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A NOTE ON TRANSFORMATION FORMULAE FOR BILATERAL BASIC HYPERGEOMETRIC SERIES

Pankaj Srivastava[1 §], Tunis Elfrgani[2],
Ajit Paul[3], Mahmoud Eltikali[4]

[1]Department of Mathematics
Motilal Nehru National Institute of Technology
Allahabad, Uttar Pradesh, 211004, INDIA

[2,3,4]Department of Mathematics
Sam Higginbottom Institute of Agriculture
Technology and Sciences
Allahabad, Uttar Pradesh, 211007, INDIA

Abstract: The present research article deals with the development of transformation formulae for bilateral basic hypergeometric series by making use of some known results.

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1. Introduction

The basic hypergeometric functions played a key role in the theory of Mathematical analysis during the end of 20th century, and it was observed that Basic hypergeometric series and Bilateral basic hypergeometric series emerged as an important tool to handle the problems of Mathematical physics, Number theory, Statistics, Complex variable, etc. The emergence of S. Ramanujan's celebrated transformation formula for ${}_1\psi_1$ not only inspired the mathematicians

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[§]Correspondence author

working in this field but also provided an important platform for further developments of a number of transformations formulae for ${}_1\psi_1$, ${}_2\psi_2$, ${}_3\psi_3$, ${}_6\psi_6$, etc. Such transformation formulae were developed by R.P. Agarwal [16], G. Andrews [4], S.N. Singh [18], R.Y. Denis [17], P. Srivastava [11], P. Srivastava and B. Prakash Mishra [14], C.S. Adiga [2], K.R. Vasuki [6], M.S.M. Naika [9], P. Srivastava and P. Gupta [15] and many others. S.N. Singh [18] had developed a very interesting transformation formula for Bilateral to Basic hypergeometric series which was extensively exploited by P. Srivastava [12], [13], P. Srivastava and M. Pathak [10] to develop Continued fractions for the ratio of ${}_2\psi_2$ and ${}_3\psi_3$, respectively.

W.Y.C. Chen and A.M. Fu [20] developed a method that can be applied to derive Ramanujan ${}_1\psi_1$ summation, Bailey ${}_2\psi_2$ transformation and Bilateral ${}_6\psi_6$ summation formulae and derived some classical summation and transformation formulae in semi finite form. Later on, V.B. Chen et al. [19] obtained a formula reducing the evaluation of ${}_2\psi_2$ series to ${}_2\phi_1$ series. Recently K.R. Vasuki and C. Chamaraju [7] established an alternative proof of W.N. Bailey's ${}_2\psi_2$ Bilateral Basic hypergeometric series transformation formula. The current development in Bilateral Basic hypergeometric series motivated us to develop certain new transformation formulae for Bilateral Basic hypergeometric series.

2. Definitions and Notations

We shall use the following notations. For real or complex q, and $|q| < 1$,

$$(aq^m)_{-m} = \frac{1}{(a)_m},\ (q^{m+1})_{-m} = \frac{1}{(q)_m} \tag{2.1}$$

$$(\frac{bq^{-m}}{a})_\infty = (-1)^m b^m a^{-m} q^{-m(m+1)/2} (\frac{aq}{b})_m (\frac{b}{a})_\infty \tag{2.2}$$

$$(a)_m = \frac{(a)_\infty}{(aq^m)_\infty} \tag{2.3}$$

$$(a)_{k-m} = (a)_{-m}(aq^{-m})_k \tag{2.4}$$

$$(aq^{-m})_m = (-a)^m q^{-\binom{m}{2}} (\frac{q}{a})_m \tag{2.5}$$

$$(a)_{m+k} = (a)_m (aq^m)_k \tag{2.6}$$

$$(a)_{-m} = \frac{(-1)^m q^{\binom{m}{2}} a^{-m}}{(\frac{q}{a})_m}. \tag{2.7}$$

A basic hypergeometric series is defined as

$${}_A\phi_{A-1}\left[\begin{array}{l} a_1, a_2, \ldots, a_A; q; z \\ b_1, b_2, \ldots, b_{A-1} \end{array}\right] = \sum_{n=0}^{\infty} \frac{(a_1, a_2, \ldots, a_A; q)_n \, z^n}{(q, b_1, b_2, \ldots, b_{A-1}; q)_n}, \tag{2.8}$$

where $|z| < 1, |q| < 1$. A generalized basic bilateral hypergeometric series is defined as

$${}_A\psi_A\left[\begin{array}{l} a_1, a_2, \ldots, a_A; q; z \\ b_1, b_2, \ldots, b_A \end{array}\right] = \sum_{n=-\infty}^{\infty} \frac{(a_1, a_2, \ldots, a_A; q)_n \, z^n}{(b_1, b_2, \ldots, b_A; q)_n}, \tag{2.9}$$

where $|\frac{b_1 b_2 \ldots b_A}{a_1 a_2 \ldots a_A}| < |z| < 1, |q| < 1$ justifies the convergence of the series.

In order to establish our main results we shall make use of the following known results:

$${}_2\phi_1\left[\begin{array}{l} a, b; q, z \\ c \end{array}\right] = \frac{[abz/c, q/c; q]_\infty}{[az/c, q/a; q]_\infty} {}_2\phi_1\left[\begin{array}{l} c/a, cq/abz; q, bq/c \\ cq/az \end{array}\right]$$

$$-\frac{[b, q/c, c/a, az/q, q^2/az; q]_\infty}{[c/q, bq/c, q/a, az/c, cq/az; q]_\infty} {}_2\phi_1\left[\begin{array}{l} aq/c, bq/c; q, z \\ q^2/c \end{array}\right]$$

[3], App. (III.31), p. 245, (2.10)

$${}_2\phi_1\left[\begin{array}{l} a, b; q, z \\ c \end{array}\right] = \frac{[b, c/a, az, q/az; q]_\infty}{[c, b/a, z, q/z; q]_\infty} {}_2\phi_1\left[\begin{array}{l} a, aq/c; q, cq/abz \\ aq/b \end{array}\right]$$

$$+\frac{[a, c/b, bz, q/bz; q]_\infty}{[c, a/b, z, q/z; q]_\infty} {}_2\phi_1\left[\begin{array}{l} b, bq/c; q, qc/abz \\ bq/a \end{array}\right]$$

[3], App. (III.32), p. 245, (2.11)

$$ {}_3\phi_2\left[\begin{array}{l} a,b,c;de/abc \\ d,e \end{array}\right] = \frac{[e/b,e/c;q]_\infty}{[e,e/bc;q]_\infty} {}_3\phi_2\left[\begin{array}{l} d/a,b,c;q,q \\ d,bcq/e \end{array}\right] $$

$$ + \frac{[d/a,b,c,de/bc;q]_\infty}{[d,e,bc/e,de/abc;q]_\infty} {}_3\phi_2\left[\begin{array}{l} e/b,e/c,de/abc;q,q \\ de/bc,eq/bc \end{array}\right] $$

[3], App. (III.34), p. 245, (2.12)

$$ {}_3\phi_2\left[\begin{array}{l} a,b,c;de/abc \\ d,e \end{array}\right] = \frac{[b,de/ab,de/bc;q]_\infty}{[d,e,de/abc;q]_\infty} $$

$$ \times {}_3\phi_2\left[\begin{array}{l} d/b,e/b,de/abc;q,b \\ de/ab,de/bc \end{array}\right] $$

[3], App. (III.10), p. 241, (2.13)

$$ {}_3\phi_2\left[\begin{array}{l} a,b,c;q,de/abc \\ d,e \end{array}\right] = \frac{[e/a,de/bc;q]_\infty}{[e,de/abc;q]_\infty} $$

$$ \times {}_3\phi_2\left[\begin{array}{l} a,d/b,d/c;q,e/a \\ d,de/bc \end{array}\right] $$

[5], Eq. (10.10.7), p. 525, (2.14)

$$ {}_3\phi_2\left[\begin{array}{l} a,b,c;q,de/abc \\ d,e \end{array}\right] = \frac{[a,de/ab,de/ac;q]_\infty}{[d,e,de/abc;q]_\infty} $$

$$ \times {}_3\phi_2\left[\begin{array}{l} d/a,e/a,de/abc;q,a \\ de/ab,de/ac \end{array}\right] $$

[5], Eq. (10.10.8), p. 525, (2.15)

$$ {}_1\psi_1\left[\; a;b;q,z \;\right] = \prod\left[\begin{array}{l} b/a,az,q/az,q; \\ \qquad\qquad\quad q \\ q/a,b/az,b,z; \end{array}\right] $$

[8], App. (IV.12), p.248, (2.16)

$$
{}_2\psi_2\left[\begin{array}{l} a,b;q;z \\ c,d \end{array}\right] = \frac{[c/b,abz/d,dq/abz,q/d,q;q]_\infty}{[c,az/d,q/a,q/b,cd/abz;q]_\infty}
$$

$$
\times {}_2\phi_1\left[\begin{array}{l} cd/abz,d/a;q,bq/d \\ dq/az \end{array}\right]
$$

$$
-\frac{[cq/d,b,d/a,az/q,q^2/az,q/d,q;q]_\infty}{[d/q,c,bq/d,az/d,dq/az,q^2/d,d/a;q]_\infty}{}_2\phi_1\left[\begin{array}{l} aq/d,bq/d;q,z \\ cq/d \end{array}\right]
$$

[19], Eq. (2.1), p. 5, (2.17)

$$
{}_2\psi_2\left[\begin{array}{l} a,b;q,z \\ c,d \end{array}\right] = \frac{[az,d/a,c/b,dq/abz;q]_\infty}{[z,d,q/b,cd/abz;q]_\infty}{}_2\psi_2\left[\begin{array}{l} a,abz/d;q,d/a \\ az,c \end{array}\right]
$$

[20], Eq. (3.1), p. 3, (2.18)

$$
{}_6\psi_6\left[\begin{array}{l} qa^{1/2},-qa^{1/2},b,c,d,e;q,\dfrac{a^2q}{bcde} \\ a^{1/2},-a^{1/2},aq/b,aq/c,aq/d,aq/e \end{array}\right]
$$

$$
= \frac{[aq,aq/bc,aq/bd,aq/be,aq/ce,aq/cd,aq/de,q,q/a;q]_\infty}{[aq/b,aq/c,aq/d,aq/e,q/b,q/c,q/d,q/e,qa^2/bcde;q]_\infty}
$$

[20], Eq. (4.2), p. 7. (2.19)

3. Main Results

In this section, we shall establish the following results:

$$ {}_2\psi_2\left[\begin{array}{l} b, c; q, de/abc \\ d, e \end{array}\right] = \frac{[q, a, aq/d, aq/e, de/ab, de/ac; q]_\infty}{[q/a, q/b, q/c, d, e; q]_\infty} \times {}_2\psi_2\left[\begin{array}{l} d/a, e/a; q, a \\ de/ab, de/ac \end{array}\right], \tag{3.1} $$

$$ {}_2\psi_2\left[\begin{array}{l} a, b; q, de/abc \\ d, e \end{array}\right] = \frac{[q, e/a, de/bc, d/b, cq/d; q]_\infty}{[c, e, de/abc, q/b; q]_\infty} \times {}_2\psi_2\left[\begin{array}{l} a, d/c; q, e/a \\ d, de/bc \end{array}\right], \tag{3.2} $$

$$ {}_2\psi_2\left[\begin{array}{l} a, b; q, de/abc \\ d, e \end{array}\right] = \frac{[q, e/b, cq/e, e/c, d/a; q]_\infty}{[q/a, e, e/bc, bcq/e; q]_\infty} {}_1\psi_1\left[\begin{array}{l} b; q, q \\ d \end{array}\right] + \frac{[q, d/a, b, c, de/bc, e/b, cq/e; q]_\infty}{[q/a, c, d, e, bc/e, eq/bc; q]_\infty} {}_1\psi_1\left[\begin{array}{l} e/c, z; q, q \\ de/bc \end{array}\right], \tag{3.3} $$

$$ {}_2\psi_2\left[\begin{array}{l} a, c; q, de/abc \\ d, e \end{array}\right] = \frac{[q, de/ab, de/bc, bq/d, bq/e; q]_\infty}{[q/a, q/c, d, e; q]_\infty} \times {}_2\psi_2\left[\begin{array}{l} d/b, e/b; q, b \\ de/ab, de/bc \end{array}\right], \tag{3.4} $$

$$ {}_1\psi_1\left[\begin{array}{l} b; q, z \\ c \end{array}\right] = \frac{[q/c, aq/c, cq/abz, abz/c, q; q]_\infty}{[a, q/b, az/c, q/a; q]_\infty} {}_1\psi_1\left[\begin{array}{l} c/a; q, bq/c \\ cq/az \end{array}\right] - \frac{[q, q/c, aq/c, b, c/a, az/q, q^2/az; q]_\infty}{[a, q^2/c, az/c, c/q, bq/c, q/a, cq/az; q]_\infty} {}_1\phi_0\left[\begin{array}{l} bq/c; q, z \end{array}\right], \tag{3.5} $$

$$ {}_1\psi_1\left[\begin{array}{l} b; q, z \\ c \end{array}\right] = \frac{[b, c/a, az, q/az, aq/c, q; q]_\infty}{[a, c, b/a, z, q/z, aq/b; q]_\infty} {}_1\phi_0\left[\begin{array}{l} a; q, cq/abz \end{array}\right] + \frac{[q, c/b, bz, bq/c; q]_\infty}{[c, z, q/z, a/b; q]_\infty} {}_1\psi_1\left[\begin{array}{l} b; q, cq/abz \\ bq/q \end{array}\right]. \tag{3.6} $$

4. Proof of Main Results

[a] In order to establish our main result (3.1), for any positive integer m, we consider the following series:

$$\sum_{n=-m}^{\infty} \frac{[aq^m, b, c]_n}{[d, e, q^{m+1}]_n} (de/abc)^n. \tag{4.1}$$

Taking $n = k - m$ in (4.1) and making use of (2.4), we obtain

$$\sum_{n=-m}^{\infty} \frac{[aq^m, b, c]_n}{[d, e, q^{m+1}]_n} (de/abc)^n = \frac{(aq^m, b, c)_{-m}}{(d, e, q^{m+1})_{-m}} (de/abc)^{-m}$$

$$\times \sum_{k=0}^{\infty} \frac{[a, bq^{-m}, cq^{-m}]_k}{[dq^{-m}, eq^{-m}, q]_k} (de/abc)^k. \tag{4.2}$$

Further, replacing b, c, d, e by bq^{-m}, cq^{-m}, dq^{-m}, eq^{-m} in (2.15), we obtain

$${}_3\phi_2\left[\begin{array}{l} a, bq^{-m}, cq^{-m}; de/abc \\ dq^{-m}, eq^{-m} \end{array}\right] = \frac{[a, deq^{-m}/ab, deq^{-m}/ac; q]_\infty}{[dq^{-m}, eq^{-m}, de/abc; q]_\infty}$$

$$\times {}_3\phi_2\left[\begin{array}{l} dq^{-m}/a, eq^{-m}/a, de/abc; q, a \\ deq^{-m}/ab, deq^{-m}/ac \end{array}\right]. \tag{4.3}$$

Now, we consider the series:

$$\sum_{n=-m}^{\infty} \frac{[dq^{-m}/a, eq^{-m}/a, de/abc; q]_n}{[deq^{-m}/ab, deq^{-m}/ac; q]_n} (a)^n. \tag{4.4}$$

Taking $n = m + k$ in (4.4) and making use of (2.6), we get

$$\sum_{n=-m}^{\infty} \frac{[dq^{-m}/a, eq^{-m}/a, de/abc; q]_n}{[deq^{-m}/ab, deq^{-m}/ac; q]_n} (a)^n =$$

$$\frac{(dq^{-m}/a, eq^{-m}/a, de/abc)_m}{(deq^{-m}/ab, deq^{-m}/ac)_m} (a)^m \sum_{k=-2m}^{\infty} \frac{[d/a, e/a, deq^m/abc]_k}{[de/ab, de/ac, q]_k} (a)^k. \tag{4.5}$$

Now making use of (4.3) and (4.5) in (4.2)and evaluating by making use of (2.1), (2.2), (2.5) and (2.7), we get

$$\sum_{n=-m}^{\infty} \frac{[aq^m, b, c]_n}{[d, e, q^{m+1}]_n}(de/abc)^n = \frac{(q, aq/d, aq/e, de/abc)_m}{(q/a, q/b, q/c)_m}$$

$$\times \frac{[a, de/ab, de/ac]_\infty}{[d, e, de/abc]_\infty} \sum_{k=-2m}^{\infty} \frac{[d/a, e/a, deq^m/abc]_k}{[de/ab, de/ac]_k}(a)^k, \quad (4.6)$$

finally taking $m \to \infty$ in (4.6), we get the result (3.1).

[b] To establish (3.2) and (3.3), for any positive integer m, we consider the series:

$$\sum_{n=-m}^{\infty} \frac{[a, b, cq^m]_n}{[d, e, q^{m+1}]_n}(de/abc)^n. \quad (4.7)$$

Taking $n = k - m$ in the above series and using (2.4), we get

$$\sum_{n=-m}^{\infty} \frac{[a, b, cq^m]_n}{[d, e, q^{m+1}]_n}(de/abc)^n = \frac{(a, b, cq^m)_{-m}}{(d, e, q^{m+1})_{-m}}(de/abc)^{-m}$$

$$\times \sum_{k=0}^{\infty} \frac{[aq^{-m}, bq^{-m}, c]_k}{[dq^{-m}, eq^{-m}, q]_k}(de/abc)^k. \quad (4.8)$$

Now replacing $a = aq^{-m}$, $b = bq^{-m}$, $d = dq^{-m}$ and $e = eq^{-m}$ in (2.14) and (2.12) and using these in (4.8) respectively, finally making use of (2.1), (2.2), (2.5) and (2.7), we get after some simplifications

$$\sum_{n=-m}^{\infty} \frac{[a, b, cq^m]_n}{[d, e, q^{m+1}]_n}(de/abc)^n = \frac{(q, d/b, cq/d)_m}{(q/b, c)_m} \frac{[e/a, de/bc]_\infty}{[e, de/abc]_\infty}$$

$$\times \sum_{k=-2m}^{\infty} \frac{[a, dq^m/b, d/c]_k}{[d, de/bc]_k}(e/a)^k \quad (4.9)$$

and

$$\sum_{n=-m}^{\infty} \frac{[a, b, cq^m]_n}{[d, e, q^{m+1}]_n}(de/abc)^n = \frac{(q, cq/e, d/a)_m}{(q/a, bcq/e)_m} \frac{[e/b, e/c]_\infty}{[e, e/bc]_\infty}$$

$$\times \sum_{k=-2m}^{\infty} \frac{[dq^m/a, b, cq^m]_k}{[d, bcq^{1+m}]_k}(q)^k + \frac{(q, e/b, cq/e, de/abc)_m}{(q/a, c, eq/bc)_m} \frac{[d/a, b, c, de/bc]_\infty}{[d, e, bc/e, de/abc]_\infty}$$

$$\times \sum_{k=-2m}^{\infty} \frac{[eq^m/b, e/c, deq^m/abc]_k}{[de/bc, eq^{1+m}/bc]_k}(q)^k. \tag{4.10}$$

Taking $m\to\infty$, in (4.9) and (4.10), we obtain the results (3.2) and (3.3).

[c] In order to establish (3.4), we consider the series $\sum_{k=-m}^{\infty} \frac{[a, bq^m, c]_k}{[q^{m+1}, d, e]_k}(de/abc)^k$, for any positive integer m.

$$\sum_{k=-m}^{\infty} \frac{[a, bq^m, c]_k}{[q^{m+1}, d, e]_k}(de/abc)^k = \frac{(a, bq^m, c)_{-m}}{(q^{m+1}, d, e)_{-m}}(de/abc)^{-m}$$

$$\times \sum_{n=0}^{\infty} \frac{[aq^{-m}, b, cq^{-m}]_n}{[q, dq^{-m}, eq^{-m}]_n}(de/abc)^n. \tag{4.11}$$

Taking $a = aq^{-m}$, $c = cq^{-m}$, $d = dq^{-m}$ and $e = eq^{-m}$ in (2.13) and using this in (4.11), finally making use of (2.1), (2.2), (2.5) and (2.7), we get

$$\sum_{k=-m}^{\infty} \frac{[a, bq^m, c]_n}{[q^{m+1}, d, e]_k}(de/abc)^k = \frac{(q, abq/de, bcq/de, bq/d, bq/e, de/abc)_m}{(b, q/a, q/c, abq/de, bcq/de)_m}$$

$$\times \frac{[b, de/ab, de/bc]_\infty}{[d, e, de/abc]_\infty} \sum_{k=-2m}^{\infty} \frac{[d/b, e/b, deq/abc]_k}{[de/ab, de/bc]_k}(b)^k. \tag{4.12}$$

Thus, taking $m\to\infty$ in (4.12), we get the result (3.4).

[d] Proceeding in the same way, we consider the series

$$\sum_{k=-m}^{\infty} \frac{[aq^m, b]_k}{[c, q^{m+1}]_k} z^k, \text{ for any positive integer } m, \tag{4.13}$$

$$\sum_{k=-m}^{\infty} \frac{[aq^m, b]_n}{[c, q^{m+1}]_n} z^n = \frac{(aq^m, b)_{-m}}{(c, q^{m+1})_{-m}} z^{-m} \sum_{n=0}^{\infty} \frac{[a, bq^{-m}]_n}{[cq^{-m}, q]_n} z^n. \tag{4.14}$$

Taking $b = bq^{-m}$, $c = cq^{-m}$ in (2.10) and (2.11), and using these in (4.14)respectively, now by making use of (2.1), (2.2), (2.5) and (2.7), we get

$$\sum_{n=-m}^{\infty} \frac{[aq^m, b]_n}{[c, q^{m+1}]_n} z^n = \frac{(q, q/c, aq/c, cq/abz)_m}{(a, q/b, az/c)_m} \frac{[abz/c, q^{1+m}/c]_\infty}{[azq^m/c, q/a]_\infty}$$

$$\times \sum_{k=-2m}^{\infty} \frac{[c/a, cq^{1+m}/abz]_k}{[cq/az]_k} (bq/c)^k - \frac{(q, q/c, aq/c)_m}{(a, q^2/c, az/c)_m}$$

$$\times \frac{[b, q^{1+m}/c, c/a, az/q, q^2/az]_\infty}{[c/q, bq/c, q/a, azq^m/c, cq/az]_\infty} {}_2\phi_1 \left[\begin{matrix} aq^{1+m}/c, bq/c; q, z \\ q^{2+m}/c \end{matrix} \right] \tag{4.15}$$

and

$$\sum_{n=-m}^{\infty} \frac{[aq^m, b]_n}{[c, q^{m+1}]_n} z^n = \frac{(q, aq/c)_m}{(a, aq/b)_m} \frac{[b, c/a, az, q/az]_\infty}{[c, b/a, z, q/z]_\infty}$$

$$\times {}_2\phi_1 \left[\begin{matrix} a, aq^{1+m}/c; cq/abz \\ aq^{1+m}/b \end{matrix} \right] + \frac{(q, q^{1+m}/bz, bq/c)_m}{(a, a/b)_m}$$

$$\times \frac{[a, c/b, bz, q^{1+m}/bz]_\infty}{[c, aq^m/b, z, q/z]_\infty} \sum_{k=-2m}^{\infty} \frac{[b, bq^{1+m}/c]_k}{[bq/a]_k} (cq/abz)^k. \tag{4.16}$$

Finally making $m \to \infty$, in (4.15) and (4.16), we obtain the results (3.5) and (3.6).

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