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WHITE NOISE FUNCTIONAL SOLUTIONS FOR THE WICK-TYPE STOCHASTIC FRACTIONAL KDV-BURGERS-KURAMOTO EQUATIONS WITH TIME-FRACTIONAL DERIVATIVES

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Abstract: The aim of this paper is to give some new approximations for the exact solutions of the Wick-type stochastic generalized fractional KdV-Burgers-Kuramoto equations with time-fractional derivatives. The homotopy analysis method (HAM) is employed to obtain approximate analytical solutions for the exact solutions of fractional KdV-Burgers-Kuramoto equations with time-fractional derivatives. Moreover, by using white noise functional analysis, Hermite transform and inverse Hermite transform we will obtained new exact solutions of the Wick-type stochastic generalized fractional KdV-Burgers-Kuramoto equations with time-fractional derivatives. Finally, by the help of the mapping relation constructed between the general formal solutions of

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the Wick-type stochastic generalized fractional KdV-Burgers-Kuramoto equations and the solutions of the auxiliary equations various types of the Wick-type stochastic generalized fractional KdV-Burgers-Kuramoto equations are derived.

AMS Subject Classification: 60H30, 60H15, 35R60
Key Words: fractional KdV-Burgers-Kuramoto equations, time-fractional derivative, white noise, stochastic fractional equation, homotopy analysis, Hermite transform

1. Introduction

In this paper we obtain white noise functional solutions for the Wick-type stochastic generalized fractional KdV-Burgers-Kuramoto equations with space-fractional derivatives. The generalized fractional KdV-Burgers-Kuramoto equations with time-fractional derivatives is given by

$$\begin{aligned}&\frac{\partial^\alpha u}{\partial t^\alpha}+u\frac{\partial u}{\partial x}+a(t)\frac{\partial^2 u}{\partial x^2}+b(t)\frac{\partial^3 u}{\partial x^3}+c(t)\frac{\partial^4 u}{\partial x^4}=0,\\ &t>0,\quad 0<\alpha\leqslant 1,\end{aligned} \tag{1}$$

where $a(t), b(t)$ and $c(t)$ are bounded measurable or integrable functions on $\mathbb{R}_+$. Nonlinear partial differential equations (PDEs) are encountered in various fields as physics, applied mathematics, engineering, biology and chemistry. Most nonlinear models of real life problems are still very difficult to solve, neither theoretically nor numerically. In the past decades, both mathematicians and physicists have devoted considerable effort to the study of explicit solutions to nonlinear integer-order differential equation. In recent years, an important progress has been made in the research of the exact solutions of nonlinear (PDEs). To seek various exact solutions of multifarious physical models described by nonlinear PDEs, various methods have been proposed. Recently, many researchers pay more attention to the study of the random waves, which are important subjects of stochastic partial differential equation (SPDE). M. Wadati [19] first answered the interesting question, "How does external noise affect the motion of solitons?" and studied the diffusion of soliton of the KdV equation under Gaussian noise, which satisfies a diffusion equation in transformed coordinates. Wadati and Akutsu also studied the behaviors of solitons under the Gaussian white noise of the stochastic KdV equations with and without damping [21]. In addition, a nonlinear partial differential equation which describes wave propagations in random media was presented by Wadati [20]. Debussche and

Printems ([6, 7]), de Bouard and Debussche ([2, 3]), Konotop and Vazquez [14], recently, Ugurlu and Kaya [18] gave the tanh function method, Xie first introduced Wick-type stochastic KdV equations on white noise space and showed the auto-Backlund transformation and the exact white noise functional solutions in [24], furthermore, Chen and Xie ([4, 5]) and Xie ([22, 23, 24, 25]) researched some Wick-type stochastic wave equations using white noise analysis method. Eq. (1) plays a significant role in many scientific applications such as solid state physics, nonlinear optics, chemical kinetics, etc. If Eq. (1) is considered in random environment, we can get random fractional KdV-Burgers-Kuramoto equations with space-fractional derivatives. In order to give the exact solutions of random fractional KdV-Burgers-Kuramoto equations with space-fractional derivatives, we only consider this problem in white noise environment. Wick-type stochastic generalized fractional KdV-Burgers-Kuramoto equations with time-fractional derivatives is given by:

$$\begin{aligned} &U_{t^\alpha} + U \diamond U_x + A(t) \diamond U_{x^2} + B(t) \diamond U_{x^3} + C(t) \diamond U_{x^4} = 0, \\ &U_{x^\beta} = \frac{\partial^\beta U}{\partial x^\beta}, \qquad \beta = \alpha, 2, 3, 4, \end{aligned} \tag{2}$$

where $a(t), b(t)$ and $c(t)$ are integrable or bounded measurable functions on $\mathbb{R}_+$, "$\diamond$" is the Wick product on the Kondratiev distribution space$(\mathcal{S})_{-1}$ which was defined in [13] and $A(t), B(t)$ and $C(t)$ are white noise functionals. Eqn.(2) can be seen as the perturbation of the coefficients $a(t), b(t)$ and $c(t)$ of Eqn.(1) by white noise functionals.

Our main interest in this work is in implementing new strategies that give White noise functional solutions of the Wick-type two-dimensional stochastic fractional KdV-Burgers-Kuramoto equations. The strategies that will be pursued in this work are based mainly on Homotopy analysis method and Hermite transform, both of which are employed to find White noise functional solutions of Eqn. (2). The proposed schemes, as we believe, are entirely new and introduce new solutions in addition to the well-known traditional solutions. The ease of using these methods, to determine shock or solitary type of solutions, shows its power.

2. White Noise Functional Solutions of Eq. (2)

Taking the Hermite transform of Eqn. (2), we get the deterministic equation:

$$\begin{aligned} &\widetilde{U}_{t^\alpha}(x,t,z) + \widetilde{U}(x,t,z)\widetilde{U}_x(x,t,z) + \widetilde{A}(t,z)\widetilde{U}_{x^2}(x,t,z) + \\ &\quad + \widetilde{B}(t,z)\widetilde{U}_{x^3}(x,t,z) + \widetilde{C}(t,z)\widetilde{U}_{x^4}(x,t,z) = 0, \end{aligned} \tag{3}$$

where $z = (z_1, z_2, ...) \in (\mathbb{C}^{\mathbb{N}})_c$ is a vector parameter. For the sake of simplicity we denote $A(t,z) = \widetilde{A}(t,z)$, $B(t,z) = \widetilde{B}(t,z)$, $C(t,z) = \widetilde{C}(t,z)$ and $u(x,t,z) = \widetilde{U}(x,t,z)$. Using the homotopy analysis method (HAM) developed for integer-order differential equation, we can find out the solution of Eqn. (3). First, we will introduce some notions of the fractional calculus and then proceed with the homotopy analysis method.

Definition 1. A real function $h(t)$, such that $h(t) = t^p h_1(t), t > 0$, is said to be in the space C_μ, $\mu \in \mathbb{R}$ if there exists a real number $p > \mu$, where $h_1(t) \in C[0,\infty)$, and it is said to be in the space C_μ^n if and only if $h^{(n)} \in C_\mu$, $n \in \mathbb{N}$.

Definition 2. The Riemann-Liouville fractional integral operator J^α of order $\alpha \geq 0$, of a function $h \in C_\mu$, $\mu \geq -1$, is defined as

$$\begin{aligned} J^\alpha h(t) &= \frac{1}{\Gamma(\alpha)} \int_0^t (t-\tau)^{\alpha-1} h(\tau) d\tau \quad (\alpha > 0), \\ J^0 h(t) &= h(t), \end{aligned} \tag{4}$$

$\Gamma(\alpha)$ is the well-known Gamma function. Some of the basic properties of the operator J^α, which we need here, are as follows:

(1) $J^\alpha J^\beta h(t) = J^{\alpha+\beta} h(t)$,

(2) $J^\alpha J^\beta h(t) = J^\beta J^\alpha h(t)$,

(3) $J^\alpha t^\gamma = \frac{\Gamma(\gamma+1)}{\Gamma(\alpha+\gamma+1)} t^{\alpha+\gamma}$, where $\beta \geqslant 0$, and $\gamma \geqslant -1$.

Definition 3. The fractional derivative D^α of $h(t)$ in the Caputo sense is defined as

$$\begin{aligned} D^\alpha h(t) &= \frac{1}{\Gamma(n-\alpha)} \int_0^t (t-\tau)^{n-\alpha-1} h^{(n)}(\tau) d\tau \quad , \\ \text{for } n-1 &< \alpha \leq n, \quad n \in \mathbb{N}\ , t > 0, \quad h \in C_{-1}^n. \end{aligned} \tag{5}$$

We mention the following two basic properties of the Caputo fractional derivative, see [17]:

(1) Let $h \in C_{-1}^n$, $n \in \mathbb{N}$. Then $D^\alpha h$, $0 \leq \alpha \leq n$ is well defined and $D^\alpha h \in C_{-1}$.

(2) Let $n-1<\alpha\leq n$, $n\in\mathbb{N}$ and $h\in C_\mu^n$, $\mu\geq -1$. Then

$$(J^\alpha D^\alpha)h(t)=h(t)-\sum_{k=0}^{n-1}h^{(k)}(0^+)\frac{t^k}{k!}. \tag{6}$$

Consider the fractional differential equation in the following general form

$$\mathcal{N}(u(x,t,z))=0, \tag{7}$$

where $\mathcal{N}$ is a fractional differential operator, x and t denote independent variables, $u(x,t,z)$ is an unknown function. For simplicity, we ignore all boundary or initial conditions, which can be treated in the same way. Based on the constructed zero-order deformation equation by Liao [15], we give the following zero-order deformation equation in the similar way,

$$(1-q)\mathcal{L}(\phi(x,t,z;q)-u_0(x,t,z))=qh\mathcal{N}[\phi(x,t,z;q)], \tag{8}$$

where $q\in[0,1]$ is the embedding parameter, h is a non zero auxiliary parameter, $\mathcal{L}$ is an auxiliary linear integer-order operator and it possesses the property $\mathcal{L}(C)=0, u_0(x,t,z)$ is an initial guess of $u(x,t,z), U(x,t,z;q)$ is a unknown function on independent variables x,t,z,q. It is important that one has great freedom to choose auxiliary parameter h in HAM. If $q=0$ and $q=1$, it holds

$$\phi(x,t,z;0)=u_0(x,t,z),\quad \phi(x,t,z,1)=u(x,t,z). \tag{9}$$

Thus as q increases from 0 to 1, the solution $\phi(x,t,z;q)$ varies from the initial guess $u_0(x,t,z)$ to the solution $u(x,t,z)$. Expanding $\phi(x,t,z;q)$ in Taylor series with respect to q, one has

$$\phi(x,t,z;q)=u_0(x,t,z)+\sum_{m=1}^{\infty}u_m(x,t,z)q^m, \tag{10}$$

where

$$u_m(x,t,z)=\frac{1}{m!}\frac{\partial^m\phi(x,t,z;q)}{\partial q^m}\mid_{q=0}. \tag{11}$$

If the auxiliary linear integer-order operator, the initial guess, and the auxiliary parameter h are so properly chosen, the series (10) converges at $q=1$, one has

$$u(x,t,z)=u_0(x,t,z)+\sum_{m=1}^{\infty}u_m(x,t,z). \tag{12}$$

According to (11), the governing equation can be deduced from the zero-order deformation equation (8). Define the vector

$$\vec{u}(x,t,z) = \{u_0(x,t,z), u_1(x,t,z), u_2(x,t,z), \ldots.., u_n(x,t,z)\}. \tag{13}$$

Differentiating Eq. (8) m times with respect to the embedding parameter q and then setting $q = 0$ and finally dividing them by $m!$, we have the so-called mth-order deformation equation

$$\mathcal{L}(u_m(x,t,z) - \varkappa_m u_{m-1}(x,t,z)) = \frac{h}{(m-1)!}\frac{\partial^{m-1}[\phi_1(x,t,z;q), \cdots, \phi_n(x,t,z;q)]}{\partial q^{m-1}}\Big|_{q=0}, \tag{14}$$

where

$$\varkappa_m = \begin{cases} 0 & m \leq 1, \\ 1 & m > 1. \end{cases} \tag{15}$$

The mth-order deformation equation (14) is linear and thus can be easily solved, especially by means of symbolic computation software such as MATHEMATICA, MAPLE, MATHLAB and so on. To demonstrate the effectiveness of the method, we consider Eqn. (1) with the following initial condition

$$u(x,0) = k.\exp\{x\}, \quad k \in \mathbb{C}. \tag{16}$$

We choose the linear integer-order operator

$$\mathcal{L}[U(x,t,z;q)] = \frac{\partial U(x,t,z;q)}{\partial t}. \tag{17}$$

Furthermore, Eq. (3) suggests to define the nonlinear fractional differential operator

$$\mathcal{N}[U(x,t,z;q)] = \frac{\partial^\alpha U(x,t,z;q)}{\partial t^\alpha} + U(x,t,z;q)\frac{\partial U(x,t,z;q)}{\partial x} + A(t)\frac{\partial^2 U(x,t,z;q)}{\partial x^2} + B(t)\frac{\partial^3 U(x,t,z;q)}{\partial x^3} + C(t)\frac{\partial^4 U(x,t,z;q)}{\partial x^4}. \tag{18}$$

Using the above definition, we construct the zeroth-order deformation equation

$$(1-q)\mathcal{L}(U(x,t,z;q) - u_0(x,t,z)) = qh\mathcal{N}[U(x,t,z;q)]. \tag{19}$$

Obviously, when $q = 0$ and $q = 1$,

$$U(x,t,z;0) = u_0(x,t,z) \quad U(x,t,z;1) = u(x,t,z). \tag{20}$$

According to (14)-(15), we gain the mth-order deformation equation

$$\begin{aligned}&\mathcal{L}(u_m(x,t,z)-\varkappa_m u_{m-1}(x,t,z))\\ =&h\{\frac{\partial^\alpha u_{m-1}(x,t,z)}{\partial t^\alpha}+\sum_{i=0}^{m-1}u_i(x,t,z)\frac{\partial u_{m-1-i}(x,t,z)}{\partial x}\\ &+A(t)\frac{\partial^2 u_{m-1}(x,t,z)}{\partial x^2}+B(t)\frac{\partial^3 u_{m-1}(x,t,z)}{\partial x^3}+C(t)\frac{\partial^4 u_{m-1}(x,t,z)}{\partial x^4}\}.\end{aligned} \tag{21}$$

Now, the solution of Eq. (21) for $m \geqslant 1$ becomes

$$\begin{aligned}u_m(x,t,z)=&\varkappa_m u_{m-1}(x,t,z)+h\mathcal{L}^{-1}\{\frac{\partial^\alpha u_{m-1}(x,t,z)}{\partial t^\alpha}\\ &+\sum_{i=0}^{m-1}u_i(x,t,z)\frac{\partial u_{m-1-i}(x,t,z)}{\partial x}+A(t)\frac{\partial^2 u_{m-1}(x,t,z)}{\partial x^2}\\ &+B(t)\frac{\partial^3 u_{m-1}(x,t,z)}{\partial x^3}+C(t)\frac{\partial^4 u_{m-1}(x,t,z)}{\partial x^4}\}.\end{aligned} \tag{22}$$

From (16), (20) and (22), we now successively obtain:

$$u_0=u(x,0,z)=k.\exp\{\mu x+z\},\quad k,\mu\in\mathbb{C},$$

$$u_1=h\mathcal{L}^{-1}\{\frac{\partial^\alpha u_0}{\partial t^\alpha}+u_0\frac{\partial u_0}{\partial x}+A(t)\frac{\partial^2 u_0}{\partial x^2}+B(t)\frac{\partial^3 u_0}{\partial x^3}+C(t)\frac{\partial^4 u_0}{\partial x^4}\},$$

$$u_1=\frac{\mu h k^2 t^\alpha}{\Gamma(\alpha+1)}.\exp\{2(\mu x+z)\}+\mu^2 hkS_1(t).\exp\{(\mu x+z)\},$$

$$\begin{aligned}u_2=u_1+h\mathcal{L}^{-1}\{&\frac{\partial^\alpha u_1}{\partial t^\alpha}+u_0\frac{\partial u_1}{\partial x}+u_1\frac{\partial u_0}{\partial x}+A(t)\frac{\partial^2 u_1}{\partial x^2}\\ &+B(t)\frac{\partial^3 u_1}{\partial x^3}+C(t)\frac{\partial^4 u_1}{\partial x^4}\},\end{aligned}$$

$$\begin{aligned}u_2=&(1+h)u_1+\frac{3\mu^2h^2k^3t^{2\alpha}}{\Gamma(2\alpha+1)}.\exp\{3(\mu x+z)\}\\ &+[2\mu^3h^2k^2J^\alpha S_1(t)+4\mu^3h^2k^2J^\alpha\{t^\alpha S_2(t)\}].\exp\{2(\mu x+z)\}\\ &+\mu^4h^2kJ^\alpha\{S_1(t)S_1(t)\}.\exp\{(\mu x+z)\},\end{aligned}$$

$$\cdots$$

where $S_i(t)=A(t)+(i\mu)B(t)+(i\mu)^2C(t),\quad i=1,2,\cdots.$

Special Case

If

$$A(t) = -\mu^2 C(t); \quad B(t) = 0; \quad h = -1;$$

this implies that

$$\begin{cases} u_0 = k.\exp\{\mu x + z\}. \\ u_1 = \frac{\mu h k^2 t^\alpha}{\Gamma(\alpha+1)}.\exp\{2(\mu x + z)\} \\ u_2 = \frac{3\mu^2 h^2 k^3 t^{2\alpha}}{\Gamma(2\alpha+1)}.\exp\{3(\mu x + z)\} \\ \qquad + \frac{12\mu^3 h^2 k^2}{\Gamma(\alpha+1)} J^\alpha\{t^\alpha C(t)\}.\exp\{2(\mu x + z)\} \\ . \quad . \quad . \\ u_n = S_n \mu^n h^n k^{n+1} t^n.\exp\{[n+1](\mu x + z)\}, \end{cases} \tag{23}$$

where S_n is given by

$$S_n = (S_{n-1} + 1) \int t^{n-1} D_x \exp\{nx\} dt|_{t=1,x=0}; \quad S_1 = 1; \quad n \geqslant 1$$

So, the solution of Eqn. (3) can be written in the form

$$\widetilde{U}(x,t,z) = \sum_{m=0}^{\infty} S_m \mu^m h^n k^{m+1} t^m.\exp\{[m+1](\mu x + z)\}. \tag{24}$$

Obviously, the solution given by Eqn. (24) belongs to the infinite class of solutions for the deterministic Eqn. (3), each solution belongs to this class can be reached by supposing some initial condition for $u_0(x,t)$ for Eqn. (1) and then by following the above steps for this initial condition we get another solution for Eqn. (3), and so on.

In order to get exact solutions of Eqn. (2), we will assume the following condition:

(A):

Suppose that $A(t), B(t)$ and $C(t)$ satisfy that there exist a bounded open set $G \subset \mathbb{R} \times \mathbb{R}_+$, $m < \infty, n > 0$ such that $u(x,t,z)$, $u_t(x,t,z)$, $u_x(x,t,z)$, $u_{xx}(x,t,z)$, $u_{xxx}(x,t,z)$ and $u_{xxxx}(x,t,z)$ are (uniformly) bounded for $(x,t,z) \in G \times K_m(n)$, continuous with respect to$(x,t) \in G$ for all$z \in K_m(n)$ and analytic with respect to $z \in K_m(n)$, for all$(x,t) \in G$, [9, 10, 11, 12, 16].

Under condition (A), Theorem 2.1 of Xie [25] implies that there exists $U(x,t) \in (\mathcal{S})_{-1}$ such that $u(x,t,z) = \widetilde{U}(x,t)(z)$ for all $(x,t,z) \in G \times K_m(n)$

and that $U(x,t)$ solves (2). From the above, we have that $U(x,t)$ is the inverse Hermite transformation of $u(x,t,z)$. Hence, Eqn. (24) yields stochastic single solitary solutions of Eqn. (2) as the following form:

$$U(x,t) = \sum_{m=0}^{\infty} S_m \mu^m h^n k^{m+1} t^{\diamond m} \diamond \exp^{\diamond}\{[m+1](\mu x + z)\}. \tag{25}$$

Clearly, applying the ratio test implies that the above summations are convergent.

3. Example

Since the Wick versions of functions are usually difficult to evaluate, we will give some non-Wick versions of the solutions of Eqn. (2) in special case:

$$\begin{cases} A(t) = f(t) + \delta_1 W(t), \\ B(t) = g(t) + \delta_2 W(t), \\ C(t) = h(t) + \delta_3 W(t), \end{cases} \tag{26}$$

with $f(t), g(t)$ and $h(t)$ being integrable or bounded measurable functions on $\mathbb{R}_+$ and $\delta_i (i = 1,2,3)$ being constants, where $W(t)$ is Gaussian white noise, i.e., $W(t) = \dot{B}(t)$, $B(t)$ is a Brown motion. We have the Hermite transforms: $A(t,z) = f(t) + \delta_1 \widetilde{W}(t,z), B(t,z) = g(t) + \delta_2 \widetilde{W}(t,z)$ and $C(t,z) = h(t) + \delta_3 \widetilde{W}(t,z)$, where $\widetilde{W}(t,z) = \sum_{i=1}^{\infty} z_i \int_0^t \eta_i(s)ds$, $\eta_i(t)$ is defined in the second section of [25]. In this case, we obtain the solution of Eqn. (2.1) as follows:

$$u(x,t,z) = \sum_{m=0}^{\infty} S_m \mu^m h^n k^{m+1} t^m . \exp\{\psi_m(x,t,z)\} \tag{27}$$

$$\psi_m(x,t,z) = m\alpha^* x + \beta^* z - 2\gamma_2 \int_0^t [g(s) + \delta_2 \widetilde{W}(s,z)]ds + x_0 + z_0,$$

where α^* and β^* are arbitrary constants. From (27) and the definition of $\widetilde{W}(t,z)$, it is easy to prove that the condition (A) is tenable for $A(t), B(t)$ and $C(t)$ in case (26). Hence, Eqn. (27) yields that the exact solution of Eqn. (2) as follows:

$$U(x,t) = \sum_{m=0}^{\infty} S_m \mu^m h^n k^{m+1} t^{\diamond m} \diamond \exp^{\diamond}\{\phi_m(x,t)\} \tag{28}$$

with

$$\phi_m(x,t) = m\alpha^* x - 2\gamma_2\left\{\int_0^t g(s)ds + \delta_1 B(t)\right\} + x_0.$$

In terms of the equality $\exp^\diamond\{B(t)\} = \exp\{B(t) - t^2/2\}$ [9], form (28), we have

$$U(x,t) = \sum_{m=0}^{\infty} S_m \mu^m h^n k^{m+1}(t + 0.5t^2)^m . \exp\{\phi_m(x,t)\} \tag{29}$$

with

$$\phi_m(x,t) = m\alpha^* x - 2\gamma_2\left\{\int_0^t g(s)ds + \delta_1(B(t) - t^2/2)\right\} + x_0.$$

4. Summary and Discussion

In general, the solution of SPDE will be a stochastic distribution, and we have to interpret possible products that occur in the equation, as one cannot in general take the product of two distributions, in our paper, products are considered to be Wick products which overcome this difficulty through white noise functional approach. Subsequently, we take the Hermite transform of the resulting equation and obtain an equation that we try to solve, where the random variables have been replaced by complex-valued functions of infinitely many complex variables. Finally, we use the inverse Hermite transform to obtain a solution of the regularized, original equation [8]. Since $\Phi^\diamond(x) = \Phi(x)$ for any non-random function $\Phi(x)$, hence (17) are solutions of the variable coefficients fractional KdV-Burgers-Kuramoto equation (1), where $a(t), b(t)$ and $c(t)$ are bounded measurable or integrable functions on $\mathbb{R}_+$. And noting that there exists a unitary mapping between the Wiener white noise space and the Poisson white noise space, we can obtain the solution of the Poisson SPDE simply by applying this mapping to the solution of the corresponding Gaussian SPDE. A nice and concise account of this connection was given by Benth and Gjerde [1]. We can see it in ([21], Section 4.9) as well. Hence, we can attain stochastic soliton solutions as we do in Section 2 if the coefficients $A(t), B(t)$ and $C(t)$ are Poisson white noise functions in Eqn. (2).

References

[1] E. Benth and J. Gjerde, A remark on the equivalence between Poisson and Gaussian stochastic partial differential equations, *Potential Analysis*, **8** (1998), 179-193.

[2] A. de Bouard and A. Debussche, A white noise driven Korteweg-de Vries equation, *J. Funct. Anal.*, **169** (1999), 532-558.

[3] A. de Bouard and A. Debussche, On the Korteweg-de Vries equation, *J. Funct. Anal.*, **154** (1998), 215-251.

[4] B. Chen and Y.C. Xie, Exact solutions for generalized stochastic Wick-type KdV-mKdV equations, *Chaos, Solitons* & *Fractals*, **23** (2005), 281-287.

[5] B. Chen and Y.C. Xie, Exact solutions for wick-type stochastic coupled Kadomtsev-Petviashili equations, *J. Phys. A*, **38** (2005), 815-822.

[6] A. Debussche J. Printems, Effect of localized random forcing term on the Korteweg-de Vries equation, *Comput. Anal. Appl.*, **3** (2001), 183-206.

[7] A. Debussche and J. Printems, Numerical simulation of the stochastic Korteweg-de Vries equation, *Physica D: Nonlinear Phenomena*, **134** (1999), 200-226.

[8] A. Hossam Ghany, Exact solutions for stochastic generalized Hirota-Satsuma coupled KdV equations, *Chinese Journal of Physics*, **49** (2011), 926-940.

[9] A. Hossam Ghany and A. Hyder, The fractional coupled KdV equations: exact solutions and white noise functional approach, *Chinese Physics B*, Accepted.

[10] A. Hossam Ghany and A. Hyder, Local and global well-posedness of stochastic Zakharov-Kuznetsov equation, *J. Computational Analysis and Applications*, **15** (2013), 1332-1343.

[11] A. Hossam Ghany and A. Fathallah, Exact solutions for KdV Burger equations with application of white noise applications, *International Journal of Pure and Applied Mathematics*, **78** (2012), 17-27.

[12] A. Hossam Ghany and M. S. Mohammed, White noise functional solutions for the Wick-type stochastic fractional KdV-Burgers-Kuramoto equations, *Chinese Journal of Physics*, **50** (2012), 618-626.

[13] H. Hölden, B. Øsendal, J. Ubøe. and T. Zhang, *Stochastic Partial Differential Equations*, Springer Science+Business Media, LLC (2010).

[14] V. V. Konotop and L. Vazquez, *Nonlinear Random Waves*, World Scientific (1994).

[15] S. J. Liao, *Beyond Perturbation: Introduction to the Homotopy Analysis Method*, Chapman & Hall/CRC Press, Boca Raton (2003).

[16] A. S. Okb El Bab, H. A. Ghany, Harmonic analysis on hypergroups, *Intern. J. of Pure and Appl. Mathematics*, **64** (2010), 9-19.

[17] I. Podlubny, *Fractional Differential Equations*, Academic Press, San Diego (1999).

[18] Y. Ugurlu and D. Kaya, Analytic method for solitary solutions of some partial differential equations, *Phys. Lett. A*, **370** (2007), 251-259.

[19] M. Wadati, Deformation of solitons in random media, *J. Phys. Soc. Jpn.*, **59** (1990), 4201-4203.

[20] M. Wadati, Stochastic Korteweg de Vries equation, *J. Phys. Soc. Jpn.*, **52** (1983), 2642-2648.

[21] M. Wadati and Y. Akutsu, Stochastic Korteweg de Vries equation, *J. Phys. Soc. Japan*, **53** (1984), 3342-3350.

[22] Y. C. Xie, Chaos, Exact solutions of the Wick-type stochastic Kadomtsev-Peviashvili equations, *Solitons* & *Fractals* **21** (2004), 473-480.

[23] Y. C. Xie, Chaos, Positonic solutions for Wick-type stochastic KdV equations, *Solitons & Fractals*, **20** (2004), 337-342.

[24] Y. C. Xie, An auto-Backlund transformation and exact solutions for Wick type stochastic generalized KdV equations, *J. Phys. A: Math. Gen.* **37** (2004), 5229-5236.

[25] Y. C. Xie, Exact solutions for stochastic KdV equations, *Phys. Lett. A*, **310** (2003), 161-167.