International Journal of Applied Mathematics

Volume 28 No. 2 2015, 165-176
ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)
doi: http://dx.doi.org/10.12732/ijam.v28i2.7

OSCILLATION CRITERIA FOR A CLASS OF SECOND-ORDER NONLINEAR DIFFERENTIAL EQUATIONS

Bülent Ayanlar

Department of Mathematics and Computer Science
Faculty of Science and Letters
Istanbul Arel University
Tepekent-Büyükçekmece, 34537, Istanbul, TURKEY

Abstract: By using the classical variational principle and averaging technique, several oscillation criteria are established for nonlinear second-order equations of the form

$$\left(r(t)\left|u'\right|^{p-2}u'\right)' + g(t,u,u')u' + a(t)f(u) = e(t),$$

where $p > 1$ is a real constant.

AMS Subject Classification: 34C10
Key Words: differential equations, second order, averaging technique, oscillation

1. Introduction

In the present paper we investigate the oscillation behavior for a class of second-order nonlinear differential equations of the form

$$\left(r(t)\left|u'\right|^{p-2}u'\right)' + g(t,u,u')u' + a(t)f(u) = e(t), \tag{1.1}$$

where

Received: January 20, 2015

- $p > 1$ is a real constant;
- $r, a \in C^1(\mathbb{R}_+, (0, \infty))$;
- $g \in C(\mathbb{R}_+ \times \mathbb{R} \times \mathbb{R}, \mathbb{R}_+)$;
- $f \in C(\mathbb{R}, \mathbb{R})$, and $e \in C(\mathbb{R}_+, \mathbb{R})$;

and where $\mathbb{R}_+$ denotes the set of all nonnegative real numbers. Throughout the paper we shall also assume that the following conditions are true for p, f, g:

(C_1) $xf(x) > 0$ for $x \neq 0$ and

(C_2) there exists a continuous function $p(t)$ such that

$$\frac{g(t,x,y)y}{f(x)} \geqslant \frac{p(t)\,|y|^{p-2}\,y}{f(x)} \quad \text{for} \quad x \neq 0, y \neq 0.$$

By a solution of (1.1), we mean a function $u \in C^1[T_u, \infty), T_u \geqslant t_0$, which has the property

$$r(t)\left|u'\right|^{p-2}u' \in C^1[T_u, \infty)$$

and satisfies Eq. (1.1). We restrict our attention only to the nontrivial solutions of Eq. (1.1), i.e., to the solutions $u(t)$ such that

$$\sup\{|u(t)| : t \geqslant T\} > 0$$

for all $T \geqslant T_u$. A nontrivial solution of Eq. (1.1) is called *oscillatory* if it has arbitrarily large zeros; otherwise, it is said to be *nonoscillatory.* Eq. (1.1) is said to be *oscillatory* if all its solutions are oscillatory.

The class of equations we are working with can be considered as a natural generalization of the class of Emden–Fowler-type equations of the form

$$\left(r(t)\left|u'\right|^{p-2}u'\right)' + c(t)\left|u'\right|^{p-2}u' + a(t)\,|u|^{p-2}\,u = e(t), \tag{1.2}$$

and of the class of the Lienard-type equations of the form

$$u'' + \Phi(u, u')u' + h(u) = e(t). \tag{1.3}$$

As in the literature, we will use an auxiliary function $H(t,s) \in C(D, \mathbb{R})$ having the following properties:

(i) $H(t,t)$=0, $H(t,s) > 0$ for $t > s$,

(ii) H has partial derivatives $\dfrac{\partial H}{\partial t}$ and $\dfrac{\partial H}{\partial s}$ on D such that

$$\frac{\partial H}{\partial t}=h_1(t,s)\sqrt{H(t,s)},\quad \frac{\partial H}{\partial s}=-h_2(t,s)\sqrt{H(t,s)},$$

where $D=\{(t,s):t_0\leqslant s\leqslant t<\infty\}$, $h_1,h_2\in L_{loc}(D,\mathbb{R}_+)$.

2. $f(x)$ is Monotone Increasing

In this section, we shall deal with the oscillation for Eq. (1.1) under the assumptions (C_1), (C_2), and the following assumption

(C_3) $f'(x)$ exists and

$$\frac{f'(x)}{|f(x)|^{(p-2)/(p-1)}}\geqslant\gamma>0,$$

for some nonnegative constant γ and for all $x\in\mathbb{R}\setminus\{0\}$.

Theorem 2.1. *Suppose that the conditions (C_1), (C_2) and (C_3) are all true and for any $T\geqslant t_0$ there exist $T\leqslant a_1<b_1\leqslant a_2<b_2$ such that*

$$e(t)\left\{\begin{array}{l}\leqslant 0,\ t\in[a_1,b_1]\\ \geqslant 0,\ t\in[a_2,b_2]\end{array}\right\}. \tag{2.1}$$

If there exist some $c_i\in(a_i,b_i)$, where $i=1,2$, a function $H(t,s)$ satisfying (i)–(ii) and a positive function $\rho\in C^1([t_0,\infty),\mathbb{R})$ such that

$$\frac{1}{H^p(c_i,a_i)}\int\limits_{a_i}^{c_i}\left[H^p(s,a_i)\,a(s)\rho(s)-\Phi r(s)\rho(s)H_1^p(s,a_i)\right]ds$$

$$+\frac{1}{H^p(b_i,c_i)}\int\limits_{c_i}^{b_i}\left[H^p(b_i,s)\,a(s)\rho(s)-\Phi r(s)\rho(s)H_2^p(b_i,s)\right]ds>0 \tag{2.2}$$

for $i=1,2$, where

$$\Phi=\frac{[(p-1)]^{p-1}}{(\gamma p)^{p-1}\,p},$$

$$H_1(t,s)=\left|ph_1(t,s)\sqrt{H(t,s)}+H(t,s)\left(\frac{\rho'(t)}{\rho(t)}-\frac{p(t)}{r(t)}\right)\right|,$$

$$H_2(t,s)=\left|ph_2(t,s)\sqrt{H(t,s)}+H(t,s)\left(\frac{\rho'(s)}{\rho(s)}-\frac{p(s)}{r(s)}\right)\right|,$$

then Eq. (1.1) *is oscillatory.*

Proof. Suppose, towards a contradiction, that $u(t)$ is a nonoscillatory solution of Eq. (1.1), say, $u(t) \neq 0$ on $[T_0, \infty)$ for some sufficiently large $T_0 \geqslant t_0$. Define

$$w(t) = \rho(t)\frac{r(t)\left|u'(t)\right|^{p-2}u'(t)}{f(u(t))},\ t \geqslant T_0. \tag{2.3}$$

Then differentiating (2.3) and making use of Eq. (1.1), assumptions (C_1), (C_2) and (C_3), we have

$$\begin{aligned} w'(t) &= -a(t)\rho(t) + \frac{e(t)}{f(u(t))}\rho(t) - \rho(t)\frac{g(t,u,u')u'}{f(u(t))} \\ &\quad -\rho(t)r(t)\frac{f'(u(t))}{f^2(u(t))}\left|u'(t)\right|^p + \frac{\rho'(t)}{\rho(t)}w(t) \\ &\leqslant -a(t)\rho(t) + \frac{e(t)}{f(u(t))}\rho(t) - \rho(t)p(t)\frac{\left|u'(t)\right|^{p-2}u'(t)}{f(u(t))} \\ &\quad -\rho(t)r(t)\frac{f'(u(t))}{f^2(u(t))}\left|u'(t)\right|^p + \frac{\rho'(t)}{\rho(t)}w(t) \\ &\leqslant -a(t)\rho(t) + \frac{e(t)}{f(u(t))}\rho(t) - \gamma\left(\rho(t)r(t)\right)^{1/(1-p)}\left|w(t)\right|^{p/(p-1)} \\ &\quad + \left(\frac{\rho'(t)}{\rho(t)} - \frac{p(t)}{r(t)}\right)w(t). \end{aligned} \tag{2.4}$$

By the conditions of the theorem, we can choose $a_i, b_i \geqslant T_0$ for $i = 1, 2$ such that $e(t) \leqslant 0$ on the interval $I_1 = [a_1, b_1]$ and $u(t) > 0$, or $e(t) \geqslant 0$ on the interval $I_2 = [a_2, b_2]$ and $u(t) < 0$.

By (2.4),

$$\begin{aligned} w'(t) &\leqslant -a(t)\rho(t) - \gamma\left(\rho(t)r(t)\right)^{1/(1-p)}\left|w(t)\right|^{p/(p-1)} \\ &\quad + \left(\frac{\rho'(t)}{\rho(t)} - \frac{p(t)}{r(t)}\right)w(t). \end{aligned} \tag{2.5}$$

on both intervals I_1 and I_2.

On one hand, multiplying $H^p(t,s)$ through (2.5) and integrating it (with t replaced by s) over $[c_i, t)$ for $t \in [c_i, b_i)$, $i = 1, 2$, by using hypotheses (i), (ii), we have for $s \in [c_i, t)$

$$\int_{c_i}^{t} H^p(t,s)\,a(s)\rho(s)ds \leqslant -\int_{c_i}^{t} H^p(t,s)\,w'(s)ds$$

$$+\int_{c_i}^{t} H^p(t,s)\left[\left(\frac{\rho'(s)}{\rho(s)}-\frac{p(s)}{r(s)}\right)w(s)-\gamma(\rho(s)r(s))^{1/(1-p)}|w(s)|^{p/(p-1)}\right]ds$$

$$=H^p(t,c_i)\,w(c_i)-\int_{c_i}^{t} pH^{p-1}(t,s)\,h_2(t,s)\sqrt{H(t,s)}w(s)ds$$

$$+\int_{c_i}^{t} H^p(t,s)\left[\left(\frac{\rho'(s)}{\rho(s)}-\frac{p(s)}{r(s)}\right)w(s)-\gamma(\rho(s)r(s))^{1/(1-p)}|w(s)|^{p/(p-1)}\right]ds$$

$$\leqslant H^p(t,c_i)\,w(c_i)+\int_{c_i}^{t}[H^{p-1}(t,s)\,H_2(t,s)\,|w(s)|$$

$$-\gamma H^p(t,s)\,(\rho(s)r(s))^{1/(1-p)}\,|w(s)|^{p/(p-1)}]ds. \tag{2.6}$$

Given t and s, set

$$F(v):=H^{p-1}H_2v-\gamma H^p(\rho r)^{1/(1-p)}\,v^{p/(p-1)},$$

where $v>0$.

Since

$$F'(v)=H^{p-1}H_2-\frac{\gamma p}{p-1}H^p(\rho r)^{1/(1-p)}\,v^{1/(p-1)},$$

$F(v)$ attains the maximum value at

$$v=r\rho\left(\frac{(p-1)\,H_2}{\gamma pH}\right)^{p-1},$$

and since

$$F(v)\leqslant F_{\max}=\Phi r\rho H_2^p, \tag{2.7}$$

we get, using (2.7),

$$\int_{c_i}^{t} H^p(b_i,s)\,a(s)\rho(s)ds\leqslant H^p(b_i,c_i)\,w(c_i)$$

$$+\,\Phi\int_{c_i}^{b_i} r(s)\rho(s)H_2^p(b_i,s)\,ds. \tag{2.8}$$

Letting $t \to b_i^-$ in (2.6) , we obtain

$$\int_{c_i}^{b_i} H^p(t,s)\,a(s)\rho(s)ds \leqslant H^p(t,c_i)\,w(c_i) + \Phi \int_{c_i}^{t} r(s)\rho(s)H_2^p(t,s)\,ds. \tag{2.9}$$

On the other hand, multiplying again by H^p both parts of (2.5), and integrating (with t replaced by s) over $(t,c_i]$ for $t \in (a_i,c_i]$, $i=1,2$, instead, by using hypotheses (i)–(ii), we yield for $s \in (t,c_i]$

$$\int_{t}^{c_i} H^p(s,t)\,a(s)\rho(s)ds \leqslant -\int_{t}^{c_i} H^p(s,t)\,w'(s)ds$$

$$+\int_{t}^{c_i} H^p(s,t)\left[\left(\frac{\rho'(s)}{\rho(s)}-\frac{p(s)}{r(s)}\right)w(s)-\gamma\left(\rho(s)r(s)\right)^{1/(1-p)}|w(s)|^{p/(p-1)}\right]ds$$

$$= -H^p(c_i,t)\,w(c_i) + \int_{t}^{c_i} pH^{p-1}(s,t)\,h_1(s,t)\sqrt{H(s,t)}w(s)ds$$

$$+\int_{t}^{c_i} H^p(s,t)\left[\left(\frac{\rho'(s)}{\rho(s)}-\frac{p(s)}{r(s)}\right)w(s)-\gamma\left(\rho(s)r(s)\right)^{1/(1-p)}|w(s)|^{p/(p-1)}\right]ds$$

$$\leqslant -H^p(c_i,t)\,w(c_i) + \int_{t}^{c_i}[H^{p-1}(s,t)\,H_1(s,t)\,|w(s)|$$

$$-\gamma H^p(s,t)\left(\rho(s)r(s)\right)^{1/(1-p)}|w(s)|^{p/(p-1)}]ds$$

$$\leqslant -H^p(c_i,t)\,w(c_i) + \Phi\int_{t}^{c_i} r(s)\rho(s)H_1^p(s,t)\,ds. \tag{2.10}$$

We get the last inequality in (2.10) by following the proof of (2.8). Letting $t \to a_i^+$ in (2.10) leads to

$$\int_{a_i}^{c_i} H^p(s,a_i)\,a(s)\rho(s)ds \leqslant -H^p(c_i,a_i)\,w(c_i)$$

$$+\Phi\int_{a_i}^{c_i} r(s)\rho(s)H_1^p(s,a_i)\,ds. \tag{2.11}$$

Finally, dividing (2.9) and (2.11) by $H^p(b_i, c_i)$ and $H^p(c_i, a_i)$, respectively, and then adding them, we obtain the inequality

$$\frac{1}{H^p(c_i,a_i)}\int_{a_i}^{c_i} H^p(s,a_i)\,a(s)\rho(s)ds + \frac{1}{H^p(b_i,c_i)}\int_{c_i}^{b_i} H^p(b_i,s)\,a(s)\rho(s)ds$$

$$\leqslant \frac{1}{H^p(c_i,a_i)}\Phi\int_{a_i}^{c_i} r(s)\rho(s)H_1^p(s,a_i)\,ds$$

$$+\frac{1}{H^p(b_i,c_i)}\Phi\int_{c_i}^{b_i} H_2^p(b_i,s)\,a(s)\rho(s)ds, \tag{2.12}$$

which contradict (2.2). The proof of Theorem 2.1 is completed. □

The following result is an easy corollary of Theorem 2.1.

Corollary 2.2. *Suppose that the hypotheses in Theorem 2.1 hold and a function $H(t,s)$ satisfies the conditions (i)–(ii). If there exist some $c_i \in (a_i, b_i)$, $i = 1, 2$, and some positive function $\rho \in C^1([t_0,\infty),\mathbb{R})$ such that*

$$\int_{a_i}^{c_i} \left[H^p(s,a_i)\,a(s)\rho(s) - \Phi\, r(s)\rho(s)H_1^p(s,a_i)\right] ds > 0 \tag{2.13}$$

$$\int_{c_i}^{b_i} \left[H^p(b_i,s)\,a(s)\rho(s) - \Phi\, r(s)\rho(s)H_2^p(b_i,s)\right] ds > 0 \tag{2.14}$$

for $i = 1, 2$, where γ, H_1, H_2, Φ are similar to ones in Theorem 2.1, then Eq. (1.1) is oscillatory.

Specifically, if a function $H := H(t,s) \in C(D,\mathbb{R})$ which satisfies (i)–(ii) is such that the following additional condition

(iii) $h_1(t-s) = h_2(t-s)$,

is true for H. Then denoting $h_k(t-s)$ where $k = 1, 2$ by $h(t-s)$, and assuming that $\rho(t) \equiv 1$, we derive one more useful corollary from Theorem 2.1:

Corollary 2.3. *Suppose that for any $T \geqslant t_0$, there exist $T \leqslant a_1 < 2c_1 - a_1 \leqslant a_2 < 2c_2 - a_2$ such that*

$$e(t)\left\{\begin{array}{l} \leqslant 0,\ t \in [a_1, 2c_1 - a_1] \\ \geqslant 0,\ t \in [a_2, 2c_2 - a_2\,] \end{array}\right\}. \tag{2.15}$$

If there exists a function $H := H(t-s)$ having the form described above and satisfying the inequality

$$\int_{a_i}^{c_i} H^p\,(s-a_i)\,[a(s) + a(2c_i - s)]\,ds$$

$$> \Phi \int_{a_i}^{c_i} [r(s) + r(2c_i - s)] \left(h\,(s-a_i)\,\sqrt{H(s-a_i)}\right)^p ds, \tag{2.16}$$

for $i = 1, 2$ and γ, and if all other hypotheses listed in Theorem 2.1 are true, then Eq. (1.1) *is oscillatory.*

The proof of the corollary is similar to the proof of Theorem 2.2 in [4], so we skip it.

Remark 2.4. It can be verified that we can replace the hypothesis concerning the function e in Theorem 2.1 with the hypothesis

$$e(t)\left\{\begin{array}{l} \geqslant 0,\ t \in [a_1, b_1] \\ \leqslant 0,\ t \in [a_2, b_2\,] \end{array}\right\}.$$

3. $f(x)$ is not Monotone Increasing

In this section, we shall mainly consider the oscillation problem for Eq. (1.1), assuming as before the conditions (C_1) and (C_2) and the condition stating that

(C_4) $f(x)$ satisfies

$$\frac{f(x)}{x} \geqslant K\,|x|^{q-2},$$

for $x \neq 0$, where $K > 0$ and $q > 1$ be constant.

Lemma 3.1 (Hölder's inequality). *If A and B are nonnegative real numbers, then*

$$\frac{1}{p}A^p + \frac{1}{q}B^q \geqslant AB, \text{ for } \frac{1}{p} + \frac{1}{q} = 1.$$

Theorem 3.2. *Suppose the conditions (C_1), (C_2), (C_3), and (C_4) are all true for the function*

$$f(x) = |x(t)|^{p-2}x(t)$$

and that for any $T \geqslant t_0$, there exist $T \leqslant a_1 < b_1 \leqslant a_2 < b_2$ such that (2.1) holds and $a(t) \geqslant 0$ for $t \in [a_1, b_1] \cup [a_2, b_2]$.

If there exist some $c_i \in (a_i, b_i)$ for $i = 1, 2$, $H(t, s)$ satisfying (i)–(ii) and a positive function $\rho \in C^1([t_0, \infty), \mathbb{R})$ such that

$$\frac{1}{H^p(c_i, a_i)} \int_{a_i}^{c_i} \left[H^p(s, a_i) Q(s)\rho(s) - \Phi r(s)\rho(s)H_1^p(s, a_i)\right] ds$$

$$+\frac{1}{H^p(b_i, c_i)} \int_{c_i}^{b_i} \left[H^p(b_i, s) Q(s)\rho(s) - \Phi r(s)\rho(s)H_2^p(b_i, s)\right] ds > 0 \qquad (3.1)$$

for $i = 1, 2$, where H_1, H_2, Φ are defined as in Theorem 2.1 and $Q(t) = [Ka(t)]^{p/q} |e(t)|^{(q-p)/q}$, then Eq. (1.1) is oscillatory.

Proof. Suppose otherwise: let $u(t)$ be a nonoscillatory solution of Eq. (1.1), say $u(t) \neq 0$ on $[T_0, \infty)$ for some sufficiently large $T_0 \geqslant t_0$. Define

$$v(t) = \rho(t)\frac{r(t)\,|u'(t)|^{p-2}\,u'(t)}{|u(t)|^{p-2}\,u(t)}, \quad t \geqslant T_0. \qquad (3.2)$$

Then differentiating (3.2) and making use of Eq. (1.1) and assumptions (C_3)–(C_4), we obtain

$$\begin{aligned} v'(t) = {}& \frac{e(t)}{|u(t)|^{p-2}\,u(t)}\rho(t) - \rho(t)\frac{g(t,u,u')u'}{|u(t)|^{p-2}\,u(t)} - \rho(t)\frac{a(t)f(u)}{|u(t)|^{p-2}\,u(t)} \\ & -(p-1)\rho(t)r(t)\frac{|u'(t)|^p}{|u(t)|^p} + \frac{\rho'(t)}{\rho(t)}v(t) \\ \leqslant {}& \frac{e(t)}{|u(t)|^{p-2}\,u(t)}\rho(t) + \left(\frac{\rho'(t)}{\rho(t)} - \frac{p(t)}{r(t)}\right)v(t) \end{aligned}$$

$$-K\rho(t)a(t)\,|u(t)|^{q-p} - (p-1)\rho(t)r(t)\frac{|u'(t)|^p}{|u(t)|^p}$$

$$\leqslant -\rho(t)\left(-\frac{e(t)}{|u(t)|^{p-2}\,u(t)} + Ka(t)\,|u(t)|^{q-p}\right)$$

$$-(p-1)\left[\rho(t)r(t)\right]^{1/(1-p)}|v(t)|^{p/(p-1)} + \left(\frac{\rho'(t)}{\rho(t)} - \frac{p(t)}{r(t)}\right)v(t). \tag{3.3}$$

By the hypotheses, we can choose $a_i, b_i \geqslant 0$ for $i = 1, 2$ such that $e(t) \leqslant 0$ on the interval $I_1 = [a_1, b_1]$ with $a_1 < b_1$ and $u(t) > 0$; or $e(t) \geqslant 0$ on the interval $I_2 = [a_2, b_2]$ with $a_2 < b_2$ and $u(t) < 0$. Thus, by Hölder's inequality, we have

$$-\frac{e(t)}{|u(t)|^{p-2}\,u(t)} + Ka(t)\,|u(t)|^{q-p} = \frac{|e(t)|}{|u(t)|^{p-1}} + Ka(t)\,|u(t)|^{q-p}$$

$$\geqslant \frac{q-p}{q}\left[\frac{|e(t)|^{(q-p)/q}}{|u(t)|^{(p-1)(q-p)/q}}\right]^{q/(q-p)}$$

$$+\frac{p}{q}\left[(Ka(t))^{p/q}\,u^{(p-1)(q-p)/q}\right]^{q/p} \geqslant Q(t) \tag{3.4}$$

on the interval $I_1 = [a_1, b_1]$. Similarly,

$$-\frac{e(t)}{|u(t)|^{p-2}\,u(t)} + Ka(t)\,|u(t)|^{q-p} = \frac{|e(t)|}{|u(t)|^{p-1}} + Ka(t)\,|u(t)|^{q-p}$$

$$\geqslant \left[Ka(t)^{p/q}e(t)^{(q-p)/q}\right] = Q(t). \tag{3.5}$$

on the interval $I_2 = [a_2, b_2]$.

It follows from (3.4), (3.5), and (3.3) that the function $v(t)$ satisfies

$$v'(t) \leqslant -\rho(t)Q(t) - (p-1)\left[\rho(t)r(t)\right]^{1/(1-p)}|v(t)|^{p/(p-1)}$$

$$+\left(\frac{\rho'(t)}{\rho(t)} - \frac{p(t)}{r(t)}\right)v(t). \tag{3.6}$$

on both intervals I_1 and I_2.

The rest of the proof is similar to that of Theorem 2.1 and hence omitted. □

The following two corollaries are similar to Corollaries 2.2–2.3.

Corollary 3.3. *Suppose that all hypotheses in Theorem 3.2 hold and $H(t,s)$ is a function satisfying the conditions (i)–(ii).*

If there exist some $c_i \in (a_i, b_i)$, $i = 1,2$, and some positive function $\rho \in C^1([t_0,\infty),\mathbb{R})$ such that

$$\int_{a_i}^{c_i} \left[H^p(s,a_i)\,Q(s)\rho(s) - \Phi r(s)\rho(s)H_1^p(s,a_i)\right] ds > 0 \tag{3.7}$$

$$\int_{c_i}^{b_i} \left[H^p(b_i,s)\,Q(s)\rho(s) - \Phi r(s)\rho(s)H_2^p(b_i,s)\right] ds > 0 \tag{3.8}$$

for $i = 1,2$, where γ, H_1, H_2, and Φ are similar to that ones in Theorem 3.2, then Eq. (1.1) *is oscillatory.*

The second corollary is an analog of Corollary 2.3.

Corollary 3.4. *Suppose that for any $T \geqslant t_0$, there exist $T \leqslant a_1 < 2c_1 - a_1 \leqslant a_2 < 2c_2 - a_2$ such that* (2.15) *holds for the function e, and $\rho \equiv 1$.*

If there exists a function $H := H(t-s)$ satisfying the conditions (i)–(iii) such that

$$\int_{a_i}^{c_i} H^p(s-a_i)\left[Q(s) + Q(2c_i - s)\right] ds$$

$$> \Phi \int_{a_i}^{c_i} \left[r(s) + r(2c_i - s)\right]\left(h(s-a_i)\sqrt{H(s-a_i)}\right)^p ds, \tag{3.9}$$

for $i = 1,2$, then Eq. (1.1) *is oscillatory.*

Remark 3.5. Likewise, we can replace the condition (2.15) in for the function e in Corollary 2.3 to the condition in Remark 2.4.

References

[1] D. Çakmak and A. Tiryaki, Oscillation criteria for certain forced second-order nonlinear differential equations, *Appl. Math. Lett.*, **17** (2004), 275-279.

[2] H.L. Hong, C.C. Yeh and C.H. Hong, Oscillation criteria for nonlinear differential equations with integrable coefficient, *Math. Nachr.*, **278**, No 1-2 (2005), 145-153.

[3] F. Jiang and F. Meng, New oscillation criteria for a class of second-order nonlinear forced differential equations, *J. Math.Anal. Appl.*, **336** (2007), 1476-1485.

[4] Q. Kong, Interval criteria for oscillation of second-order linear ordinary differential equations, *J. Math. Anal. Appl.*, **229** (1999), 258-270.

[5] W.T. Li, Interval oscillation criteria for second-order quasi-linear nonhomogeneous differential equations with damping, *Appl. Math. Comput.*, **147** (2004), 753-763.

[6] A. Tiryaki and B. Ayanlar, Oscillation theorems for certain nonlinear differential equations of second order, *Comput. Math. Appl.*, **47** (2004), 149-159.

[7] A. Tiryaki and A. Zafer, Global existence and boundedness for a class of second-order nonlinear differential equations, *Appl. Math. Lett.*, **17** (2013), 939-944.

[8] A. Tiryaki, D. Çakmak and B. Ayanlar, On the oscillation of certain second-order nonlinear differential equations, *J. Math. Anal. Appl.*, **281** (2003), 565-574.

[9] J.S. Wong, Second order nonlinear forced oscillations, *SIAM J. Math. Anal.*, **19**, No 3 (1998), 667-675.