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ON REGULAR AND IRREGULAR BEHAVIOR OF A SPRAY TOWER WITH PERIODIC COEFFICIENTS

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Abstract: This paper analyzes the dynamic of the spray system that consists of a spray tower mounted on trailer and his main contribution was to consider a gap at the junction point between trailer and spray tower. The analysis of the dynamic stability of the system, especially the spray tower, is of fundamental importance because the mechanical vibrations of the system can affect the quality and efficiency of application during the work in the field. The mathematical model of this agricultural implement has three degrees of freedom and, with periodic vibrations at the junction, the system of differential equations is nonlinear, second-order with time-periodic coefficients. For the analysis of the problem, we used a technique based on the Chebyshev polynomial expansion, the iterative Picard and transformation of Lyapunov-Floquet (LF). In the numerical simulations, we did the phase planes and the diagram of stability, varying the torsional stiffness, the amplitude and the frequency of vibration in the joint point. We verified that there is instability in the system to some set parameters, which can produce chaotic motions in the spray tower.

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1. Introduction

The application of chemical pesticides to control pests, diseases and weeds is a common practice in industrial cultivars of medium and large companies. In general, the application of defense is performed with the aid of an airblast sprayer, which travels between the plants traction for a tractor, so that the phytosanitary product distribution is uniform and optimum amount.

Moreover, the driving of spraying equipment during work in the field, is a complex task. The tractor driver must operate the tractor-implement a rate that meets the requirements of cost, range and volume of spray and also must maintain an appropriate distance from the tops of the plants which promotes the quality of the application. Depending on the conditions of the trace, the increase in speed may produce unstable vibrations in the agricultural implement, trailer sprayer and the spray tower oscillations can damage the quality of the application.

For a better understanding of the dynamics of movement of the sprayer, when it is in operation in the field, we can use the mathematical modeling. The simulation and stability analysis of the model spray-trailer when working on the field may be of great importance for increasing the accuracy and better control in plant cultivars. The improvement application procedures can reduce the amount of fungicide used and hence reduce the production costs, contamination of agricultural products and the environment.

In [1] a system in a spray tower type was studied. In this work it was obtained a physical model of spray and also the dynamics of the real system and mathematical model (Figure 1a) was analyzed. In the physical model two types of junctions were considered: rigid and flexible, and were analyzed system responses for different torsional rigidity coefficients. In the mathematical model of the spray system there is three degrees of freedom and three types of movements are considered: the lateral, vertical and rotational about a plan, based on the movements of an inverted pendulum.

By adapting the model from [1], in [2] the author studied the stability of the nonlinear model of a spray tower type stability criterion by Routh-Hurwitz. Thus, [2] investigated the behavior of the system through an external harmonic excitation of the set trailer-sprayer and in analysis developed the torsional stiffness of the joint and amplitude and frequency of the excitation were varied.

The research shows that the mechanical systems may exhibit chaos due to the imperfect nature of the assembly (joints) or clearances due to wear, friction, and fatigue crack propagation. However, in [1], [2] there were no considerations on the existence of wear on joints whose gaps can produce unwanted vibrations in the spray system.

The vibrations at the junction point of the trailer-spray tower system can be modelled by periodic parametric excitations in support. Thus, the system of nonlinear equations for the problem of trailer sprayer features linear part with periodic coefficients and the State Transition Matrix (STM) of the system can be obtained numerically, through the expansion of the Chebyshev polynomials [5]-[9] and the interactive method of Picard.

In this work, we considered the problem of oscillations, auto-excited due to the existence of gaps in the agricultural implement, introducing a small vertical vibration and periodic in sup-port of the inverted pendulum, the model spray tower type proposed in [1]. Thus, we modeled the spray system with periodic excitation in support, by Lagrange equation and analyze the local stability of the system by varying the torsional stiffness and the parameters of the periodic vertical vibration, amplitude and excitation frequency.

2. Modelling of the Spray Tower

The physical model of the trailer-spray tower system with three degrees of freedom is shown in Figure 1 and the mathematical model was obtained by the Lagrange equation.

2.1. Physical Model

Figure 1a represents the physical model of the trailer-spray tower system. The sprayer tower was modeled by inverted pendulum with a torsion spring, whose C_T and k_t represents the damper e torsional stiffness, respectively [1]. The distance between the center of mass of the trailer m_c and the joint point P is l_c. The spray tower is described by a long stem l_t with mass m_t concentrated at its end. The angular displacement of the trailer and the spray tower are given by ϕ_c and ϕ_t. The tires of the trailer are excited externally by y_{e1} and y_{e2}.

Furthermore, this paper considers a periodic parametric excitation $Asin(\omega t)$ to represent a vertical periodic vibration at the junction point P, where $\bar{A}$ is the a amplitude and ω is the vibration frequency. Figure 1b shows the oscillations

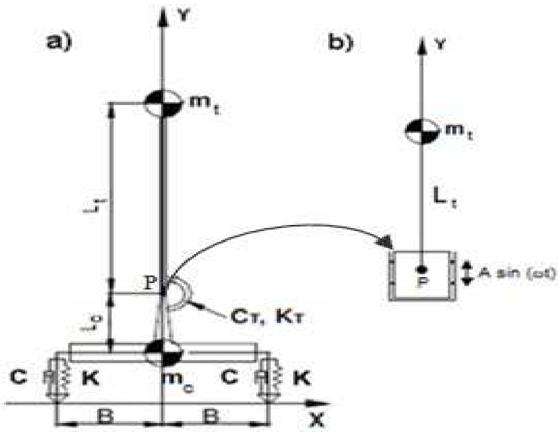

Figure 1: (a) Physical model of the trailer-spray system (modified of [1]), (b) Joint point vibration.

auto-excited due the existence of gaps in the agricultural implement.

2.2. Equations Spray System

The equations of motion of the system are obtained by the Lagrange equation. Initially, we obtained the equations of the kinetic and potential energies of the system:

$$\begin{aligned} T = (\frac{m_c + m_t}{2})(\dot{x}_c^2 + \dot{y}_c^2) + \frac{m_t}{2}(l_c^2\dot{\phi}_c^2 + l_t^2\dot{\phi}_t^2) + m_t l_c l_t \dot{\phi}_c \dot{\phi}_t (cos(\phi_c - \phi_t)) \\ - m_l \dot{x}_c l_c \dot{\phi}_c cos(\phi_c) - m_l \dot{x}_c l_t \dot{\phi}_t cos(\phi_t) - m_t \dot{y}_c l_c \dot{\phi}_c sin(\phi_c) \\ - m_t \dot{y}_c l_t \dot{\phi}_t sin(\phi_t) + \frac{I_c \dot{\phi}_c^2 + I_t \dot{\phi}_t^2}{2} + m_t \dot{y}_c \bar{A}\omega cos(\omega t) - \\ m_t l_c \dot{\phi}_c sin(\phi_c)\bar{A}\omega cos(\omega t) - m_t l_t \dot{\phi}_t sin(\phi_t)\bar{A}\omega cos(\omega t) + \frac{m_t}{2}(\bar{A}\omega)^2 cos^2(\omega t), \quad (1) \end{aligned}$$

$$\begin{aligned} V = m_c g y_c + \frac{k_{c_1}[y_c - B_1 sin(\phi_c) - y_{e_1}]^2}{2} + \frac{k_{c_2}[y_c - B_2 sin(\phi_c) - y_{e_2}]^2}{2} \\ + m_t g[y_c + l_c cos(\phi_c) + l_t cos(\phi_t) + \bar{A} sin(\omega t)] + \frac{k_c(\phi_c - \phi_t)^2}{2}. \quad (2) \end{aligned}$$

Then, using the Lagrangian function ($L = TV$), the governing equations of the motions of the trailer-spray problem can be written by the system of nonlinear equations:

$$
\begin{gathered}
(m_c+m_t)\ddot{y_c} - m_t l_c sin(\phi_c)\ddot{\phi_c} - m_t l_t sin(\phi_t)\ddot{\phi_t} - \\
m_t l_c dot\phi_c^2 cos(\phi_c) - m_t l_t \dot{\phi_t^2} cos(\phi_t) - m_t \bar{A}\omega^2 sin(\omega t) + (k_{c_1}+k_{c_2})y_c + \\
(k_{c_2}B_2 - kc_1 B_1)sin(\phi_c) + (C_1+C_2)\dot{y_c} + (C_2B_2 - B_1C_1)\dot{\phi_c}cos(\phi_c) \\
= k_{c_1}y_{e_1} + k_{c_2}y_{e_2} + C_1\dot{y_{e_1}} + C_2\dot{y_{e_2}} - (m_c+m_t)g \\
- m_t l_c sin(\phi_c)\ddot{y_c} + (m_t l_c^2 + I_c)\phi_c + m_t l_c l_t cos(\phi_c - \phi_t)\phi_t + \\
m_t l_c l_t \phi_t^2 sin(\phi_c - \phi_t) + m_t l_c \dot{x_c}\phi_c sin(\phi_c) + m_t l_c \bar{A}\omega^2 sin(\omega t) sin(\phi_c) - \\
g m_t l_c sin(\phi_c) - (k_{c_1}B_1 - k_{c_2}B_2)cos(\phi_c)y_c + \\
(k_{c_1}B_1^2 + k_{c_2}B_2^2)cos(\phi_c)sin(\phi_c) + \\
(k_{c_1}y_{e_1}B_1 - k_{c_2}y_{e_2}B_2)cos(\phi_c) + k_t\phi_c - +k_t\phi_t + C_T\dot{\phi_c} - C_T\dot{\phi_t} - \\
m_t l_c \ddot{x_c} cos(\phi_c) - m_t l_t \dot{x_c}\dot{\phi_t} sin(\phi_t) = -(C_1B_1^2 + C_2B_2^2)\dot{\phi_c}cos^2(\phi_c) - \\
(C_2B_2 - C_1B_1)\dot{y_c}cos(\phi_c) - C_1B_1\dot{y_{e_1}}cos(\phi_c) + C_2B_2\dot{y_{e_2}}cos(\phi_c) \\
- m_t l_t sin(\phi_t)\ddot{y_c} + (m_t l_t^2 + I_t)\ddot{\phi_t} + m_t l_c l_t cos(\phi_c - \phi_t)\ddot{\phi_c} \\
- m_t l_c l_t \dot{\phi_c^2} sin(\phi_c - \phi_t) - m_t l_t \dot{y_c}\dot{\phi_t} cos(\phi_t) + m_t l_t \bar{A}\omega^2 sin(\omega t) sin(\phi_t) - \\
m_t l_t \dot{x_c}\dot{\phi_t} sin(\phi_t) + m_t l_t \dot{y_c}\dot{\phi_t} cos(\phi_t) - g m_t l_t sin(\phi_t) - k_t\phi_c + k_t\phi_t - \\
C_T\dot{\phi_c} + C_T\dot{\phi_t} - m_t l_t \ddot{x_c} cos(\phi_t) + m_t l_t \dot{x_c}\dot{\phi_t} sin(\phi_t) = 0. \quad (3)
\end{gathered}
$$

In state-space form, the system of nonlinear equations of problem-trailer sprayer features linear part with periodic coefficients:

$$
\begin{pmatrix}\dot{x_1}\\ \dot{x_2}\\ \dot{x_3}\\ \dot{x_4}\\ \dot{x_5}\\ \dot{x_6}\end{pmatrix} =
\begin{pmatrix}
0 & 1 & 0 & 0 & 0 & 0 \\
-\frac{k_c}{m_{ct}} & -\frac{C}{m_{ct}} & -\frac{k_B}{m_{ct}} & -\frac{C_B}{m_{ct}} & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
-F\cdot F_2\cdot k_B & -F\cdot F_2\cdot C_B & -F\cdot a_{43} & -F\cdot a_{44} & -F\cdot a_{45} & -F\cdot a_{46} \\
0 & 0 & 0 & 0 & 0 & 1 \\
F\cdot ml_{ct}\cdot k_B & F\cdot ml_{ct}\cdot C_B & F\cdot a_{63} & F\cdot a_{64} & -F\cdot a_{65} & -F\cdot a_{66}
\end{pmatrix}
\cdot \begin{pmatrix}x_1\\ x_2\\ x_3\\ x_4\\ x_5\\ x_6\end{pmatrix} + \begin{pmatrix}0\\ F\cdot NL_1\\ 0\\ F\cdot NL_2\\ 0\\ F\cdot NL_3\end{pmatrix}, \quad (4)
$$

where

$$
\begin{aligned}
a_{43} =& F_2[m_t l_c \bar{A}\omega^2 sin(\omega t) + (k_{c1}B_1^2 + k_{c2}B_2^2) + k_t - g m_t l_c] + m l_{ct} k_t,\\
a_{44} =& F_2 C_{B_T} + m l_{ct} C_T,\\
a_{45} =& m l_{ct}[m_t l_c \bar{A}\omega^2 sin(\omega t) - g m_t l_t + k_t] + F_2 k_t,\\
a_{46} =& F_2 + m l_{ct} C_T,\\
a_{63} =& m l_{ct}(m_t l_c \bar{A}\omega^2 sin(\omega t) + (k_{c_1}B_1^2 + k_{c_2}B_1^2) + k_t - g m_t l_c) + F_1 k_t\\
a_{64} =& m l_{ct} C_{B_T} + F_1 C_T,\\
a_{65} =& F_1(m_t l_t \bar{A}\omega^2 sin(\omega t) - g m_t l_t + k_t) + m l_c k_t,\\
a_{66} =& [m l_{ct} + f_1] C_T.
\end{aligned}
$$

For the periodic solutions of the nonlinear dynamical system with periodic coefficients, such as (4), the Floquet theory provides the theoretical basis for analysis of the stability of these solutions from the eigenvalues of the Monodromy Matrix or State Transition Matrix (STM) of the linear part of the system (4), [3]-[4]. An approximation of the State Transition Matrix (STM) of the linear part of the system-trailer sprayer (4) was obtained numerically, through the expansion of Chebyshev polynomials [5]-[9] and interactive method of Picard. The Chebyshev polynomials of the first kind were used, modified with grade 20 and the number of interactions of Picard was 40. The numerical simulations, that we carried out were implemented in *Matlab* 6.1 (The Mathworks, Inc., 2001, USA).

3. Numerical Simulations Results and Dynamics Analysis

For local stability analysis and the bifurcations boundaries of the nonlinear dynamics of the sprayer system (4), the states transition matrix in function of some parameters of the linearization of the system was obtained. Control Parameters used in the simulations are torsional stiffness (k_t) and the parameters of the vertical periodic vibration, amplitude (A) and excitation frequency ω, at the junction point P.

For this purpose, the values of the parameter must be found in the Euclidian space $(A, \omega, k_t) \in R^3$, where the system is critical, ie, the values of the amplitude (A), of the excitation frequency (ω) and of the torsional stiffness (k_t) for which the Floquet multipliers approaches 1. The other values of the parameters of the spray system (4), as in [1]-[2], are:

$$g = 9.81m/s^2;\ b_1 = b_2 = 0.85m;\ l_c = 0.2m;\ i_c = 6850kgm^2;$$
$$l_t = 2.4m;\ i_t = 6250kgm^2;\ m_c = 6500kg;\ m_t = 800kg;$$
$$k_{cl} = 465000N/m;\ c_2 = 465000N/m;\ c_1 = 5600s/m;\ c_2 = 5600s/m.$$

If the Floquet transition matrix (STM), associated with the linear part of system (4), has a Floquet multiplier equal to 1 and others have magnitudes less than 1, then the spray system model describes a Flip (period doubling) bifurcation. If a Floquet multiplier is equal 1, and others have magnitudes less than 1, then the spray system model describes a Fold (transcritical and symmetry breaking) bifurcation. Finally, if it exists one complex pair on the unit circle, and others have magnitudes less than 1, then the spray system model describes a Hopf bifurcation.

Varying the torsional stiffness $1.000 \leq k_t \leq 25.000$ (N m/rad), the parameters of the periodic parametric excitation $0 \leq A \leq 1$ and $0 < \omega \leq 4\pi$ and considering the parameters values set (5), the Floquet multipliers of the STM, associated with the system (4), show that the periodic solutions of this system became nonhyperbolic at a certain localization in the state-control space.

Figure 2 presents the stability diagram obtained in the state-control space $(A, \omega, k_t) \in R^3$, where it is observed the surface generated by cyclic-fold bifurcations of the spray system. Figure 3 shows the orthogonal projections of this surface in the state-control plane $(A \times \omega)$. Elapsed time was 520480 seconds (145h or 6d).

Analyzing qualitatively the surface of the cyclic-fold bifurcations, in Figure 2, we observe that increasing the values of the torsional stiffness parameter, the periodic solutions of the system (4) become more nonhyperbolic indicating the existence of the qualitative changes in dynamic of the system.

In fact, the level curves in Figure 3, show that the curves of the cyclic-fold bifurcations moving from right to left are increasing the stability region of the solution of the system (4) in the state-control plane $(A \times \omega)$.

To analyze quantitatively the influence of the torsional stiffness on the dynamic of the trailer-spray system and determine accurately the stability regions on the state-control plane $(A \times \omega)$, we studied individually, the level curves of the cyclic-fold bifurcations 3D surface, see Figure 2. The values of the torsional stiffness analyzed, are: $k_t = 1.000; 10.000; 19.000$ and 19.363 (N m/rad).

First, we considered that torsional stiffness of the spray system model is $k_t = 1.000 (Nms/rad)$. In Figure 4, we observe that in the state-control plane $(A \times \omega)$, the system is predominantly unstable and the fold bifurcations curve borders on a small stable re-gion in the upper right of the figure.

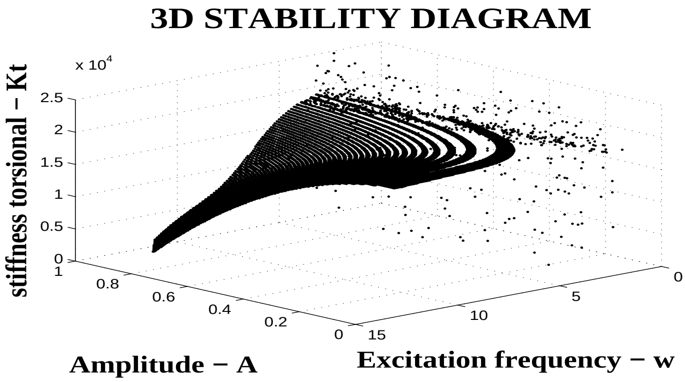

Figure 2: 3D stability diagram of the spray tower system.

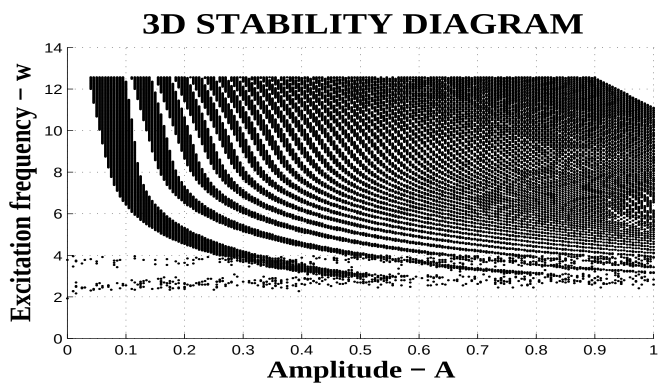

Figure 3: 2D stability diagram of the spray tower system.

In fact, for $A = 0.95$ and $\omega = 11$, there is a unstable Floquet multiplier $\mu_1 = 1.0217969$ ($|\mu_1| > 1$) and the other five Floquet multipliers: $\mu_2 = 0.0112673$; $\mu_{3,4} = 0.638056 \pm 10.095812i$ and $\mu_{5,6} = -0.144267 \pm 0.127284i$ are stable ($|\mu_i| < 1$). For $A = 0.95$ and $\omega = 12$ all six Floquet multipliers: $\mu_1 = 0.994417; \mu_2 = 0.016662; \mu_{3,4} = 0.619721 \pm 0.252547i$ and $\mu_{5,6} = -0.212155 \pm 0.063986i$ are stable.

Increasing the torsional stiffness to $k_t = 1.000$ (Nm s/rad), we obtain a new stability diagram, Figure 5, where the fold bifurcations curve was displaced to the left, increasing the stable region.

For $A = 0.85$ and $\omega = 8$ there is a Floquet multiplier unstable $\mu_1 = 1.004972$, and the other five: $\mu_2 = 0.002611$; $\mu_{3,4} = -0.457778 \pm 0.300220i$ and $\mu_{5,6} = 0.065568 \pm 0.067412i$ are stable. For $A = 0.85$ and $\omega = 9$, all

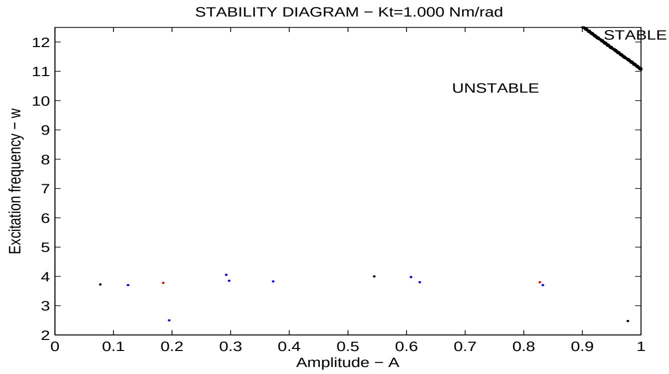

Figure 4: Stability diagram of the spray system for $k_t = 1.000$ (N m/rad).

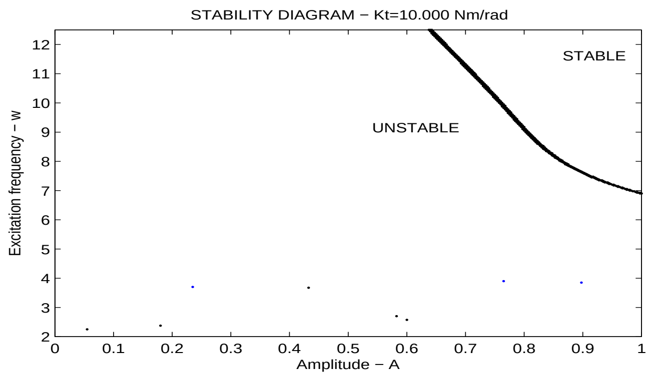

Figure 5: Stability diagram of the spray system for $k_t = 10.000$ (N m/rad).

six Floquet multipliers are unstable ($\mu_1 = 0.005114; \mu_2 = 0.985052; \mu_{3,4} = -0.004470 \pm 0.585328i$ and $\mu_{5,6} = 0.015496 \pm 0.121811i$).

Further increasing the torsional stiffness to $k_t = 19.000$ (Nm/rad), the stability diagram, Figure 6, presents the fold bifurcations curve is the boundary of the regions of stability and instability. Note that the stability region of the system, in the control space $(A \times \omega)$, is increasing. Let $A = 0.45$ and $\omega = 4.5$ in the instability region of Figure 5. There are two unstable Floquet multipliers: $\mu_1 = 800.557208$ and $\mu_2 = 440.496273$, and the others are stable ($\mu_3 = 0.000647; \mu_4 = 0.997540$ and $\mu_{5,6} = -0.009832 \pm 0.006096i$. For $A = 0.45$ and $\omega = 5.5$, in stability region, all Floquet multipliers are

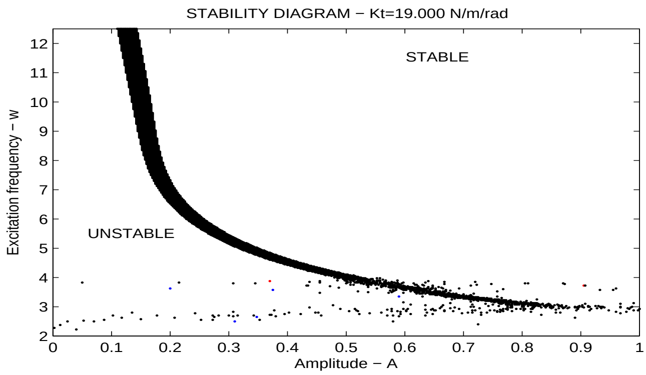

Figure 6: Stability diagram of the spray system for $k_t = 19.000$ (N m/rad).

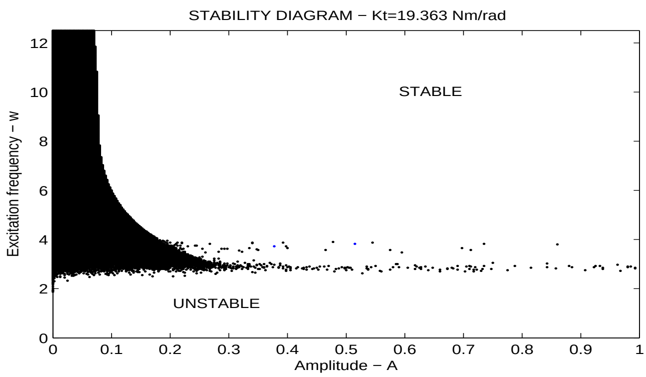

Figure 7: Stability diagram of the spray system for $k_t = 19.363$ (N m/rad).

stable ($\mu_1 = -0.520973; \mu_2 = 0.908853; \mu_3 = 0.986468; \mu_4 = 0.000248$ and $\mu_{5,6} = -0.006220 \pm 0.026495i$).

For the value of the torsional stiffness slightly larger, $k_t = 19.363$ (N m/rad), we observed, in Figure 7, that the fold bifurcation curve became a region of control parameters ($A \times \omega$). After this value, fold bifurcation curves will disappear and the dynamics of the sprayer system (4) will be independent of the parameters (A, ω, k_t).

4. Conclusions

The control parameters of the spray tower system with periodic vibrations at the junction were studied, and it was observed that its dynamics is unstable for certain values of torsional stiffness k_t and of the parameters of the periodic parametric excitation A and ω. This was shown in several stability diagrams in this work. It was observed that the vibrations at the junction point do not influence the dynamics of the system for values of the torsional stiffness larger than 20000 Nm/rad. Further studies will be needed to better analyze the dynamics of this system.

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