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SOLUTION AND STABILITY OF n-DIMENSIONAL ADDITIVE FUNCTIONAL EQUATION

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Abstract: In this paper, the authors established the solution and generalized Ulam - Hyers stability of n-dimensional additive functional equation

$$\sum_{i=1}^{n} f\left(\sum_{j=1}^{n} x_{ij}\right) = (n-2)\sum_{j=1}^{n} f\left(x_j\right)$$

in C^*-algebra, where

$$x_{ij} = \begin{cases} -x_j & if \quad i = j, \\ x_j & if \quad i \neq j, \end{cases}$$

and n is a positive integer.

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1. Introduction

The stability of functional equations had been first raised by S.M. Ulam [27] for what metric group G is it true that a $\varepsilon-$automorphism of G is necessarily near to a strict automorphism?.

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In 1941, D. H. Hyers [11] gave a positive answer to the question of Ulam for Banach spaces. In 1950, T. Aoki [2] was the second author to treat this problem for additive mappings. Th.M. Rassias [25] succeeded in extending the result of Hyers' Theorem by weakening the condition for the Cauchy difference controlled by $(||x||^p + ||y||^p)$, $p \in [0, 1)$ to be unbounded. Taking into consideration a lot of influence of Ulam, Hyers and Rassias on the development of stability problems of functional equations, the stability phenomenon that was proved by Th.M. Rassias is called Hyers-Ulam-Rassias stability one can refer [1, 8, 12, 15].

In 1982, J.M. Rassias [23] followed the innovative approach of the Th.M. Rassias theorem [25] in which he replaced the factor $||x||^p + ||y||^p$ by $||x||^p||y||^q$ for $p, q \in R$ with $p+q = 1$. A generalization of all the above results was obtained by P. Gavruta [9] in 1994 by replacing the unbounded Cauchy difference by a general control function $\phi(x, y)$ in the spirit of Rassias approach.

In 2008, a special case of Gavruta's theorem for the unbounded Cauchy difference was obtained by Ravi et al., [26] by considering the summation of both the sum and the product of two $p-$ norms in the sprit of Rassias approach. The stability problems of several functional equations have been extensively investigated by a number of authors and there are many interesting results concerning this problem (see [3, 22, 26]).

C. Park [18] applied Gavruta's result to Banach modules over a C^*-algebra. Many authors have studied the structure of C^*- algebras for different types of functional equations in various settings one can refer [5, 7, 19]. It seems that approximate derivations was first investigated by K.W. Jun and D.W. Park [14]. Recently, the stability of derivations have been investigated in [6, 10, 16, 20, 21] and references therein. Very recently the stability of cubic derivations was first time introduced and investigated by M.E. Gordji et al., [10].

The functional equation

$$f(x + y) = f(x) + f(y) \tag{1.1}$$

is called the additive functional equation and it is the most famous functional equation. Since $f(x) = kx$ is the solution of the functional equation (1.1), every solution of the additive equation is called an additive function.

The solution and stability of the following additive functional equations

$$f(x + ay) + af(x - y) = f(x - ay) + af(x + y), \quad a \neq -1, 0, 1, \tag{1.2}$$

$$f(2x - y) + f(x - 2y) = 3f(x) - 3f(y), \tag{1.3}$$

$$f(2x \pm y \pm z) = f(x \pm y) + f(x \pm z) \tag{1.4}$$

were studied by K.W. Jun and H.M. Kim [13], D.O. Lee [17], M. Arunkumar [3]. Recently the solution and generalized Ulam - Hyers - Rassias stability of the functional equation

$$f(x) = \sum_{\ell=1}^{n} \left(\frac{f(x + \ell y_\ell) + f(x - \ell y_\ell)}{2\ \ell} \right), \tag{1.5}$$

where n is a positive integer, which is originating from n consecutive terms of an arithmetic progression was investigated by M. Arunkumar and S. Karthikeyan [4].

In this paper, the authors introduce a new type of n-dimensional additive functional equation

$$\sum_{i=1}^{n} f\left(\sum_{j=1}^{n} x_{ij} \right) = (n-2) \sum_{j=1}^{n} f(x_j), \tag{1.6}$$

where

$$x_{ij} = \begin{cases} -x_j & if \quad i = j, \\ x_j & if \quad i \neq j, \end{cases}$$

and n is a positive integer.

In Section 2, the general solution of the functional equation (1.6) is given. In Section 3, the stability of homomorphisms of the additive functional equation (1.6) is present. The stability of deviations of the additive functional equation (1.6) is discussed in Section 4.

2. Solution of the Functional Equation (1.6)

In this section, the general solution of the functional equation (1.6) is given.

Theorem 2.1. *Let X and Y be real vector spaces. The mapping $f : X \to Y$ satisfies the functional equation (1.1) for all $x, y \in X$ if and only if $f : X \to Y$ satisfying the functional equation (1.6) for all $x_1, x_2, \ldots, x_n \in X$.*

Proof. The proof is omitted as trivial. □

Throughout this paper, let X and Y be normed algebra and Banach algebra, respectively. Define a mapping $F : X \to Y$ by

$$\begin{aligned} F(x_1, x_2, \ldots, x_n) = & f(-x_1 + x_2 + \ldots + x_n) + f(x_1 - x_2 + \ldots + x_n) + \ldots \\ & + f(x_1 + x_2 + \ldots - x_n) \\ & - (n-2)\left(f(x_1) + f(x_2) + \ldots + f(x_n)\right), \end{aligned}$$

for all $x_1, x_2, \cdots, x_n \in X$.

3. Stability of Homomorphisms of the Additive Functional Equation (1.6)

In this section, the authors discussed the generalized Ulam-Hyers stability of homomorphisms of the $n-$dimensional additive functional equation (1.6).

Definition 3.1. A $\mathbb{C}-$linear mapping $H : X \to X$ is called **Homomorphism** on X if H satisfies

$$H(x_1x_2\ldots x_n) = H(x_1)H(x_2)\cdots H(x_n)$$

for all $x_1, x_2, \cdots, x_n \in H$.

Theorem 3.2. *Let $j \in \{-1, 1\}$. Let $\alpha : X^n \to [0, \infty)$ be a function such that*

$$\sum_{i=0}^{\infty} \frac{\alpha\left((n-2)^{ij}x_1, (n-2)^{ij}x_2, , \cdots, (n-2)^{ij}x_n\right)}{(n-2)^{ij}} \quad converges$$

$$and \quad \lim_{i\to\infty} \frac{\alpha\left((n-2)^{ij}x_1, (n-2)^{ij}x_2, \cdots, (n-2)^{ij}x_n\right)}{(n-2)^{ij}} < \infty \tag{3.1}$$

for all $x_1, x_2, \cdots, x_n \in X$ and let $f : X \to Y$ be a function satisfying the inequality

$$\|F(x_1, x_2, \ldots, x_n)\| \leq \alpha\left(x_1, x_2, \cdots, x_n\right) \tag{3.2}$$

$$\|f(x_1x_2\ldots x_n) - f(x_1)f(x_2)\cdots f(x_n)\| \leq \alpha\left(x_1, x_2, \cdots, x_n\right) \tag{3.3}$$

for all $x_1, x_2, \cdots, x_n \in X$. Then there exists a unique homomorphism function $H : X \to Y$ such that

$$\|f(x) - H(x)\| \leq \sum_{i=\frac{1-j}{2}}^{\infty} \frac{\beta((n-2)^{ij}x)}{(n-2)^{ij}}, \tag{3.4}$$

where

$$\beta((n-2)^{ij}x) = \frac{1}{n(n-2)}\alpha((n-2)^{ij}x, (n-2)^{ij}x, \cdots, (n-2)^{ij}x) \tag{3.5}$$

for all $x \in X$. The mapping $H(x)$ is defined by

$$H(x) = \lim_{k \to \infty} \frac{f((n-2)^{kj}x)}{(n-2)^{kj}} \tag{3.6}$$

for all $x \in X$.

Proof. Assume $j = 1$. Replacing $(x_1, x_2, \cdots, x_n)$ by $(x, x, \cdots, x)$ in (3.2), we arrive

$$\|nf\left[(n-2)x\right] - n(n-2)f\left(x\right)\| \leq \alpha(x, x, \cdots, x) \tag{3.7}$$

for all $x \in X$. Hence from the above inequality, we have

$$\left\| f\left(x\right) - \frac{f\left((n-2)x\right)}{(n-2)} \right\| \leq \frac{1}{n(n-2)} \alpha(x, x, \cdots, x) \tag{3.8}$$

for all $x \in X$. Putting

$$\beta(x) = \frac{1}{n(n-2)} \alpha(x, x, \cdots, x)$$

in (3.8), we arrive

$$\left\| f\left(x\right) - \frac{f\left((n-2)x\right)}{(n-2)} \right\| \leq \beta(x) \tag{3.9}$$

for all $x \in X$. Now replacing x by $(n-2)x$ and dividing by $(n-2)$ in (3.9) and adding the resultant inequality with (3.9), we obtain

$$\left\| f(x) - \frac{f((n-2)^2 x)}{(n-2)^2} \right\| \leq \beta(x) + \frac{\beta((n-2)x)}{(n-2)} \tag{3.10}$$

for all $x \in X$. In general for any positive integer n , we get

$$\begin{aligned} \left\| f(x) - \frac{f((n-2)^k x)}{(n-2)^k} \right\| &\leq \sum_{i=0}^{k-1} \frac{\beta((n-2)^i x)}{(n-2)^i} \\ &\leq \sum_{i=0}^{\infty} \frac{\beta((n-2)^i x)}{(n-2)^i} \end{aligned} \tag{3.11}$$

for all $x \in X$. In order to prove the convergence of the sequence $\left\{\frac{f((n-2)^k x)}{(n-2)^k}\right\}$, replacing x by $(n-2)^m x$ and dividing by $(n-2)^m$ in (3.11), for any $m, k > 0$, we arrive

$$\begin{aligned} &\left\|\frac{f((n-2)^m x)}{(n-2)^m} - \frac{f((n-2)^{k+m} x)}{(n-2)^{(k+m)}}\right\| \\ &\leq \sum_{i=0}^{\infty} \frac{\beta((n-2)^{i+m} x)}{(n-2)^{i+m}} \\ &\to 0 \quad as \;\; m \to \infty \end{aligned} \tag{3.12}$$

for all $x \in X$. Hence the sequence $\left\{\frac{f((n-2)^k x)}{(n-2)^k}\right\}$ is a Cauchy sequence. Since Y is complete, there exists a mapping $H : X \to Y$ such that

$$H(x) = \lim_{k\to\infty} \frac{f((n-2)^k x)}{(n-2)^k}, \quad \forall \; x \in X.$$

Letting $k \to \infty$ in (3.11), we see that (3.4) holds for all $x \in X$. Now we need to prove that H satisfies (1.6), replacing $(x_1, x_2, \cdots, x_n)$ by $((n-2)^k x_1, (n-2)^k x_2, , \cdots, (n-2)^k x_n)$ and dividing by $(n-2)^k$ in (3.2), we arrive

$$\begin{aligned} &\frac{1}{(n-2)^k} \left\|F\left(\left((n-2)^k x_1\right), \left((n-2)^k x_2\right), \cdots, \left((n-2)^k x_n\right)\right)\right\| \\ &\qquad \leq \frac{1}{(n-2)^k} \alpha \left((n-2)^k x_1, (n-2)^k x_2, \cdots, (n-2)^k x_n\right) \end{aligned}$$

for all $x_1, x_2, \cdots, x_n \in X$. Letting k tends to infinity, we see that

$$\|H(x_1, x_2, \cdots, x_n)\| = 0.$$

Hence H satisfies (1.6) for all $x_1, x_2, \cdots, x_n \in X$. It follows form (3.3), we have

$$\begin{aligned} &\|H(x_1 x_2 \ldots x_n) - H(x_1)H(x_2)\cdots H(x_n)\| \\ &= \frac{1}{(n-2)^k} \Big\| f((n-2)^k x_1 (n-2)^k x_2 \ldots (n-2)^k x_n) \\ &\qquad - f((n-2)^k x_1) f((n-2)^k x_2) \cdots f((n-2)^k x_n) \Big\| \\ &\leq \frac{1}{(n-2)^k} \alpha \left((n-2)^k x_1, (n-2)^k x_2, \cdots, (n-2)^k x_n\right) \\ &\to 0 \quad as \;\; k \to \infty \end{aligned}$$

for all $x_1, x_2, \cdots, x_n \in X$. Hence

$$H(x_1 x_2 \ldots x_n) = H(x_1)H(x_2)\cdots H(x_n)$$

In order to prove that H is unique, we let $J(x)$ be another mapping satisfying (1.6) and (3.4). Then

$$\begin{aligned}
&\|H(x) - J(x)\| \\
&= \frac{1}{(n-2)^k}\left\|H((n-2)^k x) - J((n-2)^k x)\right\| \\
&\leq \frac{1}{(n-2)^k}\left\{\left\|H((n-2)^k x) - f((n-2)^k x)\right\| + \left\|f((n-2)^k x) - J((n-2)^k x)\right\|\right\} \\
&\leq 2\sum_{i=0}^{\infty}\frac{\beta((n-2)^{i+k}x)}{(n-2)^{(i+k)}} \\
&\to 0 \quad as \ \ k \to \infty
\end{aligned}$$

for all $x \in X$. Hence H is unique. Thus the mapping $H : X \to Y$ is a unique homomorphism mapping satisfying (3.4).

For $j = -1$, we can prove the similar type of stability result. This completes the proof of the theorem. $\square$

The following corollary is an immediate consequence of Theorem 3.2 concerning the stability of (1.6).

Corollary 3.3. *Let λ and s be nonnegative real numbers. If a function $f : X \to Y$ satisfies the inequality*

$$\|F(x_1, x_2, \ldots, x_n)\| \leq \begin{cases} \lambda, \\ \lambda\left\{\sum_{i=1}^{n}||x_i||^s\right\}, \\ \lambda\left\{\prod_{i=1}^{n}||x_i||^s\right\}, \\ \lambda\left\{\prod_{i=1}^{n}||x_i||^s + \sum_{i=1}^{n}||x_i||^{ns}\right\}, \end{cases} \tag{3.13}$$

$$\|f(x_1x_2\ldots x_n)-f(x_1)f(x_2)\cdots f(x_n)\|\leq\begin{cases}\lambda,\\ \lambda\left\{\sum_{i=1}^{n}||x_i||^s\right\},\\ \lambda\left\{\prod_{i=1}^{n}||x_i||^s\right\},\\ \lambda\left\{\prod_{i=1}^{n}||x_i||^s+\sum_{i=1}^{n}||x_i||^{ns}\right\},\end{cases}\tag{3.14}$$

for all $x_1,x_2,\cdots,x_n\in X$. *Then there exists a unique homomorphism function* $H:X\to Y$ *such that*

$$\|f(x)-H(x)\|\leq\begin{cases}\dfrac{\lambda}{n(n-3)},\\ \dfrac{||x||^s}{|(n-2)-(n-2)^s|}, & s<1 \quad or \quad s>1;\\ \dfrac{||x||^{ns}}{n|(n-2)-(n-2)^{ns}|}, & s<\frac{1}{n} \quad or \quad s>\frac{1}{n};\\ \dfrac{(n+1)||x||^{ns}}{n|(n-2)-(n-2)^{ns}|}, & s<\frac{1}{n} \quad or \quad s>\frac{1}{n};\end{cases}\tag{3.15}$$

for all $x\in X$.

4. Stability of Deviations of the Additive Functional Equation (1.6)

In this section, the authors discuss the generalized Ulam-Hyers stability of deviations of the $n-$dimensional additive functional equation (1.6).

Definition 4.1. A $\mathbb{C}-$linear mapping $D:X\to X$ is called **Deviation** on X if D satisfies

$$D(x_1x_2\ldots x_n)=(D(x_1)x_2\cdots x_n)+(x_1D(x_2)\cdots x_n)+\cdots+(x_1x_2\cdots D(x_n))$$

for all $x_1,x_2,x_3,\cdots,x_n\in D$.

Theorem 4.2. *Let* $j\in\{-1,1\}$. *Let* $\alpha:X^n\to[0,\infty)$ *be a function such* *that*

$$\sum_{i=0}^{\infty} \frac{\alpha\left((n-2)^{ij}x_1,(n-2)^{ij}x_2,,\cdots,(n-2)^{ij}x_n\right)}{(n-2)^{ij}} \quad converges$$

$$and \quad \lim_{i\to\infty} \frac{\alpha\left((n-2)^{ij}x_1,(n-2)^{ij}x_2,\cdots,(n-2)^{ij}x_n\right)}{(n-2)^{ij}} < \infty \tag{4.1}$$

for all $x_1, x_2, \cdots, x_n \in X$ *and let* $f : X \to Y$ *be a function satisfying the inequality*

$$\|F(x_1, x_2, \ldots, x_n)\| \leq \alpha\,(x_1, x_2, \cdots, x_n) \tag{4.2}$$

$$\begin{aligned}\|f(x_1x_2\ldots x_n) - (f(x_1)x_2\cdots x_n) - (x_1f(x_2)\cdots x_n) - \cdots - (x_1x_2\cdots f(x_n))\|\\ \leq \alpha\,(x_1, x_2, \cdots, x_n)\end{aligned} \tag{4.3}$$

for all $x_1, x_2, \cdots, x_n \in X$*. Then there exists a unique deviation function* $D : X \to Y$ *such that*

$$\|f(x) - D(x)\| \leq \sum_{i=\frac{1-j}{2}}^{\infty} \frac{\beta((n-2)^{ij}x)}{(n-2)^{ij}} \tag{4.4}$$

where $\beta((n-2)^{ij}x)$ *and* $D(x)$ *are respectively defined in (3.5) and (3.6) for all* $x \in X$*.*

Proof. The proof is similar to that of Theorem 3.2. It follows from (4.3), we have

$$\begin{aligned}&\|D(x_1x_2\ldots x_n) - (D(x_1)x_2\cdots x_n) + (x_1D(x_2)\cdots x_n) + \cdots + (x_1x_2\cdots D(x_n))\|\\
&\quad = \frac{1}{(n-2)^k}\Big\|f((n-2)^kx_1(n-2)^kx_2\ldots(n-2)^kx_n)\\
&\qquad - (f((n-2)^kx_1)(n-2)^kx_2\cdots(n-2)^kx_n)\\
&\qquad - ((n-2)^kx_1f((n-2)^kx_2)\cdots(n-2)^kx_n) - \cdots\\
&\qquad - ((n-2)^kx_1(n-2)^kx_2\cdots f((n-2)^kx_n))\Big\|\\
&\quad \leq \frac{1}{(n-2)^k}\alpha\Big((n-2)^kx_1,(n-2)^kx_2,\cdots,(n-2)^kx_n\Big)\\
&\quad \to 0 \quad as \;\; k\to\infty\end{aligned}$$

for all $x_1, x_2, \cdots, x_n \in X$. Hence

$$D(x_1x_2\ldots x_n) = (D(x_1)x_2\cdots x_n) + (x_1D(x_2)\cdots x_n) + \cdots + (x_1x_2\cdots D(x_n))$$

for all $x_1, x_2, \cdots, x_n \in X$. □

The following corollary is a immediate consequence of Theorem 4.2 concerning the stability of (1.6).

Corollary 4.3. *Let λ and s be nonnegative real numbers. If a function $f : X \to Y$ satisfies the inequality*

$$\|F(x_1, x_2, \ldots, x_n)\| \leq \begin{cases} \lambda, \\ \lambda \left\{ \sum_{i=1}^{n} ||x_i||^s \right\}, \\ \lambda \left\{ \prod_{i=1}^{n} ||x_i||^s \right\}, \\ \lambda \left\{ \prod_{i=1}^{n} ||x_i||^s + \sum_{i=1}^{n} ||x_i||^{ns} \right\}, \end{cases} \tag{4.5}$$

$$\|f(x_1 x_2 \ldots x_n) - (f(x_1) x_2 \cdots x_n) - (x_1 f(x_2) \cdots x_n) - \cdots - (x_1 x_2 \cdots f(x_n))\|$$
$$\leq \begin{cases} \lambda, \\ \lambda \left\{ \sum_{i=1}^{n} ||x_i||^s \right\}, \\ \lambda \left\{ \prod_{i=1}^{n} ||x_i||^s \right\}, \\ \lambda \left\{ \prod_{i=1}^{n} ||x_i||^s + \sum_{i=1}^{n} ||x_i||^{ns} \right\}, \end{cases} \tag{4.6}$$

for all $x_1, x_2, \cdots, x_n \in X$. Then there exists a unique deviation function $D : X \to Y$ such that

$$\|f(x) - D(x)\| \leq \begin{cases} \dfrac{\lambda}{n(n-3)}, & \\ \dfrac{||x||^s}{|(n-2) - (n-2)^s|}, & s < 1 \quad or \quad s > 1; \\ \dfrac{||x||^{ns}}{n|(n-2) - (n-2)^{ns}|}, & s < \frac{1}{n} \quad or \quad s > \frac{1}{n}; \\ \dfrac{(n+1)||x||^{ns}}{n|(n-2) - (n-2)^{ns}|}, & s < \frac{1}{n} \quad or \quad s > \frac{1}{n}; \end{cases} \tag{4.7}$$

for all $x \in X$.

References

[1] J. Aczel and J. Dhombres, *Functional Equations in Several Variables,* Cambridge Univ. Press (1989).

[2] T. Aoki, On the stability of the linear transformation in Banach spaces, *J. Math. Soc. Japan*, **2** (1950), 64-66.

[3] M. Arunkumar, Solution and Stability of Arun-Additive functional equations, *International Journal Mathematical Sciences and Engineering Applications*, **4**, No. 3 (2010), 33-46.

[4] M. Arunkumar, S. Karthikeyan, Solution and stability of a functional equation originating from n consecutive Terms of an arithmetic progression, *Adv. Theo. Appl. Math.*, Accepted.

[5] C. Baak, D. Boo, Th.M. Rassias, Generalized additive mapping in Banach modules and isomorphism between C^*-algebras, *J. Math. Anal. Appl.*, **314** (2006), 150-161.

[6] R. Badora, On approximate derivations, *Math. Inequal. Appl.*, **9**, No. 1 (2006), 167-173.

[7] L. Brown and G. Pedersen, C^*-algebras of real rank zero, *J. Funct. Analysis*, **99** (1991), 138-149.

[8] S. Czerwik, *Functional Equations and Inequalities in Several Variables*, World Scientific, River Edge, NJ (2002).

[9] P. Gavruta, A generalization of the Hyers-Ulam-Rassias stability of approximately additive mappings, *J. Math. Anal. Appl.*, **184** (1994), 431-436.

[10] M.E.Gordji, S. Kaboli Gharetapeh, M. B. Savadkouhi, M. Aghaei, T. Karimi, On Cubic Derivations, *Int. Journal of Math. Analysis*, **4**, No. 51 (2010), 2501-2514.

[11] D.H. Hyers, On the stability of the linear functional equation, *Proc.Nat. Acad.Sci.*, USA, **27** (1941), 222-224.

[12] D.H. Hyers, G. Isac, Th.M. Rassias, *Stability of Functional Equations in Several Variables*, Birkhauser, Basel (1998).

[13] K.W. Jun and H.M. Kim, On the Hyers-Ulam-Rassias stability of a generalized quadratic and additive type functional equation, *Bull. Korean Math. Soc.*, **42**, No. 1 (2005), 133-148.

[14] K. W. Jun and D. W. Park, Almost derivations on the Banach algebra $C^n[0,1]$, *Bull. Korean Math. Soc.*, **33**, No. 3 (1996), 359-366.

[15] S.M. Jung, *Hyers-Ulam-Rassias Stability of Functional Equations in Mathematical Analysis*, Hadronic Press, Palm Harbor (2001).

[16] Y.S. Jung, The Ulam-Gavruta-Rassias stability of module left derivations, *J. Math. Anal. Appl.*, doi: 10.1016/j.jmaa.2007.07.003, 1-9.

[17] D.O. Lee, Hyers-Ulam stability of an addtiive type functional equation, *J. Appl. Math. and Computing*, **13**, No-s: 1-2 (2003), 471-477.

[18] C. Park, On the stability of the linear mapping in Banach modules, *J. Math. Anal. Appl.*, **275** (2002), 711-720.

[19] C. Park, Linear functional equation in Banach modules over a C^*-algebra, *Acta Appl. Math.*, **77** (2003), 125-161.

[20] C. Park, Linear derivations on Banach algebras, *Nonlinear Funct. Anal. Appl.*, **9**, No. 3 (2004), 359-368.

[21] C. Park and J. Hou, Homomorphism and derivations in C^*-algebras, *Abstract Appl. Anal.* (2007), Art. Id 80630.

[22] C. Park and Th.M. Rassias, Fixed points and stability of the Cauchy functional equation, *Australian J. Math. Anal. Appl.*, **6**, No. 1 (2009), 1-9.

[23] J.M. Rassias, On approximately of approximately linear mappings by linear mappings, *J. Funct. Anal.*, USA, **46** (1982), 126-130.

[24] J.M. Rassias and M.J. Rassias, On the Ulam stability of Jensen and Jensen type mappings on the restricted domains, *J. Math. Anal. Appl.*, **281** (2003), 516-524.

[25] Th.M. Rassias, On the stability of the linear mapping in Banach spaces, *Proc. Amer. Math. Soc.*, **72** (1978), 297-300.

[26] K. Ravi, M. Arunkumar and J.M. Rassias, On the Ulam stability for the orthogonally general Euler-Lagrange type functional equation, *International Journal of Mathematical Sciences*, **3**, No. 8 (2008), 36-47.

[27] S.M. Ulam, *Problems in Modern Mathematics,* Science Editions, Wiley, New York (1964), Chapter VI, Some Questions in Analysis: 1, Stability.