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NEW COMPARISON OF INTERVAL NUMBERS

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Abstract: Unlike real numbers which are simply ordered, the problem of ordering interval numbers is a challenging problem. Several methods have been proposed to compare intervals. In this paper, we propose two new methods to compare two interval numbers. The relations $\leq_k$ and $\leq_F$ on the space of all interval numbers are proposed.

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1. Introduction

The problem of comparing interval numbers plays an important role in decision making problems under interval environment. In the last years, several methods have been proposed to compare two intervals. The foremost work was done by Moore [7] who studied the arithmetic of interval numbers. There are numbers of definitions of the ordering relation over intervals [2, 3, 4, 5, 8, 9]. Ishibuchi and Tanaka [6] suggested three order relations which depends on the endpoints of intervals, or the midpoint and radius.

In this paper, we propose two new methods for comparing two interval numbers.

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2. Interval Comparing Methods

In this section, we review some order relations. An interval number a is generally represented as $[\underline{a}, \overline{a}]$ where $\underline{a} \leq \overline{a}$. If $\underline{a} = \overline{a}$, then a will be degenerate. By $a_c = \frac{1}{2}(\underline{a} + \overline{a})$ and $a_\Delta = \frac{1}{2}(\overline{a} - \underline{a})$, we denote the center and the radius of a, respectively. Interval arithmetic is defined in [1]. First we define three order relations, [6].

Definition 2.1. Let a and b be two intervals. The order relations are defined as:

$$\begin{aligned} &1) \quad a \leq_1 b \Leftrightarrow \underline{a} \leq \underline{b} \wedge \overline{a} \leq \overline{b}, \\ &2) \quad a \leq_2 b \Leftrightarrow a_c \leq b_c \wedge a_\Delta \leq b_\Delta, \\ &3) \quad a \leq_3 b \Leftrightarrow a_c \leq b_c \wedge \overline{a} \leq \overline{b}. \end{aligned}$$

Note $a \leq_3 b$ if and only if $a \leq_1 b$ or $a \leq_2 b$.

Definition 2.2. An interval inequality $a \leq b$ is weakly feasible if and only if $\underline{a} \leq \overline{b}$.

Definition 2.3. An interval inequality $a \leq b$ is strongly feasible if and only if $\overline{a} \leq \underline{b}$.

3. New Comparison Methods

In this section, we define new relations for comparing intervals.

Definition 3.1. For two interval numbers a and b, we define

$$k(a,b) = \begin{cases} a_c - b_c & a_c \neq b_c \\ a_\Delta - b_\Delta & a_c = b_c. \end{cases}$$

Definition 3.2. For two interval numbers a and b, we define $a \leq_k b$ if and only if $k(a,b) \leq 0$.

It is clear when a and b are real numbers, then " $\leq_k$ " is the ordinary inequality relation " $\leq$ " on the set of real numbers.

Proposition 3.1. $k(a,b) = 0$ *if and only if* $a = b$.

Proof. If $k(a,b) = 0$, then $a_\Delta - b_\Delta = 0$ and $a_c = b_c$. Therefore $a = b$. If $a = b$, then $a_c = b_c$ and $a_\Delta = b_\Delta$ and hence $k(a,b) = 0$. □

Theorem 3.1. *The relation "* $\leq_k$ *" is a partial order.*

Proof. Follows from the definition of k. □

Example. $k([2,3],[1,4]) < 0$, therefore $[2,3] \leq_k [1,4]$. $k([1,5],[7,9]) < 0$, therefore $[1,5] \leq_k [7,9]$.

Proposition 3.2. *If the inequality* $a \leq b$ *is strongly feasible, then* $a \leq_k b$.

Proof. Since inequality $a \leq b$ is strongly feasible, then $\underline{a} \leq \overline{a} \leq \underline{b} \leq \overline{b}$ and so

$$k(a,b) = a_c - b_c \leq b_c - b_c = 0,$$

therefore $a \leq_k b$. □

Proposition 3.3. *If* $a \leq_i b$ *for* $i = 1,2$, *then* $a \leq_k b$.

Proof. Follows straightforward from Definition 2.1. □

Definition 3.3. Let F be some fixed interval number called criterion. We define

$$T_F = \{a : a \leq_k F\}.$$

Also, for each $a, b \in T_F$, we define the relation " $\leq_F$ " ("$\leq$" under criterion F) as follows:

$$a \leq_F b \Leftrightarrow N_F(a,b) \leq 0,$$

where

$$N_F(a,b) = k(a + a_\Delta F, b + b_\Delta F).$$

Example. Suppose $F = [5,7], a = [1,3]$ and $b = [0,6]$. We have $k(a,F) \leq 0, k(b,F) \leq 0$, therefore $a, b \in T_F$,

$$\begin{aligned} N_F(a,b) &= k(a + a_\Delta F, b + b_\Delta F) \\ &= k([6,10],[15,27]) \\ &= 8 - 21 = -13 < 0. \end{aligned}$$

Therefore $a \leq_F b$.

Theorem 3.2. *The relation " $\leq_F$ " is the reflexive, transitive and complete on T_F.*

Proof. Suppose $a, b, c \in T_F$.

(1) Reflexivity:

$$N_F(a,a) = k(a + a_{\Delta}F, a + a_{\Delta}F) = 0,$$

therefore $a \leq_F a$.

(2) Transitivity: If $a \leq_F b$ and $b \leq_F c$, then $N_F(a,b) \leq 0$ and $N_F(b,c) \leq 0$, therefore

$$\begin{cases} (a + a_{\Delta}F)_c \leq (b + b_{\Delta}F)_c & (a + a_{\Delta}F)_c \neq (b + b_{\Delta}F)_c \\ (a + a_{\Delta}F)_{\Delta} \leq (b + b_{\Delta}F)_{\Delta} & (a + a_{\Delta}F)_c = (b + b_{\Delta}F)_c, \end{cases}$$

and

$$\begin{cases} (b + b_{\Delta}F)_c \leq (c + c_{\Delta}F)_c & (b + b_{\Delta}F)_c \neq (c + c_{\Delta}F)_c \\ (b + b_{\Delta}F)_{\Delta} \leq (c + c_{\Delta}F)_{\Delta} & (b + b_{\Delta}F)_c = (c + c_{\Delta}F)_c, \end{cases}$$

There are 4 cases. It is easy to prove that

$$\begin{cases} (a + a_{\Delta}F)_c \leq (c + c_{\Delta}F)_c & (a + a_{\Delta}F)_c \neq (c + c_{\Delta}F)_c \\ (a + a_{\Delta}F)_{\Delta} \leq (c + c_{\Delta}F)_{\Delta} & (a + a_{\Delta}F)_c = (c + c_{\Delta}F)_c, \end{cases}$$

(3) Completeness: If $a \nleq_F b$, then we will prove $b \leq_F a$. Since $a \nleq_F b$, then $N_F(a,b) > 0$. Therefore

$$k(a + a_{\Delta}F, b + b_{\Delta}F) > 0.$$

So

$$\begin{cases} (a + a_{\Delta}F)_c > (b + b_{\Delta}F)_c & (a + a_{\Delta}F)_c \neq (b + b_{\Delta}F)_c \\ (a + a_{\Delta}F)_{\Delta} > (b + b_{\Delta}F)_{\Delta} & (a + a_{\Delta}F)_c = (b + b_{\Delta}F)_c, \end{cases}$$

then $k(b + b_{\Delta}F, a + a_{\Delta}F) < 0$, and hence $N_F(b,a) \leq 0$. Therefore $b \leq_F a$. □

4. Conclusion

The problem of ordering interval numbers is studied. For any two intervals, there is not a natural ordering among the set of all intervals. There are several methods to compare two intervals. In this paper, we propose two new methods to compare interval numbers. The relations " $\leq_k$ " and " $\leq_F$ " on the space of all interval numbers are proposed.

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