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ON SOME WEAKER FORMS OF SOFT CLOSED SETS IN SOFT TOPOLOGICAL SPACES

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Abstract: In this paper, the notions of weakly soft g-closed sets, soft g^*-closed sets and weakly soft g^*-closed sets in soft topological spaces are introduced and studied. Also, the inter-relationships between these new classes of soft sets with existing soft sets in soft topological spaces are studied.

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1. Introduction

In 1999, Molodtsov [9] introduced the soft set theory and showed how soft set theory is better than fuzzy set theory, rough set theory and game theory. Based on the work of Molodtsov [9], Maji et al [7] initiated the study of soft set theory which includes several basic definitions and basic operations of soft sets. Further, Shabir and Naz [14] introduced soft topological spaces which are

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defined over an initial universe with a fixed set of parameters and studied the basic notions such as soft open sets, soft closed sets, soft closure, soft separation axioms. Sabir and Bashir [11] and Naim Cagman et al. [10] continued the study of properties of soft topological spaces. S.S. Benchalli et al [1] studied the properties of soft regular spaces and soft normal spaces. The study of soft sets and related aspects was also undertaken in [16]. Mahanta et al [6] made a study of soft topology via soft semi open sets. Later in 2012, Kannan [5] studied soft g-closed sets. Recently in 2013, Bin Chen [2, 3] introduced and explored the properties of soft semi open sets and soft semi closed sets in soft topological spaces. Further, Gnanambal and Mrudula [4] and J. Subhashini and C. Sekar [15] introduced soft pre-open sets in soft topological spaces and studied its properties. Soft β-open sets are introduced by Metin Akdag [8] and soft regular open and soft regular closed sets are introduced by Saziye Yuksel [12]. In the present study, weakly soft g-closed sets, soft g^*-closed sets and weakly soft g^*-closed sets in soft topological spaces are introduced and studied. Also, the inter-relationships between these new classes of soft sets with existing soft sets in soft topological spaces are studied.

2. Preliminaries

The following preliminaries are required for subsequent work.

Definition 2.1. ([9]) Let U be an initial universe and E be a set of parameters. Let $P(U)$ denote the power set of U and A be a non-empty subset of E . A pair (F, A) is called a soft set over U, where F is a mapping given by $F : A \to P(U)$. In other words, a soft set over U is a parameterized family of subsets of the universe U. For $e \in A, F(e)$ may be considered as the set of e-approximate elements of the soft set (F, A). Clearly, every set is a soft set but not conversely.

Definition 2.2. ([7]) For two soft sets (F, A) and (G, B) over a common universe U, we say that (F, A) is a soft subset of (G, B) if (i)$A \subset B$ and (ii) for all $e \in A, F(e)$ and $G(e)$ are identical approximations.

Definition 2.3. ([14]) Let τ be the collection of soft sets over X. Then τ is said to be a soft topology on X if
(1) $\emptyset, \widetilde{X}$ belong to τ.
(2) The union of any number of soft sets in τ belongs to τ.

(3) The intersection of any two soft sets in τ belongs to τ.
The triplet (X, τ, E) is called a soft topological space.

Here the members of τ are called soft open sets in X and the relative complements of soft open sets are called soft closed sets.

Theorem 2.4. ([14]) *Arbitrary union of soft open sets is a soft open set and finite intersection of soft closed sets is a soft closed set.*

Definition 2.5. ([14]) Let (X, τ, E) be a soft space over X and (F, E) be a soft set over X. Then, the soft closure of (F, E) denoted by $\overline{(F, E)}$ is the intersection of all soft closed super sets of (F, E). Clearly, $\overline{(F, E)}$ is the smallest soft closed set over X containing (F, E).

The soft neighbourhood, soft relative topology, soft T_0- spaces, soft T_1- spaces and soft T_2- spaces are defined and studied by Shabir and Naz in [14].

Definition 2.6. ([17]) The soft interior of (G, E) is the soft set defined as $(G, E)^o = \text{int}(G, E) = \bigcup\{(S, E) : (S, E) \text{ is soft open and } (S, E) \subseteq (G, E)\}$. Here $(G, E)^o$ is largest soft open set contained in (G, E).

Definition 2.7. In a soft topological space (X, τ, E), a soft set (F, E) is said to be:

(i) soft generalized closed (briefly soft g-closed) [5] in X if $cl(F, E) \subseteq (G, E)$, whenever $(F, E) \subseteq (G, E)$ and (G, E) is soft open in X;

(ii) soft semi-open [2] if $(F, E) \subseteq cl(int(F, E))$;

(iii) soft preopen [4],[15] set if $(F, E) \subseteq int(cl(F, E))$;

(iv) soft β-open [8] if $(F, E) \subseteq cl(int(cl(F, E)))$;

(v) soft regular open [13] if $int(cl((F, E))) = (F, E)$ and soft regular closed if $cl(int((F, E))) = (F, E)$.

3. Weakly soft g-closed sets

Definition 3.1. In a soft topological space (X, τ, E), a soft set (A, E) is said to be weakly soft g-closed if $cl(int((A, E))) \subseteq (U, E)$, whenever $(A, E) \subseteq (U, E)$ and (U, E) is soft open in X.

Example 3.2. Let $X = \{a, b\}, E = \{e_1, e_2\}$
$(F,E)_1 = \{(e_1, \phi), (e_2, \phi)\}$, $(F,E)_2 = \{(e_1, \phi), (e_2, \{a\})\}$
$(F,E)_3 = \{(e_1, \phi), (e_2, \{b\})\}$, $(F,E)_4 = \{(e_1, \phi), (e_2, \{a, b\})\}$
$(F,E)_5 = \{(e_1, \{a\}), (e_2, \phi)\}$, $(F,E)_6 = \{(e_1, \{a\}), (e_2, \{a\})\}$
$(F,E)_7 = \{(e_1, \{a\}), (e_2, \{b\})\}$, $(F,E)_8 = \{(e_1, \{a\}), (e_2, \{a, b\})\}$
$(F,E)_9 = \{(e_1, \{b\}), (e_2, \phi)\}$, $(F,E)_{10} = \{(e_1, \{b\}), (e_2, \{a\})\}$
$(F,E)_{11} = \{(e_1, \{b\}), (e_2, \{b\})\}$, $(F,E)_{12} = \{(e_1, \{b\}), (e_2, \{a, b\})\}$
$(F,E)_{13} = \{(e_1, \{a, b\}), (e_2, \phi)\}$, $(F,E)_{14} = \{(e_1, \{a, b\}), (e_2, \{a\})\}$
$(F,E)_{15} = \{(e_1, \{a, b\}), (e_2, \{b\})\}$, $(F,E)_{16} = \{(e_1, \{a, b\}), (e_2, \{a, b\})\}$ be all possible soft sets defined over X with parameter set E.
Then $\tau = \{(F,E)_1, (F,E)_5, (F,E)_7, (F,E)_8, (F,E)_{16}\}$ is a soft topology over X.

Here, soft open sets are: $(F,E)_1, (F,E)_5, (F,E)_7, (F,E)_8, (F,E)_{16}$.

Soft closed sets are: $(F,E)_{16}, (F,E)_9, (F,E)_{10}, (F,E)_{12}, (F,E)_1$.

Soft g-closed sets are: $(F,E)_1, (F,E)_9, (F,E)_{10}, (F,E)_{11}, (F,E)_{12}, (F,E)_{13}, (F,E)_{14}, (F,E)_{15}, (F,E)_{16}$.

Weakly soft g-closed sets are: $(F,E)_1, (F,E)_2, (F,E)_3, (F,E)_4, (F,E)_9, (F,E)_{10}, (F,E)_{11}, (F,E)_{12}, (F,E)_{13}, (F,E)_{14}, (F,E)_{15}, (F,E)_{16}$.

Theorem 3.3. *Every soft closed set is weakly soft g-closed set.*

Proof. Let (A, E) be a soft closed set and $(A, E) \subseteq (U, E)$ where (U, E) is soft open. We have $int(A, E) \subseteq (A, E)$, which implies $cl(int(A, E)) \subseteq cl(A, E)$. Since (A, E) is soft closed, $cl(int(A, E)) \subseteq (A, E) \subseteq (U, E)$. Thus, $cl(int(A, E)) \subseteq (U, E)$, whenever $(A, E) \subseteq (U, E)$ and (U, E) is soft open. Therefore, (A, E) is weakly soft g-closed set.

Remark 3.4. Every weakly soft g-closed set need not be soft closed.

Example 3.5. In Example 3.2,

$$(F,E)_2, (F,E)_3, (F,E)_4, (F,E)_{11}, (F,E)_{13}, (F,E)_{14}, (F,E)_{15}$$

are weakly soft g-closed sets but not soft closed.

Theorem 3.6. *Every soft g-closed set is weakly soft g-closed.*

Proof. Let (A, E) be a soft g-closed set and $(A, E) \subseteq (U, E)$ where (U, E) is soft open. Then, $cl(A, E) \subseteq (U, E)$. But, $int(A, E) \subseteq (A, E)$, which implies $cl(int(A, E)) \subseteq cl(A, E) \subseteq (U, E)$. Then, $cl(int(A, E)) \subseteq (U, E)$, whenever

$(A, E) \subseteq (U, E)$ and (U, E) is soft open. Therefore, (A, E) is weakly soft g-closed.

Remark 3.7. Every weakly soft g-closed set needs not be soft g-closed.

Example 3.8. Let $X = \{a, b, c\}, E = \{e_1, e_2\}$, $\tau = \{\phi, X, (A, E), (B, E)\}$ where $(A, E) = \{(e_1, \{a\}), (e_2, \{a\})\}, (B, E) = \{(e_1, \{a, b\}), (e_2, \{a, b\})\}$.

Then $(C, E) = \{(e_1, \{b\}), (e_2, \{b\})\}$ is a weakly soft g-closed set but not soft g-closed set.

Theorem 3.9. *If a subset (A, E) of a soft topological space (X, τ, E) is both soft open and weakly soft g-closed, then it is soft closed.*

Proof. Suppose (A, E) is both soft open and weakly soft g-closed. Then $cl(int(A, E)) \subseteq (U, E)$, whenever $(A, E) \subseteq (U, E)$ and (U, E) is soft open. Thus, $cl(int(A, E)) \subseteq (A, E)$——(i). Since (A, E) is soft open, $int(A, E) = (A, E)$. Thus, (i) becomes $cl(A, E) \subseteq (A, E)$. But $(A, E) \subseteq cl(A, E)$ is always true. Hence, $cl(A, E) = (A, E)$. Therefore (A, E) is soft closed.

Corollary 3.10. *If a subset (A, E) of a soft topological space (X, τ, E) is both soft open and weakly soft g-closed then it is both soft regular open and soft regular closed in X.*

Proof. From Theorem 3.9, if (A, E) is both soft open and weakly soft g-closed then it is closed. Thus, $cl(A, E) = (A, E)$ and $int(A, E) = (A, E)$. Hence, $cl(int(A, E)) = (A, E) = int(cl(A, E))$. Therefore it is both soft regular open and soft regular closed.

Theorem 3.11. *If a subset (A, E) of a soft topological space (X, τ, E) is both weakly soft g-closed and soft semi open then it is soft g-closed.*

Proof. Since (A, E) is weakly soft g-closed then $cl(int(A, E)) \subseteq (U, E)$, whenever $(A, E) \subseteq (U, E)$ and (U, E) is soft open. Also, since (A, E) is soft semi open, we have $(A, E) \subseteq cl(int(A, E))$. Thus, $cl(A, E) \subseteq cl(int(A, E)) \subseteq (U, E)$, which implies $cl(A, E) \subseteq (U, E)$. Therefore, (A, E) is soft g-closed.

Theorem 3.12. *If a subset (A, E) of a soft topological space (X, τ, E) is both weakly soft g-closed and soft open then it is soft g-closed in X.*

Proof. Since every soft open set is soft semi-open, the result follows from the previous Theorem 3.11.

Theorem 3.13. *If a subset (A,E) of a soft topological space (X,τ,E) is weakly soft g-closed then $cl(int(A,E))-(A,E)$ contains no non empty soft closed set.*

Proof. Suppose (F,E) is a soft closed set such that $(F,E)\subseteq cl(int(A,E))-(A,E)$. Then, $(F,E)\subseteq cl(int(A,E))\cap(A,E)'$, which implies

$$(F,E)\subseteq cl(int(A,E)) \qquad (i)$$

and $(F,E)\subseteq(A,E)'$. Now, $(F,E)\subseteq(A,E)'$ implies $(A,E)\subseteq(F,E)'$, where $(F,E)'$ is a soft open set. Therefore, $cl(int(A,E))\subseteq(F,E)'$, $(F,E)'$ is soft open. Then,

$$(F,E)\subseteq(cl(int(A,E)))'. \qquad (ii)$$

Hence, from (i) and (ii), $(F,E)\subseteq cl(int(A,E))\cap(cl(int(A,E))'=\phi$. Therefore, $cl(int(A,E))-(A,E)$ contains no non empty soft closed set.

Theorem 3.14. *If (A,E) is weakly soft g-closed set and $(A,E)\subseteq(B,E)\subseteq cl(int(A,E))$, then (B,E) is also weakly soft g-closed set.*

Proof. Let $(B,E)\subseteq(U,E)$, (U,E) be a soft open set. Then, $(A,E)\subseteq(U,E)$, where (U,E) is soft open.Since (A,E) is weakly soft g-closed, we have $cl(int(A,E))\subseteq(U,E)$. Now, $cl(int(B,E))\subseteq cl(B,E)\subseteq cl(int(A,E))\subseteq(U,E)$. Which implies $cl(int(B,E))\subseteq(U,E)$, whenever $(B,E)\subseteq(U,E)$ and (U,E) is soft open. Therefore, (B,E) is weakly soft g-closed.

Theorem 3.15. *Every soft pre-closed set is weakly soft g-closed but not conversely.*

Proof. Let $(A,E)\subseteq(U,E)$, where (U,E) is soft open and (A,E) be soft pre-closed. Then, $cl(int(A,E))\subseteq(A,E)$. This implies $cl(int(A,E))\subseteq(U,E)$, whenever $(A,E)\subseteq(U,E)$ and (U,E) is soft open. Hence, (A,E) is weakly soft g-closed.

The converse needs not be true, as shown from the following example.

Example 3.16. Let $X=\{a,b,c\}, E=\{e_1,e_2\}$
Let $\tau=\{\phi,X,(A,E),(B,E)\}$ be a soft topology over X,

where $(A, E) = \{(e_1, \{a\}), (e_2, \{a\})\}, (B, E) = \{(e_1, \{a, c\}), (e_2, \{a, c\})\}$.
Let $(C, E) = \{(e_1, \{a, b\}), (e_2, \{a, b\})\}$. Then, (C, E) is weakly soft g-closed but not soft pre-closed set over X.

4. Soft g^*-closed sets

Definition 4.1. In a soft topological space (U, τ, E), a soft set (G, E) is said to be soft g^*-closed if $cl(A, E) \subseteq (U, E)$, whenever $(A, E) \subseteq (U, E)$ and (U, E) is soft g-open in X.

A subset (B, E) of a soft topological space (X, τ, E) is soft g^*-open if $(B, E)'$ is soft g^*-closed set in X.

Theorem 4.2. *Every soft closed set is soft g^*-closed set.*

Proof. Let (A, E) be a soft closed set and $(A, E) \subseteq (U, E)$ and (U, E) be soft g-open. We have $cl(A, E) = (A, E) \subseteq (U, E)$, whenever $(A, E) \subseteq (U, E)$ and (U, E) is soft g-open. Thus (A, E) is soft g^*-closed set.

Remark 4.3. The converse of the above theorem needs not be true in general.

Example 4.4. Let $X = \{a, b, c\}, E = \{e_1, e_2\}, \tau = \{\phi, X, (A, E), (B, E)\}$, where $(A, E) = \{(e_1, \{a\}), (e_2, \{a\})\}, (B, E) = \{(e_1, \{a, c\}), (e_2, \{a, c\})\}$.

Then, $(C, E) = \{(e_1, \{a, b\}), (e_2, \{a, b\})\}$ is soft g^*-closed set but not soft closed.

Theorem 4.5. *Every soft g^*-closed set is soft g-closed.*

Proof. Let (A, E) be a soft g^*-closed set and $(A, E) \subseteq (U, E)$, where (U, E) is soft g-open. Then, $cl(A, E) \subseteq (U, E)$, whenever $(A, E) \subseteq (U, E)$ and (U, E) is soft g-open. Since, every soft g-open set is soft open, (A, E) is soft g-closed.

Remark 4.6. Every soft g-closed set needs not be soft g^*-closed.

Example 4.7. Let $X = \{a, b, c\}, E = \{e_1, e_2\}$, $\tau = \{\phi, X, (A, E), (B, E)\}$, where $(A, E) = \{(e_1, \{a\}), (e_2, \{a\})\}, (B, E) = \{(e_1, \{b, c\}), (e_2, \{b, c\})\}$.

Then $(C,E) = \{(e_1,\{b\}),(e_2,\{b\})\}$ is a soft g-closed set but not soft g^*-closed set.

Theorem 4.8. *If (A,E) and (B,E) are soft g^*-closed sets, then $(A,E)\cup(B,E)$ is also a soft g^*-closed set.*

Proof. Let (A,E) and (B,E) be soft g^*-closed sets. Then, $cl(A,E) \subseteq (U,E)$ and $cl(B,E) \subseteq (U,E)$, whenever $(A,E) \subseteq (U,E), (B,E) \subseteq (U,E), (U,E)$ is soft g-open. Then, $(A,E)\cup(B,E) \subseteq (U,E)$, which implies $cl((A,E)\cup(B,E)) = cl(A,E)\cup cl(B,E) \subseteq (U,E)$. Hence, $(A,E)\cup(B,E)$ is also soft g^*-closed.

Theorem 4.9. *If a subset (A,E) of a soft topological space (X,τ,E) is both soft g-open and soft g^*-closed, then it is soft closed.*

Proof. Suppose (A,E) is both soft g-open and soft g^*-closed. Since, (A,E) is soft g-open $(A,E) \subseteq (A,E)$ and since (A,E) is soft g^*-closed, $cl(A,E) \subseteq (A,E)$, whenever $(A,E) \subseteq (A,E)$, (A,E) is soft g-open. But $(A,E) \subseteq cl(A,E)$ is always true. Thus, $cl(A,E) = (A,E)$, implies (A,E) is soft closed.

Theorem 4.10. *If a subset (A,E) of a soft topological space (X,τ,E) is soft g^*-closed, then $cl(A,E)-(A,E)$ contains no non empty soft g-closed set.*

Proof. Suppose (F,E) is a soft g-closed set such that $(F,E) \subseteq cl(A,E) - (A,E)$. Then, $(F,E) \subseteq cl(A,E) \cap (A,E)'$, which implies $(F,E) \subseteq cl(A,E)$—–(i) and $(F,E) \subseteq (A,E)'$. Now, $(F,E) \subseteq (A,E)'$ implies $(A,E) \subseteq (F,E)'$, where $(F,E)'$ is a soft g-open set. Since (A,E) is a soft g^*-closed set, then $cl(A,E) \subseteq (F,E)'$, where $(A,E) \subseteq (F,E)'$, (F,E) is soft g-open. Thus, $(F,E) \subseteq (cl(A,E))'$—-(ii). From (i) and (ii), $(F,E) \subseteq cl(A,E) \cap (cl(A,E))' = \phi$. Therefore, $cl(A,E)-(A,E)$ contains no non empty soft g-closed set.

Theorem 4.11. *If (A,E) is soft g^*-closed set and $(A,E) \subseteq (B,E) \subseteq cl(A,E)$, then (B,E) is also soft g^*-closed set.*

Proof. Let $(B,E) \subseteq (U,E)$, (U,E) be a soft g-open set. Then, $(A,E) \subseteq (U,E)$, where (U,E) is soft g-open. Since (A,E) is soft g^*-closed, we have $cl(A,E) \subseteq (U,E)$. Now, $cl(B,E) \subseteq cl(cl(A,E)) = cl(A,E) \subseteq (U,E)$, which implies $cl(B,E) \subseteq (U,E)$, whenever $(B,E) \subseteq (U,E)$ and (U,E) is soft g-open.

Therefore, (B, E) is soft g^*-closed.

Theorem 4.12. *Every soft g^*-closed set is weakly soft g-closed but not conversely.*

Proof. The proof follows from the definition.

The converse needs not to be true, which follows from the following example.

Example 4.13. Let $X = \{a, b, c\}, E = \{e_1, e_2\}$.

Let $\tau = \{\phi, X, (A, E), (B, E)\}$ be a soft topology over X, where $(A, E) = \{(e_1, \{a\}), (e_2, \{a\})\}, (B, E) = \{(e_1, \{b, c\}), (e_2, \{b, c\})\}$.

Let $(C, E) = \{(e_1, \{b\}), (e_2, \{b\})\}$. Then, (C, E) is weakly soft g-closed but not soft g^*-closed set.

5. Weakly soft g^*-closed sets

Definition 5.1. A subset (A, E) of a soft topological space (X, τ, E) is said to be weakly soft g^*-closed set if $cl(int(A, E)) \subseteq (U, E)$, whenever $(A, E) \subseteq (U, E)$ and (U, E) is soft g-open.

Theorem 5.2. *Every soft closed set is weakly soft g^*-closed set.*

Proof. Let (A, E) be a soft closed set and $(A, E) \subseteq (U, E)$, (U, E) is soft g-open. We have $int(A, E) \subseteq (A, E)$, implies $cl(int(A, E)) \subseteq cl(A, E) = (A, E) \subseteq (U, E)$. Then, $cl(int(A, E)) \subseteq (U, E)$, whenever $(A, E) \subseteq (U, E)$, (U, E) is soft g-open. Hence, (A, E) is weakly soft g^*-closed set.

Remark 5.3. The converse of above theorem needs not to be true.

Example 5.4. Let $X = \{a, b, c\}, E = \{e_1, e_2\}$. Let $\tau = \{\phi, X, (A, E), (B, E)\}$ be a soft topology over X, where $(A, E) = \{(e_1, \{a\}), (e_2, \{a\})\}, (B, E) = \{(e_1, \{a, c\}), (e_2, \{a, c\})\}$.

Then, $(C, E) = \{(e_1, \{c\}), (e_2, \{c\})\}$ is weakly soft g^*-closed set but not soft closed set.

Theorem 5.5. *Every soft g^*-closed set is weakly soft g^*-closed set.*

Proof. Let (A, E) be a soft g^*-closed set and (U, E) be a soft g-open set containing (A, E). Then, $cl(A, E) \subseteq (U, E)$. Also, $int(A, E) \subseteq (A, E)$, implies $cl(int(A, E)) \subseteq cl(A, E) \subseteq (U, E)$. Hence, (A, E) is weakly soft g^*-closed set.

Remark 5.6. The converse of the above theorem needs not be true.

Example 5.7. Let $X = \{a, b, c\}, E = \{e_1, e_2\}$. Let $\tau = \{\phi, X, (A, E), (B, E)\}$ be a soft topology over X, where $(A, E) = \{(e_1, \{a\}), (e_2, \{a\})\}, (B, E) = \{(e_1, \{a, b\}), (e_2, \{a, b\})\}$.

Then, $(C, E) = \{(e_1, \{b\}), (e_2, \{b\})\}$ is weakly soft g^*-closed but not soft g^*-closed.

Theorem 5.8. *If (A, E) is a soft subset of a soft topological space (X, τ, E) and (A, E) is both soft open and weakly soft g^*-closed, then it is soft closed.*

Proof. Suppose (A, E) is both soft open and weakly soft g^*-closed. Now, $cl(int(A, E)) \subseteq (U, E)$, whenever $(A, E) \subseteq (U, E)$ and (U, E) is soft g-open. Since (A, E) is soft open and $cl(int(A, E)) \subseteq (A, E)$, which implies $cl(A, E) \subseteq (A, E)$. But, $(A, E) \subseteq cl(A, E)$ is always true. Thus, $cl(A, E) = (A, E)$. Therefore, (A, E) is soft closed.

Corollary 5.9. *If (A, E) is both soft open and weakly soft g^*-closed, then it is both soft regular open and soft regular closed.*

Proof. As (A, E) is soft open then $(A, E) = int(A, E)$, which implies $(A, E) = int(cl(A, E))$, since (A, E) is soft closed from Theorem 5.8. Thus, (A, E) is soft regular open. Again, since (A, E) is soft open and soft closed from Theorem 4.8, so $cl(int(A, E)) = cl(A, E) = (A, E)$. Thus, (A, E) is soft regular closed.

Theorem 5.10. *If a subset (A, E) of a soft topological space (X, τ, E) is both weakly soft g^*-closed and soft semi open, then it is soft g^*-closed set.*

Proof. Suppose (A, E) is weakly soft g^*-closed and soft semi open. Let (U, E) be a soft open set containing (A, E). As (A, E) is weakly soft g^*-closed, $cl(int(A, E)) \subseteq (U, E)$. Now, since (A, E) is soft semi-open, $(A, E) \subseteq cl(int(A, E))$. Then, $cl(A, E) \subseteq cl(cl(int(A, E))) = cl(int(A, E)) \subseteq (U, E)$, which implies $cl(A, E) \subseteq (U, E)$. Therefore (A, E) is soft g^*-closed.

Corollary 5.11. *If a subset (A, E) of a soft topological space (X, τ, E) is both soft open and weakly soft g^*-closed, then it is soft g^*-closed.*

Proof. Since every soft open set is soft semi-open, the proof follows from the previous Theorem 5.10.

Theorem 5.12. *If a soft set (A, E) is weakly soft g^*-closed, then $cl(int(A, E)) - (A, E)$ contains no non empty soft closed set.*

Proof. Suppose that $(F, E) \subseteq cl(int(A, E)) - (A, E)$ and (F, E) is a soft closed set. Then, $(F, E) \subseteq cl(int(A, E)) \cap (A, E)'$, which implies $(F, E) \subseteq cl(int(A, E))$—-(i) and $(F, E) \subseteq (A, E)'$. Now, $(F, E) \subseteq (A, E)'$, implies $(A, E) \subseteq (F, E)'$, where $(F, E)'$ is a soft open. Since (A, E) is a strongly soft g^*-closed, we have $cl(int(A, E)) \subseteq (F, E)'$, implies $(F, E) \subseteq (cl(int(A, E)))'$——(ii). Hence, from (i) and (ii), $(F, E) \subseteq (cl(int(A, E))) \cap (cl(int(A, E)))' = \phi$. Thus, $(F, E) = \phi$, which implies,$cl(int(A, E)) - (A, E)$, contains no non empty soft closed set.

Corollary 5.13. *A weakly soft g^*-closed set (A, E) is soft regular closed iff $cl(int(A, E)) - (A, E)$ is soft closed.*

Proof. Assume that (A, E) is soft regular closed. Then, $cl(int(A, E)) = (A, E)$, implies $cl(int(A, E)) - (A, E) = \phi$ and ϕ is soft regular closed and hence soft closed.

Conversely, assume that $cl(int(A, E)) - (A, E)$ is soft closed. By previous Theorem 5.12, $cl(int(A, E)) - (A, E)$ contains no non empty soft closed set. Therefore, $cl(int(A, E)) - (A, E) = \phi$, implies $cl(int(A, E)) = (A, E)$. Thus, (A, E) is soft regular closed.

6. Conclusion

In this paper, the notions of weakly soft g-closed sets, soft g^*-closed sets and weakly soft g^*-closed sets in soft topological spaces are introduced and studied. Also, the relationships between these new classes of soft sets with existing soft sets are studied. The results obtained in this paper may be used for further applications of topology on soft sets and also in the developments of information system and various fields in engineering applications.

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