  $a(\phi_i, \phi_j) = \int_{\Omega} A\nabla\phi_i.\nabla\phi_j$ . Since functions  $F$  and  $g$  are of class  $C^1$ , the application which associates the vector of components  $(S_{ki})$  to the second member of the differential system (10) is also of class  $C^1$ . Therefore they are locally Lipschitz. Thanks to the Cauchy-Lipschitz theorem, the system (10) admits a unique solution  $S_{ki}(t)$ . Thus there exists a unique sequence  $(S^k(t)) \in H^1(\Omega)$  which satisfies the differential Cauchy system (8)-(9).  $\square$

**Proposition 3.2.** Under hypothesis of Proposition 3.1, there exists a positive constant  $C$  such that:

$$\|\nabla S^k\|_{L^2(t_0, +\infty; L^2(\Omega))}^2 \leq C. \left( \|S_0\|_{L^2(\Omega)}^2 + \|g\|_{L^2(t_0 + \infty; L^2(\partial\Omega))}^2 \right), \quad (11)$$

where  $S^k(t) \in H^1(\Omega)$  is solution of the system (8)-(9) for  $S_0 \in H^1(\Omega)$ .

*Proof.* Since  $S^k(t) \in H^1(\Omega)$ , then considering the total family  $(\phi_i)_i$ , we can write  $S^k(t) = \sum_{i=1}^k S_{ki}(t)\phi_i$ . Multiplying equation (8) by each term  $S_{ki}(t)$ , it follows

$$\frac{1}{2} \frac{d}{dt} \int_{\Omega} (S^k(t))^2 + a(S^k(t), S^k(t)) + \int_{\Omega} F(S^k(t)).S^k(t) = \int_{\partial\Omega} g.S^k(t). \quad (12)$$

According to the positivity of  $a(.,.)$  and if  $F(.)$  is increasing function, there exists a constant  $\alpha > 0$  such that:

$$\frac{d}{dt} \|S^k(t)\|_{L^2(\Omega)}^2 + \alpha \int_{\Omega} |\nabla S^k(t)|^2 \leq 2 \int_{\partial\Omega} g.S^k(t).$$

And by the Cauchy-Schwartz and Young inequalities we can deduct:

$$\frac{d}{dt} \|S^k(t)\|_{L^2(\Omega)}^2 + \alpha \int_{\Omega} |\nabla S^k(t)|^2 \leq \|g\|_{L^2(\partial\Omega)}^2 + \|S^k(t)\|_{L^2(\Omega)}^2. \tag{13}$$

Integrating this inequality over the interval  $[t_0, T]$  for any constant  $T > t_0$  with respect to the variable  $t$ , we obtain

$$\begin{aligned} \|S^k(T)\|_{L^2(\Omega)}^2 + \alpha \int_{t_0}^T \|\nabla S^k(t)\|_{L^2(\Omega)}^2 &\leq \|S_0^k\|_{L^2(\Omega)}^2 + \int_{t_0}^T \|g\|_{L^2(\partial\Omega)}^2 \\ &+ \int_{t_0}^T \|S^k(t)\|_{L^2(\Omega)}^2. \end{aligned} \tag{14}$$

Note that from (9) it is easy to see that  $\|S_0^k\|_{L^2(\Omega)}^2 \leq \|S_0\|_{L^2(\Omega)}^2$  and knowing that

$$\|S^k(T)\|_{L^2(\Omega)}^2 \leq \int_{t_0}^T \|S^k(t)\|_{L^2(\Omega)}^2,$$

from the inequality (14) its follows that

$$\alpha \int_{t_0}^T \|\nabla S^k(t)\|_{L^2(\Omega)}^2 \leq \|S_0\|_{L^2(\Omega)}^2 + \int_{t_0}^T \|g\|_{L^2(\partial\Omega)}^2 \quad \forall T > t_0.$$

Consequently, we finally obtain

$$\int_{t_0}^{+\infty} \|\nabla S^k(t)\|_{L^2(\Omega)}^2 \leq C. \left( \|S_0\|_{L^2(\Omega)}^2 + \int_{t_0}^{+\infty} \|g\|_{L^2(\partial\Omega)}^2 \right).$$

□

**Proposition 3.3.** *Let  $S_0 \in H^1(\Omega)$ . Assuming that hypotheses of the proposition 3.1 hold, if  $S^k(t) \in H^1(\Omega)$  is solution of the system (8)-(9), then*

$$\|S^k(t)\|_{L^2(\Omega)}^2 \leq \|S_0\|_{L^2(\Omega)}^2 + \|g\|_{L^2(t_0, +\infty; L^2(\partial\Omega))}^2. \tag{15}$$

*Proof.* One can easily see that, as in Proposition 3.2, equation (12) yields

$$\frac{1}{2} \frac{d}{dt} \|S^k(t)\|_{L^2(\Omega)}^2 \leq \int_{\partial\Omega} g \cdot S^k(t) \leq \|g\|_{L^2(\partial\Omega)} \cdot \|S^k(t)\|_{L^2(\Omega)}.$$

This gives

$$\frac{d}{dt} \|S^k(t)\|_{L^2(\Omega)} \leq \|g\|_{L^2(\partial\Omega)}$$

and thus, by integrating relatively to  $t$ , we obtain for any  $T > t_0$ ,

$$\|S^k(t)\|_{L^2(\Omega)} \leq \|S_0\|_{L^2(\Omega)} + \int_{t_0}^T \|g\|_{L^2(\partial\Omega)}.$$

The proposition is then proved. □

We are now in the position to prove the existence and uniqueness result.

**Theorem 3.2.** *Under the hypotheses of the proposition 3.1, if  $S_0 \in H^1(\Omega)$ , then there exists  $S \in L^2(t_0, \infty; H^1(\Omega))$  a unique weak solution of (3)-(5), that is:*

$$\int_{\Omega} \frac{d}{dt} S(t)\phi + \int_{\Omega} A \nabla S(t) \cdot \nabla \phi = \int_{\partial\Omega} g\phi - \int_{\Omega} F(S(t)) \cdot \phi \tag{16}$$

$$\int_{\Omega} S(t_0)\phi = \int_{\Omega} S_0\phi \tag{17}$$

for all  $\phi \in H^1(\Omega)$ .

*Proof.* The proof of this theorem will be done in two steps. In the first step we have to prove the existence and in the second ones the uniqueness of solutions.

**Step 1: The existence.**

First, according to Proposition 3.1, one can construct a sequence  $(S^k(t)) \subset H^1(\Omega)$  satisfying (8)-(9) which is defined by  $S^k(t) = \sum_{i=1}^k S_{ki}\phi_i$ ,  $(\phi_i)_i$  being a total family of  $H^1(\Omega)$ . From Propositions 3.2 and 3.3 it is easy to see that there exists a positive constant  $C$  such that

$$\|S^k\|_{L^2(t_0, +\infty; H^1(\Omega))} \leq C.$$

The sequence  $(S^k)_k$  is then bounded in  $L^2(t_0, +\infty; H^1(\Omega))$ . Consequently there exists a sub-sequence also denoted by  $(S^k)_k$  in  $L^2(t_0, T; H^1(\Omega))$  such that

$$S^k \rightharpoonup S \text{ weakly in } L^2(t_0, \infty; H^1(\Omega)). \tag{18}$$

Hence, knowing that the injection of  $H^1(\Omega)$  in  $L^2(\Omega)$  is compact, it follows

$$S^k \longrightarrow S \text{ strongly in } L^2(t_0, \infty; L^2(\Omega)). \tag{19}$$

Secondly, we have to prove that  $S \in L^2(t_0, \infty; L^2(\Omega))$  satisfies the equation (16). We notice that for any  $\phi \in H^1(\Omega)$ , one can write  $\phi = \lim_{k \rightarrow +\infty} \sum_{i=1}^k \mu_i \phi_i$ .

Since  $S^k$  satisfies equation (8), one obtains

$$\begin{aligned} \int_{\Omega} \frac{dS^k}{dt} \left( \sum_{i=1}^k \mu_i \phi_i \right) + a \left( S^k, \sum_{i=1}^k \mu_i \phi_i \right) + \int_{\Omega} F(S^k) \left( \sum_{i=1}^k \mu_i \phi_i \right) \\ = \int_{\partial\Omega} g \left( \sum_{i=1}^k \mu_i \phi_i \right) \end{aligned} \tag{20}$$

then, thanks to (18) and (19), and passing to the limit to the equation (20), we finally deduce that  $S$  satisfies the equation (16). This is what we have to prove.

Thirdly, to achieve the prove of the existence we have to show that the solution  $S$  satisfies  $S(t_0) = S_0$ .

Let us consider  $\psi \in C^1(t_0, T; \mathcal{D}(\Omega))$  which satisfies  $\psi(T) = 0$ . Knowing that the family  $(\phi_i)_i$  is total in  $H^1(\Omega)$ , we can write

$$\psi(t) = \lim_k \sum_{i=1}^k \psi_i(t) \phi_i$$

and, setting  $\psi^k(t) = \sum_{i=1}^k \psi_i(t) \phi_i$  it follows from (8) that

$$\int_{\Omega} \frac{dS^k}{dt} \psi^k dx + a(S^k, \psi^k) + \int_{\Omega} F(S^k) \cdot \psi^k dx = \int_{\partial\Omega} g \psi^k d, \quad j = 1, \dots, k.$$

Integrating this equation by parts over  $[t_0, T]$  and taking into account the fact that  $\psi(T) = 0$ , we establish:

$$\begin{aligned} \int_{t_0}^T \int_{\partial\Omega} g \psi^k &= - \int_{\Omega} S^k(t_0) \psi^k(t_0) - \int_{t_0}^T \int_{\Omega} S^k \frac{d}{dt} \psi^k + \int_{t_0}^T a(S^k, \psi^k) \\ &+ \int_{t_0}^T \int_{\Omega} F(S^k) \psi^k. \end{aligned}$$

Noting that  $\int_{\Omega} S^k(t_0)\psi(t_0) = \sum_{i=1}^k \psi(t_0) \int_{\Omega} S^k(t_0)\phi_i$  and knowing that  $\int_{\Omega} S^k(t_0)\phi_i = \int_{\Omega} S_0\phi_i$ , we obtain

$$\int_{\Omega} S^k(t_0)\psi(t_0) = \sum_{i=1}^k \psi_i^k(t_0); \quad \int_{\Omega} S_0\phi_i = \int_{\Omega} S_0\psi^k(t_0).$$

As a consequence we have

$$\begin{aligned} \int_{t_0}^T \int_{\partial\Omega} g\psi^k &= - \int_{\Omega} S_0\psi^k(t_0) - \int_{t_0}^T \int_{\Omega} S^k \frac{d\psi^k}{dt} + \int_{t_0}^T a(S^k, \psi^k) \\ &+ \int_{t_0}^T \int_{\Omega} F(S^k)\psi^k. \end{aligned}$$

Passing to the limit in this expression it occurs

$$- \int_{\Omega} S_0\psi(t_0) - \int_{t_0}^T S \frac{d\psi}{dt} + \int_{t_0}^T a(S, \psi) + \int_{t_0}^T F(S)\psi = \int_{t_0}^T \int_{\partial\Omega} g\psi. \tag{21}$$

Moreover, as equation (16) is satisfied, we have

$$\int_{\Omega} \frac{dS}{dt} \psi + a(S, \psi) + \int_{\Omega} F(S)\psi = \int_{\partial\Omega} g\psi.$$

Then integrating by parts on  $[t_0, T]$  with  $\psi(T) = 0$ , we obtain:

$$- \int_{\Omega} S(t_0)\psi(t_0) - \int_{t_0}^T \int_{\Omega} S \frac{d\psi}{dt} + \int_{t_0}^T a(S, \psi) + \int_{t_0}^T \int_{\Omega} F(S)\psi = \int_{\partial\Omega} g\psi. \tag{22}$$

Comparing equations (21) and (22), we deduct that

$$\int_{\Omega} S(t_0)\psi(t_0) = \int_{\Omega} S_0\psi(t_0) \quad \forall \psi \in C^1(t_0, T; \mathcal{D}(\Omega)) \text{ with } \psi(T) = 0. \tag{23}$$

We now choose  $\psi(t) = (T - t)w$  with  $w \in \mathcal{D}(\Omega)$ . It is obviously to see that  $\psi(T) = 0$ . Thus equation (23) becomes

$$\int_{\Omega} S(t_0)w = \int_{\Omega} S_0w \quad \forall w \in \mathcal{D}(\Omega),$$

from which it follows that  $S(t_0) = S_0$  in  $\Omega$ . This achieve the proof of the existence of solutions of (16)- (17 ).

**Step 2: The uniqueness.**

Suppose that there are two solutions  $S^1$  and  $S^2$  for the same initial condition  $S_0 \in H^1(\Omega)$ . We have

$$\int_{\Omega} \frac{\partial S^i}{\partial t} v + a(S^i, v) = \int_{\partial\Omega} g.v - \int_{\Omega} F(S^i).v \quad i = 1, 2,$$

for each  $v \in H^1(\Omega)$ . Thus, setting  $v = S^1 - S^2$ , and considering equation (16), one obtains

$$\frac{1}{2} \frac{d}{dt} \int_{\Omega} (S^1(t) - S^2(t))^2 + \alpha \|S^1(t) - S^2(t)\|_{L^2(\Omega)}^2 \leq 0.$$

This clearly implies that

$$\frac{1}{2} \frac{d}{dt} \int_{\Omega} (S^1(t) - S^2(t))^2 \leq 0.$$

Hence, the application  $t \mapsto \|S^1(t) - S^2(t)\|_{L^2(\Omega)}^2$  is decreasing. Thus for all  $t \geq t_0$  we have  $\|S^1(t) - S^2(t)\|_{L^2(\Omega)}^2 \leq \|S^1(t_0) - S^2(t_0)\|_{L^2(\Omega)}^2 = 0$ . Consequently  $S^1(t) = S^2(t)$  a.e. We then have verified the uniqueness of the solution of the system (16) -(17). □

The examination of the problem  $(P_2)$  leads to the following result.

**Proposition 3.4.** *For a weak solution  $(\Lambda, S)$  to be admissible, it is necessary that*

$$\left\{ \begin{array}{ll} E \leq F(S) & \text{in } ]t_0, +\infty[ \times \Omega \\ g \leq 0 & \text{in } ]t_0, +\infty[ \times \partial\Omega \\ A \nabla S . \nabla (\frac{\partial S}{\partial t} + E)^- \geq 0 & \text{in } ]t_0, +\infty[ \times \Omega \end{array} \right. \quad (24)$$

with  $(\frac{\partial S}{\partial t} + E)^- = \max(0, -\frac{\partial S}{\partial t} - E)$ .

To prove this proposition, it should be enough to set  $\phi = (\frac{\partial S}{\partial t} + E)^-$ , in the equation (16) and to establish that  $\left\| (\frac{\partial S}{\partial t} + E)^- \right\|_{L^2(\Omega)} \leq 0$ . □

#### 4. Conclusion

We have presented in this paper a study of a mathematical model which describes complex factors governing the evolution of lakes sediment bottom. The known today factors are mainly sediment inputs from watersheds and climatic factors.

In this study, we have established, under reasonable geological hypothesis, the existence and uniqueness of solutions and derived conditions for obtained admissible solutions that ensure a sedimentation process on a long term. Such conditions will be useful to identify unknown parameters of the model in the case of lakes drying up process.

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