International Journal of Applied Mathematics
Volume 25 No. 2 2012, 207-218

TWO POINT TAYLOR SERIES FOR TWO POINT HIGHER-ORDER BOUNDARY VALUE PROBLEMS USING WEIGHTED RESIDUAL METHOD

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Abstract: We implement an efficient numerical method for solving higher order linear and non-linear two point boundary value problems, which is based on the weighted residual via the partition method. With this method, Two Point Taylor polynomial is used as a trial function to obtain the residual function. Comparisons are made with other methods and with the exact solution where it exists, to verify the reliability and accuracy of the method. Several examples are presented and it is observed that weighted residual method is more effective in each case.

AMS Subject Classification: 35Qxx
Key Words: higher order boundary value problems, Two Point Taylor polynomial, weighted residual method, partition method

1. Introduction

Many attempts have been made to solve boundary value problems that arise in models of mathematical physics and applied mathematics. Such set of problems also occurs in solid state physics, astrophysics, nuclear charge in heavy

Received: February 11, 2012

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atoms, thermal behaviour of a spherical cloud of gas, thermodynamics, population models, chemical kinetics and fluid mechanics. Researchers have used several methods to tackle this type of problems. These include Adomian decomposition methods, Variational decomposition method, Differential transform method and Modified Laplace decomposition method, and these can be found in [1,2,3,7] and other references therein.

In this article, the weighted residual method [5] is applied to solving fourth, fifth, and sixth order linear and non-linear two-point boundary value problems. Two-point Taylor polynomials [6] of order 6 and 8 are used as trial functions, which on substitution into the differential equation give the residual. The interval of each problem is subdivided into smaller intervals, and the residual is minimized over these smaller intervals.

2. Analysis of the Method

Definition 2.1 ([6]): Let a and b be two distinct points. If $f(x) \in C^{2m}[a,b]$, then $f(x)$ can be approximated by the polynomial $P_{2m-1}(x)$ given as

$$P_{2m-1}(x) = (x-a)^m \sum_{k=0}^{m-1} \frac{B_k(x-b)^k}{k!} + (x-b)^m \sum_{k=0}^{m-1} \frac{A_k(x-a)^k}{k!}, \tag{1}$$

where

$$A_k = \frac{d}{dx^k}\Big[\frac{f(x)}{(x-b)^m}\Big]_{x=a}, \quad B_k = \frac{d^k}{dx^k}\Big[\frac{f(x)}{(x-a)^m}\Big]_{x=b}, \tag{2}$$

and the remainder is given as

$$f(x) - P_{2m-1}(x) = \frac{f^{2m}(\theta)}{(2m)!}(x-a)^m(x-b)^m \tag{3}$$

with $a < \theta < b$.

Equation (1) is referred to as two-point Taylor polynomial approximation.

Representing the evaluated derivatives with constants and with $m = 6$ and $m = 8$, we have equations (4) and (5) respectively, as follows:

$$F_6 = A1(x-a)^6 + A2(x-a)^6(b-x) + A3(x-a)^6(b-x)^2 + A4(x-a)^6(b-x)^3$$
$$+A5(x-a)^6(b-x)^4 + A6(x-a)^6(b-x)^5 + A7(x-a)(b-x)^6 + A8(x-a)^2(b-x)^6$$
$$+A9(x-a)^3(b-x)^6 + A10(x-a)^4(b-x)^6 + A11(x-a)^5(b-x)^6 + A12(b-x)^6, \tag{4}$$

and

$$F_8 = A1(x-a)^8 + A2(x-a)^8(b-x) + A3(x-a)^8(b-x)^2 + A4(x-a)^8(b-x)^3$$
$$+A5(x-a)^8(b-x)^4 + A6(x-a)^8(b-x)^5 + A7(x-a)^8(b-x)^6 + A8(x-a)^8(b-x)^7$$
$$+A9(x-a)(b-x)^8 + A10(x-a)^2(b-x)^8 + A11(x-a)^3(b-x)^8 + A12(x-a)^4(b-x)^8$$
$$+A13(x-a)^5(b-x)^8 + A14(x-a)^6(b-x)^8 + A15(x-a)^7(b-x)^8 + A16(b-x)^8, \tag{5}$$

where $Ai's$ and the $Bi's$ are constants to be determined.

Weighted residual method: Suppose we have a differential equation

$$L[u(x)] = f \quad \text{in} \quad \text{the} \quad \text{domain} \quad \Omega \tag{6}$$

$$B_\mu[u] = \Omega \quad \text{on} \quad \partial\Omega, \tag{7}$$

where $L[u]$ denotes a general differential operator (linear or non-linear) involving spatial derivatives of dependent variable u, f is a known function of position, $B_\mu[u]$ represents the appropriate number of boundary conditions and Ω is the domain with the boundary $\partial\Omega$.

Equation (1) is forced to satisfy the boundary conditions which gives sets of equations, the residual obtained by substituting equation (1) into the original differential equation is then minimized by integrating (The Simpson $\frac{1}{3}$ rule) the subdivided domain within the subdivision points.

We then obtain sets of equations through:

- Forcing the Taylor polynomial to satisfy the boundary conditions
- Forcing the residual function to be zero at both end points
- Integrating the residual function within subintervals and equating the results to zero.

These equalities are then solved simultaneously to obtain the constants and consequently the solution.

3. Numerical Examples

Example 1. (see [2])

$$F^{iv} = F(x) + F''(x) + e^x(x-3), \tag{8}$$

subject to the boundary conditions

$$F(0) = 1, \quad F'(0) = 0, \quad F(1) = 0, \quad F'(1) = -e.$$

The exact solution is

$$F(x) = (1-x)e^x.$$

Using equation (4) or (5) as the trial function, partitioning the domain within $[0,1]$ into equal number of subintervals and following the procedure discussed in Section 2, we have

$$\begin{aligned} F_6 = {} & 2.71828182845905x^6(1-x) + 13.5914091422362x^6(1-x)^2 \\ & + 42.133368340772x^6(1-x)^3 + 102.841662514335x^6(1-x)^4 \\ & + 216.216667086844x^6(1-x)^5 + 5.99999999999999x(1-x)^6 \\ & + 20.4999999999904x^2(1-x)^6 + 52.6666666666264x^3(1-x)^6 \\ & + 113.375000003840x^4(1-x)^6 + 216.216666650445x^5(1-x)^6 + 1.0(1-x)^6. \end{aligned}$$

Table 1 shows the comparison of the absolute error between the exact values and the computed values of each method for Example 1. WRMTPT6 and WRMTPT8 are the results obtained by the WRM using Two-point Taylor series 6 and 8 respectively.

Example 2. (see [2])

$$F^{iv}(x) = F(x) - 8x\cos(x) - 12\sin(x), \tag{9}$$

subject to the boundary conditions

$$F(-1) = 0, \quad F'(-1) = 2\sin(1), \quad F(1) = 0, \quad F'(1) = 2\sin(1).$$

The exact solution is

$$F(x) = (x^2-1)\sin(x).$$

Using equation (4) or (5) as the trial function, partitioning the domain within $[-1,1]$ into equal number of subintervals and following the procedure discussed in Section 2, we have

x	WRMTPT6	RK4	RK4Butcher	DTM
0.0	0	0	0	0
0.1	$3.28 * 10^{-13}$	$3.576 * 10^{-7}$	$4.351 * 10^{-6}$	$1.788138 * 10^{-7}$
0.2	$6.13 * 10^{-13}$	$7.748 * 10^{-7}$	$6.967 * 10^{-5}$	$4.172325 * 10^{-7}$
0.3	$8.00 * 10^{-13}$	$1.251 * 10^{-6}$	$3.592 * 10^{-4}$	$1.132488 * 10^{-6}$
0.4	$9.33 * 10^{-13}$	$1.907 * 10^{-6}$	$1.158 * 10^{-3}$	$1.847744 * 10^{-6}$
0.5	$9.75 * 10^{-13}$	$2.682 * 10^{-6}$	$2.890 * 10^{-3}$	$2.384186 * 10^{-6}$
0.6	$9.52 * 10^{-13}$	$3.635 * 10^{-6}$	$6.126 * 10^{-3}$	$3.159046 * 10^{-6}$
0.7	$8.34 * 10^{-13}$	$4.827 * 10^{-6}$	$1.160 * 10^{-2}$	$3.516674 * 10^{-6}$
0.8	$6.51 * 10^{-13}$	$6.258 * 10^{-6}$	$2.026 * 10^{-2}$	$3.457069 * 10^{-6}$
0.9	$3.51 * 10^{-13}$	$8.016 * 10^{-6}$	$3.322 * 10^{-2}$	$2.846122 * 10^{-6}$
1.0	0	0	0	0

Table 1

$$
\begin{gathered}
F_6 = -0.0262959682752467(x+1)^6(1-x) - 0.0488554791783125(x+1)^6(1-x)^2 \\
-0.0432507927467939(x+1)^6(1-x)^3 - 0.0216723182467204(x+1)^6(1-x)^4 \\
-0.00000134124365696353(x+1)^6(1-x)^5 + 0.0262959682752467(x+1)(1-x)^6 \\
+0.0488554364597430(x+1)^2(1-x)^6 + 0.0432506478613727(x+1)^3(1-x)^6 \\
+0.0216725191821991(x+1)^4(1-x)^6 + 0.00000117075190483(x+1)^5(1-x)^6
\end{gathered}
$$

$$
\begin{gathered}
F_8 = -0.00657399206881168(x+1)^8(1-x) - 0.0018787860476 2474(x+1)^8(1-x)^2 \\
-0.0279570572711995(x+1)^8(1-x)^3 - 0.0286781668772835(x+1)^8(1-x)^4 \\
-0.0216892316667008(x+1)^8(1-x)^5 + 0.0108446218582043(x+1)^8(1-x)^6 \\
-0.7919635610 * 10^{-10}(x+1)^8(1-x)^7 + 0.00657399206881168(x+1)(1-x)^8 \\
-0.0187878604762474(x+1)^2(1-x)^8 + 0.0279570572711996(x+1)^3(1-x)^8 \\
+0.0286781668772885(x+1)^4(1-x)^8 + 0.0108446218582244(x+1)^6(1-x)^8 \\
+0.791874 * 10^{-10}(x+1)^7(1-x)^8.
\end{gathered}
$$

Table 2 shows the comparison of the absolute error between the exact values and the computed values of each method for Example 2. WRMTPT6 and WRMTPT8 are the results obtained by the WRM using Two-point Taylor series 6 and 8, respectively.

x	WRMTPT8	WRMTPT6	RK4	RK4Butcher	DTM
0.1	$5 * 10^{-16}$	$3.120 * 10^{-10}$	$1.7 * 10^{-6}$	$9.611 * 10^{-7}$	$5.66 * 10^{-4}$
0.2	0.0	$6.031 * 10^{-10}$	$3.4 * 10^{-6}$	$8.493 * 10^{-7}$	$1.06 * 10^{-3}$
0.3	0.0	$8.502 * 10^{-10}$	$5.1 * 10^{-6}$	$6.854 * 10^{-7}$	$1.48 * 10^{-3}$
0.4	$6 * 10^{-15}$	$1.035 * 10^{-9}$	$6.6 * 10^{-6}$	$4.172 * 10^{-7}$	$1.76 * 10^{-3}$
0.5	$6 * 10^{-15}$	$1.152 * 10^{-9}$	$8.0 * 10^{-6}$	$8.940 * 10^{-8}$	$1.88 * 10^{-3}$
0.6	$5 * 10^{-15}$	$1.192 * 10^{-9}$	$9.2 * 10^{-6}$	$2.980 * 10^{-7}$	$1.60 * 10^{-3}$
0.7	$5 * 10^{-15}$	$1.105 * 10^{-9}$	$1.0 * 10^{-5}$	$7.748 * 10^{-7}$	$1.21 * 10^{-3}$
0.8	$6 * 10^{-15}$	$8.003 * 10^{-10}$	$1.0 * 10^{-5}$	$1.341 * 10^{-6}$	$7.75 * 10^{-4}$
0.9	$2 * 10^{-15}$	$3.011 * 10^{-10}$	$1.1 * 10^{-5}$	$2.011 * 10^{-6}$	$2.5 * 10^{-4}$
1.0	0	0	0	0	0

Table 2

Example 3. (see [1])

$$F^{iv} + 4F(x) = 1, \tag{10}$$

subject to the conditions

$$F(-1) = F(1) = 0, \quad F'(-1) = -F'(1) = \frac{\sinh(2) - \sin(2)}{4(\cosh(2) - \cos(2))}.$$

The exact solution is

$$F(x) = 0.25(1 - 2\frac{(\sin(1)\sinh(1)\sin(x)\sinh(x) + \cos(1)\cosh(1)\cos(x)\cosh(x))}{(\cos(2) + \cosh(2))})$$

Using equation (4) or (5) as the trial function, partitioning the domain within $[-1, 1]$ into equal number of subintervals and following the procedure discussed in Section 2, we have

$$F_6 = 0.00317254196101244(x+1)^6(1-x) + 0.0095176262687379(x+1)^6(1-x)^2$$

$$+0.0148906434416637(x+1)^6(1-x)^3 + 0.0175632190857846(x+1)^6(1-x)^4$$

$$+0.0175638412117282(x+1)^6(1-x)^5 + 0.00317254196101244(x+1)(1-x)^6$$

$$+0.00951762626873787(x+1)^2(1-x)^6 + 0.0148906434416637(x+1)^3(1-x)^6$$

$$+0.0175632190857847(x+1)^4(1-x)^6 + 0.0175638412117280(x+1)^5(1-x)^6$$

$$F_8 = -0.000793135490253109(x+1)^8(1-x) + 0.00317254196100911(x+1)^8(1-x)^2$$

x	WRMPTP8	WRMPTP6	RK4	RK4Butcher	DTM
0.1	$3.9 * 10^{-14}$	$1.568 * 10^{-9}$	$7.450581 * 10^{-9}$	$2.980232 * 10^{-8}$	$2.06 * 10^{-4}$
0.2	$3.6 * 10^{-14}$	$1.534 * 10^{-9}$	$1.490116 * 10^{-8}$	$6.705523 * 10^{-8}$	$2.14 * 10^{-4}$
0.3	$3.5 * 10^{-14}$	$1.462 * 10^{-9}$	$5.960464 * 10^{-8}$	$1.043081 * 10^{-7}$	$2.28 * 10^{-4}$
0.4	$3.5 * 10^{-14}$	$1.349 * 10^{-9}$	$1.192093 * 10^{-7}$	$1.192093 * 10^{-7}$	$2.47 * 10^{-4}$
0.5	$2.99 * 10^{-14}$	$1.214 * 10^{-9}$	$2.011657 * 10^{-7}$	$1.490116 * 10^{-7}$	$2.70 * 10^{-4}$
0.6	$2.262 * 10^{-14}$	$1.069 * 10^{-9}$	$2.831221 * 10^{-7}$	$1.713634 * 10^{-7}$	$2.92 * 10^{-4}$
0.7	$2.10 * 10^{-14}$	$8.817 * 10^{-10}$	$3.762543 * 10^{-7}$	$1.937151 * 10^{-7}$	$3.07 * 10^{-4}$
0.8	$1.51 * 10^{-14}$	$5.873 * 10^{-10}$	$4.731119 * 10^{-7}$	$2.123415 * 10^{-7}$	$2.93 * 10^{-4}$
0.9	$7.0 * 10^{-15}$	$2.084 * 10^{-10}$	$5.550683 * 10^{-7}$	$2.253801 * 10^{-7}$	$2.14 * 10^{-4}$
1.0	0	0	0	0	0

Table 3

$$+0.00669691865944694(x+1)^8(1-x)^3 + 0.0102945897593486(x+1)^8(1-x)^4$$

$$+0.0130113165273924(x+1)^8(1-x)^5 + 0.0143696844442951(x+1)^8(1-x)^6$$

$$+0.0143696843388379(x+1)^8(1-x)^7 + 0.000793135490253109(x+1)(1-x)^8$$

$$+0.00317254196100912(x+1)^2(1-x)^8 + 0.00669691865944683(x+1)^3(1-x)^8$$

$$+0.0102945897593499(x+1)^4(1-x)^8 + 0.0130113165273886(x+1)^5(1-x)^8$$

$$+0.0143696844443006(x+1)^6(1-x)^8 + 0.0143696843388338(x+1)^7(1-x)^8.$$

Table 3 shows the comparison of the absolute error between the exact values and the computed values of each method for Example 3. WRMTPT6 and WRMTPT8 are the results obtained by the WRM using Two-point Taylor series 6 and 8, respectively.

Example 4. (see [3])

$$32F^v(x) = e^{-x}F(x)^3, \tag{11}$$

subject to the conditions

$$F(0) = 1,\;\; F'(0)\frac{1}{2},\;\; F''(0) = \frac{1}{4},\;\; F(1) = e^{\frac{1}{2}},\;\; F'(1) = \frac{1}{2}e^{\frac{1}{2}}.$$

The exact solution is

$$F(x) = e^{\frac{x}{2}}.$$

x	Exact	WRMPTP6	DTM($N = 14$)	ADM($N = 14$)
0.0	1.0	1.0	1.0	1.0
0.1	1.05127109637602	1.05127109637602	1.051278920	1.051304392
0.2	1.10517091807565	1.10517091807565	1.105220776	1.105376925
0.3	1.16183424272828	1.16183424272828	1.161964144	1.162354804
0.4	1.22140275816017	1.22140275816017	1.221630872	1.22288426
0.5	1.28402541668774	1.28402541668774	1.284337420	1.285197759
0.6	1.34985880757600	1.34985880757601	1.284337420	1.351122659
0.7	1.41906754859326	1.41906754859326	1.284337420	1.420166655
0.8	1.49182469764127	1.49182469764127	1.284337420	1.492533672
0.9	1.56831218549017	1.56831218549017	1.284337420	1.568557204
1.0	1.64872127070013	1.64872127070013	1.284337420	1.648721285

Table 4

Using equation (4) or (5) as the trial function, partitioning the domain within $[0, 1]$ into equal number of subintervals and following the procedure discussed in Section 2, we have

$$\begin{aligned} F_6 = {} & 1.64872127070013x^6 + 9.06796698885072x^6(1-x) \\ & + 29.8830730314399x^6(1-x)^2 + 76.2190104100750x^6(1-x)^3 \\ & + 165.700781250341x^6(1-x)^4 + 322.453385416652x^6(1-x)^5 \\ & + 6.5x(1-x)^6 + 24.125x^2(1-x)^6 + 67.2708333333338x^3(1-x)^6 \\ & + 156.752604166667x^4(1-x)^6 + 322.453385416671x^5(1-x)^6 + 1.0(1-x)^6. \end{aligned}$$

Table 4 shows the comparison in the solution of weighted residual method with other methods for example 4.

Example 5. (see [1])

$$F^{vi}(x) = e^{-x}F(x)^2, \tag{12}$$

subject to the conditions

$$F(0) = 1, \quad F''(0) = 1, \quad F^{iv}(0) = 1, \quad F(1) = e, \quad F^{iv}(1) = e.$$

Using equation (4) or (5) as the trial function, partitioning the domain within $[0, 1]$ into equal number of subintervals and following the procedure discussed in Section 2, we have

x	WRMPTP8	WRMPTP6	VDM	HPM	ADM
0	0.	0	0	0	0
0.1	$2.1 * 10^{-13}$	$3.845 * 10^{-11}$	$1.233 * 10^{-4}$	$1.33 * 10^{-4}$	$1.33 * 10^{-4}$
0.2	$4.1 * 10^{-13}$	$7.282 * 10^{-11}$	$2.354 * 10^{-4}$	$2.354 * 10^{-4}$	$2.354 * 10^{-4}$
0.3	$5.7 * 10^{-13}$	$9.978 * 10^{-11}$	$3.257 * 10^{-4}$	$3.257 * 10^{-4}$	$3.257 * 10^{-4}$
0.4	$6.6 * 10^{-13}$	$1.169 * 10^{-10}$	$3.855 * 10^{-4}$	$3.855 * 10^{-4}$	$3.855 * 10^{-4}$
0.5	$6.9 * 10^{-13}$	$1.229 * 10^{-10}$	$4.086 * 10^{-4}$	$4.086 * 10^{-4}$	$4.086 * 10^{-4}$
0.6	$6.6 * 10^{-13}$	$1.172 * 10^{-10}$	$3.919 * 10^{-4}$	$3.919 * 10^{-4}$	$3.919 * 10^{-4}$
0.7	$5.7 * 10^{-13}$	$1.001 * 10^{-10}$	$3.361 * 10^{-4}$	$3.361 * 10^{-4}$	$3.361 * 10^{-4}$
0.8	$4.1 * 10^{-13}$	$7.324 * 10^{-11}$	$2.459 * 10^{-4}$	$2.459 * 10^{-4}$	$2.459 * 10^{-4}$
0.9	$2.3 * 10^{-13}$	$3.871 * 10^{-11}$	$1.299 * 10^{-4}$	$1.299 * 10^{-4}$	$1.299 * 10^{-4}$
1.0	0	0	0	0	0

Table 5

$$\begin{aligned}F_6 = {} & 2.71828182845905x^6 + 13.5914091419008x^6(1-x) \\ & + 42.1333683387487x^6(1-x)^2 + 102.841662502483x^6(1-x)^3 \\ & + 216.216667087648x^6(1-x)^4 + 409.758333257274x^6(1-x)^5 \\ & + 6.99999999960847x(1-x)^6 + 27.4999999976508x^2(1-x)^6 \\ & + 80.1666666591541x^3(1-x)^6 + 193.54166668998x^4(1-x)^6 \\ & + 409.758333298866x^5(1-x)^6 + 1.0(1-x)^6.\end{aligned}$$

Table 5 shows the comparison of the absolute error between the exact values and the computed values of each method for Example 5. WRMTPT6 and WRMTPT8 are the results obtained by the WRM using Two-point Taylor series 6 and 8, respectively.

Example 6. (see [4])

$$F^{iv}(x) = F(x)^2 + 1, \quad 0 < x < 2, \tag{13}$$

subject to the boundary conditions

$$F(0) = F'(0) = F(2) = F'(2) = 0.$$

x	WRMPTP6	VIM	EADM	TRAP
0	0.	0	0	0
0.1	0.00150565470199870	0.001514138	0.001505652	0.00150566
0.2	0.00540548149359746	0.005437536	0.005405472	0.00540548
0.3	0.0108487758087503	0.010916675	0.108487561	0.01084878
0.4	0.0170848365359378	0.017198041	0.017084801	0.01708483
0.5	0.0234629742386607	0.023628128	0.023462922	0.02346295
0.6	0.0294325290495738	0.029653459	0.029432459	0.02943250
0.7	0.0345428972315356	0.034820601	0.034542813	0.03454287
0.8	0.0384435627776815	0.038776177	0.038443468	0.03844355
0.9	0.0408841291894837	0.041266870	0.040884029	0.04088413
1.0	0.0417143464742697	0.042139406	0.041714244	0.04171435

Table 6

Using equation (4) or (5) as the trial function, partitioning the domain within $[0, 2]$ into equal number of subintervals and following the procedure discussed in Section 2, we have

$$F = 0.00260667580139449x^6(2-x)^2 + 0.00521401926906739x^6(2-x)^3$$

$$+0.0065180652186395x^6(2-x)^4 + 0.00651841294803352x^6(2-x)^5$$

$$+0.00260667580139444x^2(2-x)^6 + 0.00521401926906736x^3(2-x)^6$$

$$+0.00651806521863958x^4(2-x)^6 + 0.00651841294803341x^5(2-x)^6.$$

Table 6 shows the comparison of weighted residual method (WRM) with other methods, though the there is no exact solution of the problem but for the weighted residual method, the residual to be minimized to zero is plotted in Fig. 1 to show the efficiency of the method.

4. Conclusion

In this paper, we have used two-point Taylor polynomials of order six and eight as trial functions in the weighted residual via partition method to solve two-point boundary value problems of higher order. The computed results were compared with other methods referenced, to the exact solution where they are available. Where they are not available, a plot of the residual function is

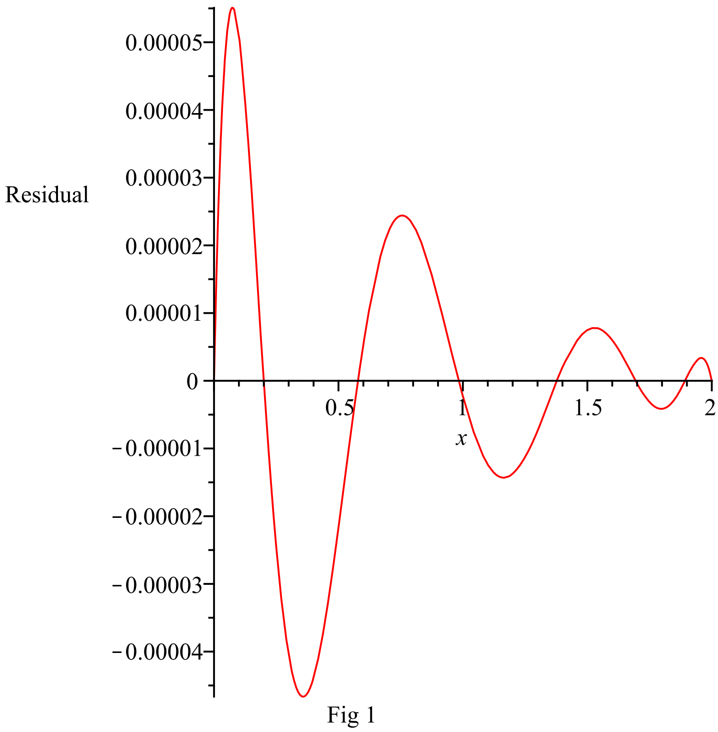

Figure 1

presented to show the extent to which the residual function is minimized to very close to zero.

From the numerical examples provided, the weighted residual method (WRM) seems to be more accurate than other methods such as Adomian decomposition method (ADM), Homotopy pertubation method (HPM), Differential transform method (DTM), Runge Kutta method (RK4) and Runge Kutta-Butcher method (RK-Butcher). This is evident from the error of the computational results obtained. The WRM is more accurate and efficient as displayed in the Tables, for two-point boundary value problems more so when the two-p0int Taylor series are used for the trial functions. The method of weighted residual has the sole advantage presenting the solution in polynomial forms.

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