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SPECTRAL CONTINUITY OF (p,k)-QUASIPOSINORMAL OPERATOR AND (p,k)-QUASIHYPONORMAL OPERATOR

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Abstract: An operator $T \in B(\mathcal{H})$ is said to be (p,k)-quasiposinormal operator, if $T^{*k}(c^2(T^*T)^p - (TT^*)^p)T^k \geq 0$ for a positive integer $0 < p \leq 1$, some $c > 0$ and a positive integer k. In this paper, we prove that, the (p,k) quasi-posinormal operator is a pole of resolvent of T^*. Then we prove that if $\{T_n\}$ is a sequence of operators in the class $(p,k) - \mathcal{Q}$ and $(p,k) - \mathcal{QP}$ which converges in the operator norm topology to an operator T in the same class, then the functions spectrum, Weyl spectrum, Browder spectrum and essential surjectivity spectrum are continuous at T.

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1. Introduction and Preliminaries

Let $\mathcal{H}$ be an infinite dimensional complex Hilbert space and $B(\mathcal{H})$ denote the algebra of all bounded linear operators acting on $\mathcal{H}$. Every operator T can be decomposed into $T=U|T|$ with a partial isometry U, where $|T|=\sqrt{T^*T}$. In [8], H.C. Rhaly Jr. introduced and studied posinormal operators. He showed a characterization of posinormality and spectral properties of posinormal operators. Moreover, he gave many fruitful examples of posinormal operators for the Casáro operator. As a further generalization of posinormal operators, M. Itoh [16] introduced p-posinormal operators and he proved that a p-posinormal

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operator is M-paranormal operator. An operator $T \in B(\mathcal{H})$ is positive, $T \geq 0$, if $(Tx, x) \geq 0$ for all $x \in \mathcal{H}$, and posinormal if there exists a positive $\lambda \in B(\mathcal{H})$ such that $TT^* = T^*\lambda T$. Here λ is called interrupter of T. In other words, an operator T is called posinormal if $TT^* \leq c^2 T^*T$, where T^* is the adjoint of T and $c > 0$, see [8]. An operator T is said to be heminormal, if T is hyponormal and T^*T commutes with TT^*. An operator T is said to be p-posinormal if $(TT^*)^p \leq c^2(T^*T)^p$ for some $c > 0$. It is clear that 1-posinormal is posinormal. In [19], M. Y. Lee and S. H. Lee have studied a structure theorem and some properties for (p, k)-quasi-posinormal operators. They have proved that if T is invertible, then T is (p, k)-quasiposinormal. Also T and T^* are (p, k)-quasi-posinormal for invertible T.

An operator T is said to be (p, k)-quasi-posinormal, if

$$T^{*k}(c^2(T^*T)^p - (TT^*)^p)T^k \geq 0,$$

where k is a positive integer, $0 < p \leq 1$ and $c > 0$. (p, k)-quasi-posinormal operated is denoted by $(p, k) - \mathcal{QP}$ a $(p, 1)$-quasi-posinormal is p-posinormal.

An operator $T \in B(\mathcal{H})$ is said to be (p, k)-quasihyponormal operator, denoted by $(p, k) - \mathcal{Q}$, for some $0 < p \leq 1$ and integer $k \geq 1$ if $T^{*k}(|T|^{2p} - |T^*|^{2p})T^k \geq 0$. Evidently, a $(1, k) - \mathcal{Q}$ operator is k-quasihyponormal, a $(1, 1) - \mathcal{Q}$ operator is quasihyponormal.

If $T \in B(\mathcal{H})$, we write $N(T)$ and $R(T)$ for null space and range of T, respectively. Let $\alpha(T) = \dim N(T) = \dim\,(T^{-1}(0))$, $\beta(T) = \dim N(T^*) = \dim\,(\mathcal{H}/T(\mathcal{H}))$, $\sigma(T)$ denote the spectrum and $\sigma_a(T)$ denote the approximate point spectrum. Then $\sigma(T)$ is a compact subset of the set $\mathbb{C}$ of complex numbers. The function σ viewed as a function from $B(\mathcal{H})$ into the set of all compact subsets of $\mathbb{C}$, with its Hausdorff metric, is know to be an upper semi-continuous function [14, Problem 103], but it fails to be continuous [14, Problem 102]. Also we know that σ is continuous on the set of normal operators in $B(\mathcal{H})$ extended to hyponormal operators [14, Problem 105]. The continuity of σ on the set of quasihyponormal operators (in $B(\mathcal{H})$) has been proved by Erevenko and Djordjevic [10], the continuity of σ on the set of p-hyponormal has been proved by Duggal and Djordjevic [9], and the continuity of σ on the set of G_1 - operators has been proved by Luecke [20].

An operator $T \in B(\mathcal{H})$ is called Fredholm, if it has closed range, finite dimensional null space and its range has finite co - dimension. The index of a Fredholm operator is given by $i(T) = \alpha(T) - \beta(T)$. The ascent of T, asc(T), is the least non-negative integer n such that $T^{-n}(0) = T^{-(n+1)}(0)$ and the descent of T, dsc(T), is the least non-negative integer n such that $T^n(\mathcal{H}) = T^{(n+1)}(\mathcal{H})$. We say that T is of finite ascent (resp., finite descent) if asc$(T - \lambda I) < \infty$ (resp., dsc$(T-\lambda I)<\infty$) for all complex numbers λ. T is said to be left semi-Fredholm (resp., right semi-Fredholm), $T\in\Phi_+(\mathcal{H})$ (resp., $T\in\Phi_-(\mathcal{H})$) if $T\mathcal{H}$ is closed and the deficiency index $\alpha(T)=\dim(T^{-1}(0))$ is finite (resp., the deficiency index $\beta(T)=\dim(\mathcal{H}\backslash T\mathcal{H})$ is finite); T is semi-Fredholm if it is either left semi-Frdholm or right semi-Fredholm, and T is Fredholm if it is both left and right semi-Fredholm. The semi-Fredholm index of T, ind(T), is the number ind(T) $=\alpha(T)-\beta(T)$. T is called Weyl, if it is Fredholm of index zero and Browder if it is Fredholm of finite ascent and descent. Let $\mathbb{C}$ denote the set of complex numbers. The Weyl spectrum $\sigma_w(T)$ and the Browder spectrum $\sigma_b(T)$ of T are the sets $\sigma_w(T)=\{\lambda\in\mathbb{C}:T-\lambda$ is not Weyl$\}$ and $\sigma_b(T)=\{\lambda\in\mathbb{C}:T-\lambda$ is not Browder$\}$.

Let $\pi_0(T)$ denote the set of Riesz points of T (i.e., the set of $\lambda\in\mathbb{C}$ such that $T-\lambda$ is Fredholm of finite ascent and descent [7]) and let $\pi_{00}(T)$ and iso$\sigma(T)$ denotes the set of eigen values of T of finite geometric multiplicity and isolated points of the spectrum. The operator $T\in B(\mathcal{H})$ is said to satisfy Browder's theorem, if $\sigma(T)\backslash\sigma_w(T)=\pi_0(T)$ and T is said to satisfy Weyl's theorem if $\sigma(T)\backslash\sigma_w(T)=\pi_{00}(T)$. In [15], Weyl's theorem for T implies Browder's theorem for T, and Browder's theorem for T is equivalent to Browder's theorem for T^*.

Berkani [5] has called an operator $T\in B(X)$ as B-Fredholm if there exists a natural number n for which the induced operator $T_n:T^n(X)\to T^n(X)$ is Fredholm. We say that the B-Fredholm operator T has stable index if ind$(T-\lambda)$ ind$(T-\mu)\geq 0$ for every λ,μ in the B-Fredholm region of T.

The essential spectrum $\sigma_e(T)$ of $T\in B(\mathcal{H})$ is the set $\sigma_e(T)=\{\lambda\in\mathbb{C}:T-\lambda$ is not Fredholm$\}$. Let acc$\sigma(T)$ denote the set of all accumulation points of $\sigma(T)$, then $\sigma_e(T)\subseteq\sigma_w(T)\subseteq\sigma_b(T)\subseteq\sigma_e(T)\cup acc\sigma(T)$. Let $\pi_{a0}(T)$ be the set of $\lambda\in\mathbb{C}$ such that λ is an isolated point of $\sigma_a(T)$ and $0<\alpha(T-\lambda)<\infty$, where $\sigma_a(T)$ denotes the approximate point spectrum of the operator T. Then $\pi_0(T)\subseteq\pi_{00}(T)\subseteq\pi_{a0}(T)$. We say that a-Weyl's theorem holds for T if

$$\sigma_{aw}(T)=\sigma_a(T)\backslash\pi_{a0}(T),$$

where $\sigma_{aw}(T)$ denotes the essential approximate point spectrum of T (i.e., $\sigma_{aw}(T)=\bigcap\{\sigma_a(T+K):K\in K(\mathcal{H})\}$ with $K(\mathcal{H})$ denoting the ideal of compact operators on $\mathcal{H}$). Let $\Phi_+(\mathcal{H})=\{T\in B(\mathcal{H}):\alpha(T)<\infty$ and $T(\mathcal{H})$ is closed$\}$ and $\Phi_-(\mathcal{H})=\{T\in B(\mathcal{H}):\beta(T)<\infty\}$ denote the semigroup of upper semi-Fredholm and lower semi-Fredholm operators in $B(\mathcal{H})$ and let $\Phi_+^-(\mathcal{H})=\{T\in\Phi_+(\mathcal{H}):\text{ind}(T)\leq 0\}$. Then $\sigma_{aw}(T)$ is the complement in $\mathbb{C}$ of all those λ for which $(T-\lambda)\in\Phi_+^-(\mathcal{H})$, see [22]. The concept of a-Weyl's theorem was introduced by Rakocvic [23]. The concept states that a-Weyl's theorem for T $\Rightarrow$ Weyl's theorem for T, but the converse is generally false. Let $\sigma_{ab}(T)$ denote the Browder essential approximate point spectrum of T.

$$\begin{aligned}\sigma_{ab}(T) &= \bigcap\{\sigma_a(T+K) : TK = \text{K T and K} \in K(\mathcal{H})\} \\ &= \{\lambda \in \mathbb{C} : T-\lambda \notin \Phi_+^-(\mathcal{H}) \text{ or asc(T} - \lambda) = \infty\},\end{aligned}$$

then $\sigma_{aw}(T) \subseteq \sigma_{ab}(T)$. We say that T satisfies a Browder's theorem, if $\sigma_{ab}(T) = \sigma_{aw}(T)$, see [22].

An operator $T \in B(\mathcal{H})$ has the single valued extension property at $\lambda_0 \in \mathbb{C}$, if for every open disc D_{λ_0} centered at λ_0 the only analytic function $f : D_{\lambda_0} \to \mathcal{H}$ which satisfies

$$(T-\lambda)f(\lambda)=0 \text{ for all } \lambda \in D_{\lambda_0}$$

is the function $f \equiv 0$. Trivially, every operator T has SVEP at points of the resolvent $\rho(T) = \mathbb{C}/\sigma(T)$; also T has SVEP at $\lambda \in \text{iso}\sigma(T)$. We say that T has SVEP if it has SVEP at every $\lambda \in \mathbb{C}$. In this paper, we prove that the continuity of the set theoretic functions spectrum, Weyl spectrum, Browder spectrum and essential surjectivity spectrum on the classes consisting of (p,k) - quasihyponormal operators and (p,k) - quasi-posinormal operators. Note that if an operator T has finite ascent, then it has SVEP and $\alpha(T-\lambda) \leq \beta(T-\lambda)$ for all λ [2, Theorem 3.8 and 3.4]. For a subset S of the set of complex numbers, let $\overline{S} = \{\overline{\lambda} : \lambda \in S\}$ where λ denotes the complex number and $\overline{\lambda}$ denotes the conjugate.

2. Main Results

Lemma 2.1. *Let $T \in (p,k)$-quasiposinormal operator. If $\overline{\lambda} \in \pi_{00}(T^*)$, then it is a pole of the resolvent of T^*.*

Proof. If $0 \neq \overline{\lambda} \in \pi_{00}(T^*)$, then $\lambda \in iso\sigma(T) \Rightarrow \lambda$ is a normal of eigen value of T ([17], Lemma 2.3) and hence a simple pole of the resolvent of T ([17], Cor. 2.8). If instead, $\lambda = 0$ then dim ker$T^* < \infty \Rightarrow$ ran T^* is closed and hence $T^* \in \Phi_+(\mathcal{H})$ implies $T \in \Phi_-(\mathcal{H})$. Since both T and T^* have SVEP at 0, it follows that, $asc(T) = dsc(T) < \infty$ (See [1], Theorem 2.3). Hence 0 is a pole of the resolvent of T implies 0 is the pole of the resolvent of T^* □

Lemma 2.2. *(i) If $T \in (p,k) - \mathcal{Q}$, then $asc(T-\lambda) \leq k$ for all λ.*

(ii) If $T \in (p,k) - \mathcal{QP}$, then T has SVEP.

Proof. (i) Proof of (i) is [13, Page 146] or [25].

(ii) Proof of (ii) is [24, Lemma 2.3]. □

Lemma 2.3. *If $T \in (p,k) - \mathcal{Q} \cup (p,k) - \mathcal{QP}$ and $\lambda \in iso\sigma(T)$, then λ is a pole of the resolvent of T.*

Proof. Proof of this lemma is [25, Theorem 6] and [17, Corollary 2.8]. □

Lemma 2.4. *If $T \in (p,k) - \mathcal{Q} \cup (p,k) - \mathcal{QP}$, then T^* satisfies a-Weyl's theorem.*

Proof. If $T \in (p,k) - \mathcal{Q}$, then T has SVEP, which implies that $\sigma(T^*) = \sigma_a(T^*)$ by [2, Corollary 2.45]. Then T satisfies Weyl's theorem i.e., $\sigma(T)\backslash\sigma_w(T) = \pi_0(T) = \pi_{00}(T)$ by [13, Corollary 3.7]. Since $\pi_{00}(T) = \overline{\pi_{00}(T^*)} = \overline{\pi_{a0}(T^*)}$, $\sigma(T) = \overline{\sigma(T^*)} = \overline{\sigma_a(T^*)}$ and $\sigma_w(T) = \overline{\sigma_w(T^*)} = \overline{\sigma_{ea}(T^*)}$ by [3, Theorem 3.6(ii)], $\sigma_a(T^*)\backslash\sigma_{ea}(T^*) = \pi_{a0}(T^*)$. Hence if $T \in (p,k) - \mathcal{Q}$, then T^* satisfies a-Weyl's theorem.

If $T \in (p,k) - \mathcal{QP}$, then by [24, Theorem 3.4], T^* satisfies a-Weyl's theorem. □

Corollary 2.5. *If $T \in (p,k) - \mathcal{Q} \cup (p,k) - \mathcal{QP}$, then $\lambda \in \sigma_a(T^*)\backslash\sigma_{ea}(T^*) \Rightarrow \lambda \in iso\sigma_a(T^*)$.*

Lemma 2.6. *If $T \in (p,k) - \mathcal{Q} \cup (p,k) - \mathcal{QP}$, then $asc(T - \lambda) < \infty$ for all λ.*

Proof. Since $T - \lambda$ is lower semi-Fredholm, it has SVEP. We know that from [2, Theorem 3.16] the SVEP implies finite ascent. Hence the proof follows. □

Lemma 2.7. *[6, Proposition 3.1] If σ is continuous at a $T^* \in B(\mathcal{H})$, then σ is continuous at T.*

Lemma 2.8. *[12, Theorem 2.2] If an operator $T \in B(\mathcal{H})$ has SVEP at points $\lambda \notin \sigma_w(T)$, then σ is continuous at $T \Leftrightarrow \sigma_w$ is continuous at $T \Leftrightarrow \sigma_b$ is continuous at T.*

Lemma 2.9. *If $\{T_n\}$ is a sequence in $(p,k) - \mathcal{Q}$ or $(p,k) - \mathcal{QP}$ which converges in norm to T, then T^* is a point of continuity of σ_{ea}.*

Proof. We have to prove that the function σ_{ea} is both upper semi-continuous and lower semi-continuous at T^*. But by [11, Theorem 2.1], we have that the function σ_{ea} is upper semi-continuous at T^*. So we have to prove that σ_{ea} is lower semi-continuous at T^* i.e., $\sigma_{ea}(T^*) \subset \lim \inf \sigma_{ea}(T_n^*)$. Assume the contradiction that σ_{ea} is not lower semi - continuous at T^*. Then there exists an $\epsilon > 0$, an integer n_0, a $\lambda \in \sigma_{ea}(T^*)$ and an ϵ-neighbourhood $(\lambda)_\epsilon$ of λ such that $\sigma_{ea}(T_n^*) \cap (\lambda)_\epsilon = \emptyset$ for all $n \geq n_0$. Since $\lambda \notin \sigma_{ea}(T_n^*)$ for all $n \geq n_0$ implies $T_n^* - \lambda \in \Phi_+^-(\mathcal{H})$ for all $n \geq n_0$, the following implications hold:

$$\begin{aligned} ind(T_n^* - \lambda) \leq 0, \alpha(T_n^* - \lambda) < \infty \text{ and } (T_n^* - \lambda)\mathcal{H} \text{ is closed} \\ \Rightarrow ind(T_n - \overline{\lambda}) \geq 0, \beta(T_n - \overline{\lambda}) < \infty \\ \Rightarrow ind(T_n - \overline{\lambda}) = 0, \alpha(T_n - \overline{\lambda}) = \beta(T_n - \overline{\lambda}) < \infty \\ (\text{Since } T_n \in (p,k) - \mathcal{Q} \cup (p,k) - \mathcal{QP} \Rightarrow ind(T_n - \overline{\lambda}) \leq 0 \\ \text{by Lemma 2.2 and Lemma 2.6}) \end{aligned}$$

for all $n \geq n_0$. The continuity of the index implies that $\text{ind}(T - \overline{\lambda}) = \lim_{n\to\infty} \text{ind}(T_n - \overline{\lambda}) = 0$, and hence that $(T - \overline{\lambda})$ is Fredholm with $\text{ind}(T - \overline{\lambda}) = 0$. But then $T^* - \lambda$ is Fredholm with $\text{ind}(T^* - \lambda) = 0 \Rightarrow T^* - \lambda \in \Phi_+^-(\mathcal{H})$, which is a contradiction. Therefore σ_{ea} is lower semi - continuous at T^*. Hence the proof follows. □

Theorem 2.10. *If $\{T_n\}$ is a sequence in $(p,k) - \mathcal{Q}$ or $(p,k) - \mathcal{QP}$ which converges in norm to T, then σ is continuous at T.*

Proof. Since T has SVEP by Lemma 2.2, $\sigma(T^*) = \sigma_a(T^*)$. Evidently, it is enough if we prove that $\sigma_a(T^*) \subset \lim \inf \sigma_a(T_n^*)$ for every sequence $\{T_n\}$ of operators in $(p,k) - \mathcal{Q}$ or $(p,k) - \mathcal{QP}$ such that T_n converges in norm to T. Let $\lambda \in \sigma_a(T^*)$. Then either $\lambda \in \sigma_{ea}(T^*)$ or $\lambda \in \sigma_a(T^*) \backslash \sigma_{ea}(T^*)$.

If $\lambda \in \sigma_{ea}(T^*)$, then the proof follows, since

$$\sigma_{ea}(T^*) \subset \lim \inf \sigma_{ea}(T_n^*) \subset \lim \inf \sigma_a(T_n^*).$$

If $\lambda \in \sigma_a(T^*) \backslash \sigma_{ea}(T^*)$, then $\lambda \in \text{iso}\sigma_a(T^*)$ by Corollary 2.5. Consequently, $\lambda \in \lim \inf \sigma_a(T_n^*)$ i.e., $\lambda \in \lim \inf \sigma(T_n^*)$ for all n by [18, Theorem IV. 3.16], and there exists a sequence $\{\lambda_n\}$, $\lambda_n \in \sigma_a(T_n^*)$, such that λ_n converges to λ. Evidently $\lambda \in \lim \inf \sigma_a(T_n^*)$. Hence $\lambda \in \lim \inf \sigma(T_n^*)$. Now by applying Lemma 2.7, we obtain the result. □

Corollary 2.11. *If $\{T_n\}$ is a sequence in $(p,k) - \mathcal{Q}$ or $(p,k) - \mathcal{QP}$ which converges in norm to T, then σ, σ_w and σ_b are continuous at T.*

Proof. Combining Theorem 2.10 with Lemma 2.8 and Lemma 2.9, we obtain the result. $\square$

Let $\sigma_s(T) = \{\lambda : T - \lambda \text{ is not surjective}\}$ denote the surjectivity spectrum of T and let $\Phi_-^+(\mathcal{H}) = \{\lambda : T-\lambda \in \Phi_-(\mathcal{H}),\ \text{ind}(T-\lambda) \geq 0\}$. Then the essential surjectivity spectrum of T is the set $\sigma_{es}(T) = \{\lambda : T - \lambda \notin \Phi_-^+(\mathcal{H})\}$.

Corollary 2.12. *If $\{T_n\}$ is a sequence in $(p,k) - \mathcal{Q}$ or $(p,k) - \mathcal{QP}$ which converges in norm to T, then σ_{es} is continuous at T.*

Proof. Since T has SVEP by Lemma 2.2, $\sigma_{es}(T) = \sigma_{ea}(T^*)$ by [2, Theorem 3.65 (ii)]. Then by applying Lemma 2.9, we obtain the result. $\square$

Let $\mathcal{K} \subset B(\mathcal{H})$ denote the ideal of compact operators, $B(\mathcal{H})/\mathcal{K}$ the Calkin algebra and let $\pi : B(\mathcal{H}) \to B(\mathcal{H})/\mathcal{K}$ denote the quotient map. Then $B(\mathcal{H})/\mathcal{K}$ being a C^* - algebra, there exists a Hilbert space $\mathcal{H}_\prime$ and an isometric $*$ - isomorphism $\nu : B(\mathcal{H})/\mathcal{K} \to B(\mathcal{H}_\prime)$ such that the essential spectrum $\sigma_e(T) = \sigma(\pi(T))$ of $T \in B(\mathcal{H})$ is the spectrum of $\nu \circ \pi(T)$ ($\in B(\mathcal{H}_\prime)$). In general, $\sigma_e(T)$ is not a continuous function of T.

Corollary 2.13. *If $\{\pi(T_n)\}$ is a sequence in $(p,k) - \mathcal{Q}$ or $(p,k) - \mathcal{QP}$ which converges in norm to $\pi(T)$, then σ_e is continuous at T.*

Proof. If $T_n \in B(\mathcal{H})$ is essentially $(p,k) - \mathcal{Q}$ or $(p,k) - \mathcal{QP}$, i.e., if $\pi(T_n) \in (p,k) - \mathcal{Q}$ or $(p,k) - \mathcal{QP}$, and the sequence $\{T_n\}$ converges in norm to T, then $\nu \circ \pi(T) \in B(\mathcal{H}_\prime)$ is a point of continuity of σ by Theorem 2.10. Hence σ_e is continuous at T, since $\sigma_e(T) = \sigma(\nu \circ \pi(T))$. $\square$

Let $\mathcal{H}(\sigma(T))$ denote the set of functions f that are non-constant and analytic on a neighbourhood of $\sigma(T)$.

Lemma 2.14. *Let $T \in B(X)$ be an invertible $(p,k) - \mathcal{QP}$ has SVEP, then ind$(T - \lambda) \leq 0$ for every $\lambda \in \mathcal{C}$ such that $T - \lambda$ is B-Fredholm.*

Proof. T has SVEP by [24, Lemma 2.3]. Then $T|_M$ has SVEP for every invariant subspaces $M \subset X$ of T. From [4, Theorem 2.7], we know that if $T - \lambda$ is a B-Fredholm operator, then there exist $T - \lambda$ invariant closed subspaces M and N of X such that $X = M \oplus N$, $(T - \lambda)|_M$ is a Fredholm operator with SVEP and $(T - \lambda)|_N$ is a Nilpotent operator. Since $\text{ind}(T - \lambda)|_M \leq 0$ by [21, Proposition 2.2], it follows that $\text{ind}(T - \lambda) \leq 0$. $\square$

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