ENVIRONMENTAL ANALYSIS OF
IMPACT OF TRANSGENIC CROPS

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Abstract: To address the problem of the environmental impact of transgenic crops, we start by studying the intra-specific competition between the natural and transgenic variants of some particular species being planted. We use analytical modeling to analyze the dynamics of the production process. Analytical Trophodynamics (AT), based on biological mechanisms, offers a powerful tool to analyze the behavior of the system away from equilibrium points. AT together with Diffusion Theory in Finsler spaces, provide an appropriate mathematical framework for the study of ecological systems under the influence of environmental changes, produced naturally or not.

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1. Introduction

In the recent years we have seen the increasing usage of transgenic organisms in agriculture. Genetically modified to develop features from other species meant to confer resistance to natural predators, these organisms offer the prospect of increased production. Initially hailed as the solution to world hunger, these variants also brought about problems once inserted into the environment. Questions about nutritional content and consequences of the characteristics artificially inserted to the human body, for instance, are still being raised. Not being the product of a natural evolutionary process, the impact of these transgenic variants on the environment is of particular concern. The classical principle of competitive exclusion [1, 2] alone already indicates a probable decrease in biodiversity, with direct consequences on the quantity and quality in the human food chain.

Analytical Trophodynamics [1], based on biological mechanisms, allows for the study of nonlinear dynamical systems far from equilibrium points. Together with the theory of diffusion on allometric spaces, where production processes of biomass or allometric related quantities are represented, provides the appropriate mathematical context for the study of ecological systems under the influence of environmental changes, naturally produced or not.

As a first step towards accessing the impact of the newly developed variants, we will address their effect on similar crops in the neighborhood. Later, we will extend this work to consider the possible interactions among those artificially created variants of some plant in the context provided by the natural biome of the original species, such as a rain forest [3, 4], using the already developed notion of ecostrain as a perturbation tool [5].

In this work we focus on the competition between natural and transgenic crops close by. In particular, we address the intra-specific competition between the natural organism and its transgenic variant. As variants of the same species, and therefore searching the same nutrients and shelter in the environment, these populations are pure competitors. As such, we take the population dynamics to be represented by the classical Gause-Witt model (see [1]). As well known, this pure-competitors population dynamics model, widely used both in ecology and evolution, leads to the Principle of Competitive Exclusion, mechanism that entails the weaker variant's extinction.

This first step towards analyzing the impact of artificially enhanced crops introduced into the environment, however simple, already signals the prospect of biodiversity loss. As a second step, we introduce secondary effects in order to produce more realistic models for the interaction between the variants. We

propose to incorporate at this point the organic development of the planted seeds, estimating the impact of their competition into each variant's characteristic net biomass production, generated over a finite time between planting and harvesting.

The effect of the competition between the variants occurs in the population levels, while the organic growth happens at the individual level. The total biomass, or the fraction of it that represents the product of commercial interest, such as grains, leaves, fruits, etc, is given as the sum over all individual products in each population. More specifically, using Harper's idea [8], according to which an organism is seen as a collection of modules, each consisting of a sub-population (such as leaves, roots, grains, etc, in the case of plants), then competition occurs at roots' level, while the economically significant production is estimated at seeds or fruits level, according to the particular culture at hand.

Organic growth is extensively represented by the Gompertz's curve, and can be combined with a competitive population dynamics, given by Gause-Witt equations. But the actual harvest is better represented by the notion of net growth, or the difference between the total biomass production and what was used in repair and maintenance, as it was proposed by Whittaker [14] in the context of forest dynamics. This concept in fact generalizes Gompertz's equation, and allows composing the population dynamics and the organic production process into a single system of differential equations.

Finally, we introduce seed dispersal, that mix originally distinct crops and causes competition between transgenic and naturally occurring seeds as they grow. Mechanisms such as wind, water or birds, cause mixing populations even when not adjacent, affecting their production rates. Dispersion is introduced in the mathematical model as diffusion terms in the differential equations. In a simplistic way, we can imagine two adjacent crops of a single species, composed by natural and transgenic variants. Assuming the environment dispersion mechanism to be strong enough, or the field to be small enough, the different seed populations mix until the two are uniformly distributed over entire region. Note that, in a later stage, we may assume these mixed fields to be geographically distributed according to the dispersion source, and we may divide the region between mixed and no-mixed categories.

The methodology of mathematical modeling proposed here is Analytical Trophodynamics, making use of ordinary differential equations representing the mechanisms associated with the biological system, both in the ecological level (interaction between species, population dynamics), as well as in the production level (biomass, or fractions of it). Models that incorporate both these aspects are called Volterra-Hamilton, with extensive use in Ecology, Evolution,

Epidemiology, Embryology and other areas of biology.

The stability of the production process, given by the dynamical system, is based on Lyapunov ideas, whose Jacobi's geometrical description is conveniently analyzed within KCC theory (as developed by Cartan, Chern and Kosambi). In this context, the metric usually expresses the conservation of nutrients. Riemannian geometry [9] results when interactions depend explicitly on the products created / consumed by the species, when they are called metabolic, as opposed to simply ecological, when they are taken to be constants, or geographical and/or time averages. Finsler geometry represents interactions which depend on proportions between the different populations, known as social interactions in the biological context. Such considerations will be addressed in a following paper.

It is worth saying that the algebraic manipulations and calculations associated with these geometries, of increasing complexity with the number of dimensions, or populations, are largely facilitated through the use of software Finsler [12], a computer algebra package based on MAPLE [10] that derives most of the geometrical invariants of interest, given a metric or interaction pattern.

2. Mathematical Model

Let (x^i, N^i) be natural coordinates in a local chart on the tangent bundle TM. Consider the second-order system of differential equations

$$\begin{cases} \mathrm{d}x^i/\mathrm{d}t = k_{(i)} N^i \ \ \text{(not summed)}, \\ \\ \mathrm{d}N^i/\mathrm{d}t = -\Gamma^i_{jk} N^j N^k + r^i_j N^j + e^i, \end{cases} \tag{1}$$

where all coefficients (possibly) depend on x^i, N^i, t; the n^3 function Γ^i_{jk} are homogeneous of degree zero in the N^i and with smooth initial conditions x^i_0, N^i_0, t_0. For almost 35 years, this system has played a major role in mathematical theories of Ecology, Evolution and Development. The coordinates x^i are Volterra production variables, whose constant per capita rate is k_i, while the second part of this system is a description of how the different populations $N^i(t) \geq 0$ grow (r^i_j), interact (Γ^i_{jk}) and react (e^i) to external influences. For our present purpose, these equations represent two crops of variants of the same species, the quantity x^i being the gross primary production of some modular unit, like leaves, flower parts, etc, for the i-th variant, or population. The production variables x^i are usually given as log-biomass or allometric related fractions of

it (see [11]). The number of types of modular units is less than or equal to the number of populations, and will remain unspecified until stated otherwise. But we should point out that the interaction between populations usually occurs among the same modular units, that is, roots versus roots, leaves versus leaves, etc.

Following the classical constant-coefficients interaction scheme, consider two varieties of a (small) seed, one natural and the other genetically modified. The two types of seeds are planted in two adjacent rectangular fields at an given initial time. Type #1 refers to field #1 and likewise for #2. However, wind and rain (or other similar factors, like birds, atmospheric humidity, etc) may transport the small seeds from their original fields, so that competition arises wherever they end up mixed together. Such mechanism takes place in both fields, where they continue to grow. We now proceed according to Conway, Hoff and Smoller (CHS) diffusion-reaction-transport theory [7, 13], which provides the main mathematical foundation for our model.

Using rectangular cartesian coordinates X and Y to locate positions within the fields, taken to form a square, and assuming there is a barrier around the perimeter of the two-field compound, so that the Neumann condition called *no-flux boundary* [7, 13] is enforced, our model equations then become

$$
\begin{cases}
\partial_t N^1 = D_1 \Delta_{X,Y}(N^1) + \beta_2 k \partial_X N^1 + \lambda N^1 \\
\qquad - (\lambda \alpha_1 + \beta_1)(N^1)^2 - \beta_2 N^1 N^2, \\
\\
\partial_t N^1 = D_2 \Delta_{X,Y}(N^2) + \beta_1 k \partial_X N^2 + \lambda N^2 \\
\qquad - (\lambda \alpha_2 + \beta_2)(N^2)^2 - \beta_1 N^1 N^2,
\end{cases}
\tag{2}
$$

where the euclidean Laplacian $\Delta_{X,Y}$ is the same in both equations. In the above equations, the constant k was introduced in the transport coefficients $\beta_1 k$ and $\beta_2 k$, to indicate that the competition, $\beta_2 N^1 N^2$ and $\beta_1 N^1 N^2$, is *induced* by the transport, but do not necessarily have the same coefficients. Another point is that the competitive system above entails the existence of a positive stable equilibrium, or that no variant eliminates the other in time. This represents the fact that generally the GM crops affect nearby ones, but do not annihilate them.

In order to justify the auto-inhibition terms in the reaction part of the system above, one must consider that, although the GM variant is an artificial one, it must, nevertheless, follow a natural process once introduced into the environment. As such, it must be cost-effective. The theorem and remarks in what follows show how the concept of cost may be associated with such dynamical systems, and more can be found in [1] and references therein.

The proposed diffusion-reaction-transport system will reach, over a long enough time, a spatially homogeneous stable solution, as a result of CHS theory, as follows. If $C^i(t)$ denotes the spatial average over the compound fields $N^1(t), N^2(t)$, then CHS theory states that

$$\frac{dC^i}{dt} = f^i(C^1, C^2) + O(e^{-\sigma t}) \tag{3}$$

with f^i being the same as the reaction-part (no diffusion and no transport, just the competitive deterministic terms) above but with N^i replaced by C^i. In other words, the diffusion and transport processes die out in time. Thus, if σ is large the system reduces to the study of a two dimensional, first order, nonlinear ODE characterizing $C^i(t)$.

Using Volterra production equations $dx^i/dt = C^i$, ([1]) the equations are converted to autoparallels of a Wagner connection in Finsler geometry. They are not geodesics of a Finsler metric, however. But, since the Wagner connection is metrical, there is a cost functional that is constant along the (averaged) production trajectories $(x^1(s), x^2(s))$, where s is the production parameter given by $ds = e^{\lambda t}\, dt$. It is the affine path parameter and is intrinsically defined just as in affine differential geometry.

Theorem 2.1. *Cost of production, $F(x, C)$, is constant along trajectories of (3). That is to say, $dF/ds = 0$, along solutions, where*

$$F(x, C) = e^{Q(x)} \frac{(dx^2/ds)^{1+1/\lambda}}{(dx^1/ds)^{1/\lambda}},$$
$$\text{and } Q(x) = -(\alpha_1 + \beta_1)(x^1) + [(1+\lambda)\alpha_2 + \beta_2]x^2.$$

Remark 2.2. *The intrinsic growth rate λ is the same for each of #1 and #2, so that $ds = e^{-\lambda t}\, dt$ leads to reparameterization of production curves by s. In fact, (3) is quadratic in C^i.*

Remark 2.3. *Without the effects of wind, rain and diffusion, the cost is $F(x, C)$ as given in the theorem, but with β's both vanishing. The dynamics is reduced to two separate logistics each with growth rate λ and carrying capacities $\dfrac{1}{\alpha_1}$ and $\dfrac{1}{\alpha_2}$, respectively.*

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[1]The production variables x here are similar to the ones in (1), but are actually averages over the fields.

The constant σ, as in CHS theory, is given by

$$\sigma = d.A - M - 2aA^{1/2}, \tag{4}$$

where $d = \min(D_1, D_2)$, A is the first non-zero eigenvalue of $\Delta_{X,Y}$, which is well approximated by $1/q^2$, with q being the length of the diagonal of the square two-field compound, $M = \max(Jf)$, the maximum value of the Jacobian Jf of f over the compound, and, finally, $a = \max(\beta_1, \beta_2)$.

It is now clear that, if d is large and the area of the square compound is small, then σ will be large, relatively speaking. Thus large diffusion over a small area yields rapid convergence of the original diffusion-reaction-transport to the equations (3) above. We explicitly make these assumptions. Note that the smaller the β's are, the quicker the convergence, as well. This agrees with the supposition of the coexistence of the pure-competitor variants.

The system of ODEs (3) are competition equations. In fact, in the special case here, we have a stable steady state. There are 4 cases to consider in the Gauss-Witt equations and only one case yields a linearly stable equilibrium. Competitive exclusion does not happen here. So the competition is mild, more like a skirmish rather than open war.

Yet, over the longer time-scale the carrying capacity for each type has been reduced because of the competition. Without it, that is, without mixing, the resulting system would be logistic and the carrying capacities would be obtained in the long term. The carrying capacities are $\dfrac{1}{[\alpha_1 + \beta_2/\lambda]}$ and $\dfrac{1}{[\alpha_2 + \beta_1/\lambda]}$. The reason for their reduced value is that the competition between the plants takes a toll on the production of the compound when compared to original isolated fields of #1 and #2.

A more useful measure of productivity is Whittaker's *net ecosystem productivity*, dy^i/dt, which, expressed in terms of modular units, is given by

$$\frac{\mathrm{d}x^i}{\mathrm{d}t} = R_0^i + k_{(i)}\ \frac{\mathrm{d}y^i}{\mathrm{d}t} = k_{(i)}\ N^i, \tag{5}$$

where the LHS is the gross production rate and R_0^i is the i-th respiration, repair and maintenance rate, assumed constant. When N^i approaches a stable steady–state N_*^i in (1), then the net ecosystem productivity approaches zero [14]. Substitution of (5) into (1) gives the equations for *net ecosystem productivity* in the ecosystem,

$$\frac{\mathrm{d}^2 y^i}{\mathrm{d}t^2} + \Gamma_{jk}^i\ \frac{\mathrm{d}y^j}{\mathrm{d}t}\frac{\mathrm{d}y^k}{\mathrm{d}t} + \left(2\Gamma_{jk}^i N_*^k - \lambda\delta_j^i\right)\ \frac{\mathrm{d}y^j}{\mathrm{d}t} = 0, \tag{6}$$

where we have assumed $r^i_j = \lambda \delta^i_j$, $\lambda > 0$ and $k_i = 1$. We obtain *net growth rates* $\tilde{r}^i_j$ as the coefficients of the linear term, $\tilde{r}^i_j = 2\Gamma^i_{jk} N^k_* - \lambda \delta^i_j$.

These equations, in fact, generalizes the Gompertz equations [1][2] and allow composed population dynamics and process of organic production in a single system of differential equations above. Here #1 denotes the transgenic crops while and #2 denotes the natural crops.

The net growth solution to (6) for large time is that of (6) with the quadratic terms approximately zero and is given by

$$
\begin{cases}
y^1(t) = A^1 - \dfrac{a^1_1}{\lambda} \left(1 + \dfrac{\beta_2}{\alpha_2} + \dfrac{\beta_1}{\alpha_1} \right) \mathrm{e}^{-\lambda^2 \alpha_1 \alpha_2 t / (\lambda \alpha_1 \alpha_2 + \beta_1 \alpha_2 + \alpha_1 \beta_2)} \\[2mm]
\qquad - \dfrac{a^1_2}{\lambda} \mathrm{e}^{-\lambda t}, \\[4mm]
y^2(t) = A^2 - \dfrac{a^2_1}{\lambda} \left(1 + \dfrac{\beta_2}{\alpha_2} + \dfrac{\beta_1}{\alpha_1} \right) \mathrm{e}^{-\lambda^2 \alpha_1 \alpha_2 t / (\lambda \alpha_1 \alpha_2 + \beta_1 \alpha_2 + \alpha_1 \beta_2)} \\[2mm]
\qquad - \dfrac{a^2_2}{\lambda} \mathrm{e}^{-\lambda t}.
\end{cases}
\tag{7}
$$

Note that $\mathrm{d}y^1/\mathrm{d}t(0)$ is the maximum value of $\mathrm{d}y^1/\mathrm{d}t(t)$ and is positive, while the second derivatives at time zero are negative. Similar statements hold for $y^2(t)$ and $\mathrm{d}y^2/\mathrm{d}t(0)$.

It is the wind and rain (transport terms) which drives the large diffusion (D^1 and D^2 are relatively large over the small area of the adjacent fields where the two crops reside). This diffusion-reaction-transport will come to a homogeneous steady-state according to the CHS theory. It is at this point in time that C^1 and C^2 are actually averages over the two fields. Additionally, after further time passes, the net growth (averaged) becomes small enough that the quadratic terms in (3) are negligible. This is how we arrive at the linear approximation for (averaged over space) net growth $y^1(t)$ given above. Likewise, similar results hold for $y^2(t)$. At this stage, almost all of the available energy (cost) goes into repair and maintenance.

3. Conclusion

In order to estimate the impact of the GM crop on its natural occurring variant, we will evaluate the difference between the net growth in (7) above and the same

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[2]The term *real growth* is used here, instead of net growth.

considering without competition. We do this by setting β_1 and β_2 small, but not zero, obtaining

$$\begin{cases} \hat{y}^1(t) \approx A^1 - \frac{a_1^1}{\lambda}\,\mathrm{e}^{-\lambda t} - \frac{a_2^1}{\lambda}\mathrm{e}^{-\lambda t}, \\[2mm] \hat{y}^2(t) \approx A^2 - \frac{a_1^2}{\lambda}\,\mathrm{e}^{-\lambda t} - \frac{a_2^2}{\lambda}\,\mathrm{e}^{-\lambda t}. \end{cases} \tag{8}$$

Then the effect of the competition between GM $\times$ natural crops, for large time, is $\hat{y}^i - y^i$, that is,

$$\hat{y}^i - y^i \approx \left(\frac{\beta_1}{\alpha_1} + \frac{\beta_2}{\alpha_2}\right)\,\mathrm{e}^{-\lambda^2 \alpha_1 \alpha_2 t/(\lambda \alpha_1 \alpha_2 + \beta_1 \alpha_2 + \alpha_1 \beta_2)}. \tag{9}$$

From this, we conclude that, as a result of transport-induced competition, both crops will be equally affected, but we expect the carrying capacity of the GM crop to be larger than the natural one, and therefore the later will be more impacted.

We may also observe from the above result that, if $\lambda \gg 1$, then the rate constant in exponent will be approximately λ, and, since t in monotonically growing, the difference tends to zero. But, biologically speaking, the interval between planting and harvesting is small, and, species that makes economic sense to produce GM variants to improve production do not have a characteristically high intrinsic growth rate λ. We will then observe significant differences in these cases.

On the other hand, if λ is small and/or β_i are large, then the exponential term is close to 1. This is the case of interest. Noting that α_i are the reciprocal of the carrying capacities of the original logistic system, and that β_i are the interaction coefficients for the j variant, we conclude that the difference above will be chiefly given by the large interaction coefficient, or impact of the competition of GM on the natural variant, times the also large GM carrying capacity.

In a following article we will consider the addition of noise [6] to the Finsler production system associated to (3) as a way of dealing with environmental noise, or effects not explicitly considered here.

Acknowledgments

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